In-Class Questions from A1142, Autumn 2021

In-Class Questions, 8/25/21

Write down two things that you know about black holes.

Write down two questions that you have about black holes.

In-Class Questions, 8/30/21

1. You drop a bowling ball from a great height (20,000 km, say) onto the surface of the Moon.
How fast is the bowling ball moving just before it hits the surface?
Explain your reasoning.

2. You drop a ping pong ball from a great height (20,000 km, say) onto the surface of the Moon.
How fast is the ping pong ball moving just before it hits the surface?
Explain your reasoning, relative to your answer to Question 1.

3. You drop a bowling ball from a great height (20,000 km, say) onto the surface of the Earth.
How fast is the bowling ball moving just before it hits the Earth's surface?
Explain your reasoning, relative to your previous answers.

In-Class Questions, 9/1/21

My introductory slides presented several examples of empirical evidence for the existence of black holes. Four of them were:

1. Disappearance of a luminous star in Hubble Space Telescope images of another galaxy.
2. Motion of stars near the center of the Milky Way over the course of about 20 years.
3. Detection of gravitational waves from a pair of merging black holes in a distant galaxy.
4. Image of hot gas around the supermassive black hole in the galaxy M87, showing the shadow cast by the black hole's event horizon.

Which of these four observations do you find the most striking, and why?

In-Class Questions, 9/8/21

The distance from the sun to the earth is 1 Astronomical Unit, or 1 AU. This statement is the definition of an AU. Observations show that 1 AU = 150 million km, but for this quiz you don't need to use that fact.

1. The velocity of the earth in its orbit is 30 km/sec.
What is the earth's orbital acceleration, in units of $(\text{km/sec})^2/\text{AU}$?
[Give your answer as a number.]

2. The distance from the sun to Jupiter is 5 AU.
The velocity of Jupiter in its orbit is 13.42 km/sec.
What is Jupiter's orbital acceleration, in units of $(\text{km/sec})^2/\text{AU}$.
[For reference, $13.42^2 = 180$]

3. The distance from the sun to Neptune is 30 AU.

The velocity of Neptune in its orbit is 5.48 km/sec.
What is Neptune's orbital acceleration, in units of $(\text{km/sec})^2/\text{AU}$.
[For reference, $5.48^2 = 30$]

4. What is the ratio of the earth's orbital acceleration to Jupiter's orbital acceleration?
What is the ratio of the earth's orbital acceleration to Neptune's orbital acceleration?
What do you notice about these ratios?
(Hint: also think about the distances from the sun to Jupiter and to Neptune.)

In-Class Questions, 9/13/21

These questions refer to diagrams of Sun-Jupiter-Saturn configurations shown in class.

1. Choose an answer and give a 1-sentence explanation
In Configuration 1:
A) Jupiter is speeding up and Saturn is slowing down
B) Jupiter is slowing down and Saturn is speeding up
C) Jupiter and Saturn are both speeding up
D) Jupiter and Saturn are both slowing down
E) Neither planet is changing speed

2. Choose one or more answers and give a 1-sentence explanation
In Configuration 2:
A) Jupiter is moving faster than its average orbital speed
B) Jupiter is moving slower than its average orbital speed
C) Saturn is moving faster than its average orbital speed
D) Saturn is moving slower than its average orbital speed
E) Both planets are moving at their average orbital speed

3. Choose an answer and give a 1-sentence explanation
A) Jupiter is speeding up and Saturn is slowing down
B) Jupiter is slowing down and Saturn is speeding up
C) Jupiter and Saturn are both speeding up
D) Jupiter and Saturn are both slowing down
E) Neither planet is changing speed

4. Choose an answer and give a 1-sentence explanation:
Saturns average orbital speed is about 10 km/sec.
The gravity of Jupiter probably changes Saturns orbital speed by roughly:
A) 1 km/sec
B) 10 meters/sec
C) 10 millimeters/sec
D) No change at all

In-Class Questions, 9/15/21

We have now completed our whirlwind overview of Newton's theory of motion and gravity and the empirical evidence that supports this theory. What did you learn that most surprised you? Answer in 2-3 sentences.

In-Class Questions, 9/20/21

1. You are standing on a train car moving at 50 miles per hour. You throw a ball forward (in the same direction that the train is moving) at 50 miles per hour. As seen by someone standing on the ground by the tracks, how fast is the ball moving?

2. You are standing on a train car moving at 50 miles per hour. You drop a ball from your hand. In your frame of reference, what is the path of the ball?

3. You are standing on a train car moving at 50 miles per hour. You drop a ball from your hand. In the frame of reference of someone standing on the ground by the tracks, what is the path of the ball? (Describe it in words, or give the mathematical term for this path if you know it.)

4. You are standing on a train car moving at 50 miles per hour. You throw a ball backward (opposite to the direction that the train is moving) at 50 miles per hour. As seen by someone standing on the ground by the tracks, what is the path of the ball?

5. You are standing on a train car moving at 0.5c, where c is the speed of light. You shine a flashlight forward (in the same direction that the train car is moving). In the frame of reference of someone standing on the ground by the tracks, what is the speed of the flashlight beam? What, if anything, is bothersome about your answer?

In-Class Questions 9/22/21

These questions were illustrated with diagrams drawn on the board.

1. A box of mass m with mirrored interior walls contains a mix of hot atoms and EM radiation.
Through a briefly opened shutter, the box emits an amount of EM radiation with energy E, upwards.
If the box is initially at rest, what is its velocity (direction and magnitude) after it emits the radiation?

2. Same setup as before, but this time the box emits EM radiation with energy E/2 upward and an equal amount of EM radiation with energy E/2 downward.
If the box is initially at rest, what is its velocity after emitting the radiation?

3. Same as Case 2, but this time the radiation is emitted at a forward angle, so that it travels up and to the right (from the top of the box) or down and to the right (from the bottom of the box).
What is the direction of motion of the box after it emits the radiation?

4. Return to Case 2, but now observe it from a reference frame that is moving backward (to the left) with speed v. In this reference frame, the box initially moves to the right with speed v, and the radiation travels at an angle to the right after it is emitted.
Apply your reasoning from Case 2. In the new reference frame, what is the velocity (direction and speed) of the box after it emits the radiation?
What is puzzling about this result?
What can you think of that could resolve this puzzle?

In-Class Questions, 9/27/21

These questions refer to the diagram shown in class.

To make numbers easier, we'll make our skateboarder's light-clock 3-feet tall and use the fact that the speed of light is very close to one foot per nano-second.

Our skateboarder is moving at 80% of the speed of light.

The distance from A to B is 4 feet, and the distance from B to C is 4 feet.

1. In the frame of the observer standing on the ground: As the skateboarder goes from A to C, how far does the light travel? How long does this take (in nano-seconds)?

2. In the frame of the skateboarder, how far does the light travel as she goes from A to C? How long does this take?

3. The skateboarder knows that she is moving at 80% of the speed of light relative to the observer on the ground, and to the chalk marks at positions A and C. Using your answer to Question 2: What does the skateboarder conclude about the distance between A and C?

4. According to Einstein's principle of relativity, which observer (man on the ground, woman on the skateboard) is correct about the time required to go from A to C and the distance between A and C?

In-Class Questions, 10/1/21

1. You are hunting a monkey with a high-powered rifle. You spy the monkey hanging from the branch of a tree 100 meters away. From experience, you know that the monkey will let go of the branch and drop to the ground as soon as he sees the muzzle flash from your rifle. Where should you aim in order to hit the monkey? (You can ignore the effects of air resistance.)

(A) You should aim slightly above the monkey's head

(B) You should aim straight at the monkey

(C) You should aim slightly below the monkey's feet

2. An object moving in a circle at constant speed

(A) is not accelerating

(B) is accelerating towards the center of the circle

(C) is accelerating away from the center of the circle

(D) is accelerating in the same direction as its velocity

(E) is accelerating in the direction opposite to its velocity

3. With very powerful rocket engines, you accelerate your spaceship to 99speed of light (relative to earth). You still have half your fuel left. Why won't firing your engines with this remaining fuel get you moving faster than the speed of light?

(A) time dilation means that the fuel comes out very slowly

(B) at this speed, normal (chemical) rocket fuel is ineffective, and only nuclear fuel (using $E = mc^2$) could accelerate you

(C) your inertial mass is much higher than when you started, so thrust produces less acceleration

(D) in your reference frame, you will be at rest, but in the earth's reference frame you will be moving faster than the speed of light after you use the rest of your fuel

4. If the sun suddenly collapsed to a black hole without changing its mass, what would happen to the period and radius of the earth's orbit?
(A) the period and radius would increase
(B) the period and radius would be unchanged
(C) the period and radius would decrease
(D) the radius would decrease, but the period would get longer
(E) the earth would fly off in a straight line

In-Class Questions, 10/6/21

1. Refer to the diagram on the blackboard, which shows a simplified version of a light ray passing a distance r from a mass M. We approximate this situation by imagining that the light is traveling across a box of length $2r$ and that the gravitational acceleration within this box is $g = GM/r^2$.

Applying the equivalence principle, what is the distance h that the light drops as it crosses the box? Give your answer as an equation for h in terms of G, M, and c.

For what value of r is $h = r$? Give your answer as an equation for r in terms of G, M, and c.

Where have we previously encountered this formula for r?

2. A body of mass m that drops a distance L in a gravitational field of acceleration g gains an amount of kinetic energy equal to mgL. We haven't shown this in class, but it isn't hard to do using our formulas $d = (1/2)at^2$ and $\mathrm{KE} = (1/2)mv^2$.

Consider a photon that is emitted with energy E_e from the top of a box and "drops" a distance L to the bottom of the box. In our discussion of gravitational redshift, we showed that the wavelength of the photon at the bottom of the box is $\lambda_o = \lambda_e(1 - gL/c^2)$. In concert with our formula for photon energy, $E = hc/\lambda$, this means that the energy of the photon at the bottom of the box is

$$E_o = E_e + (E_0/c^2)gL.$$

What is interesting about this result?

In-Class Questions, 10/18/21

What is the sun?

Describe the nature of the sun as succinctly and accurately as you can.

In-Class Questions, 10/22/21

For these questions, we will use the equation $\theta = L/D$ for the angle subtended by an object of length L at distance D.

We will use radians as our unit of angle to make the calculations simpler.

The smallest angle you can resolve by eye is about 10^{-3} radians.

The distance to the moon is about 400,000 km (4×10^5 km).

What is the diameter L of the smallest crater you could resolve by eye on the surface of the moon? Give your answer in km.

1. The smallest angle you can resolve with a ground-based telescope is about 10^{-5} radians.

What is the diameter L of the smallest crater you could resolve with a telescope on the surface of the moon? Give your answer in km.

2. A golf ball has a diameter of about 4 cm, or 0.04 meters.

What is the angle θ subtended by a golf ball at the distance of the moon?

Give your answer in radians, and remember to convert the distance to the moon from km to meters.

3. We think that there is a black hole of 4 million solar masses at the center of the Milky Way galaxy.

The diameter of its event horizon would be 24 million km, which is about 2.5×10^{-6} light years.

The distance to the center of the galaxy is about 25,000 light years (2.5×10^4 light years).

What is the angle θ subtended by the event horizon of the black hole at the center of the Milky Way?

Give your answer in radians.

In-Class Questions, 10/25/21

A star known as S2 orbits the black hole at the center of the Milky Way. Along with other stars, it has been monitored since the mid-1990s by two groups of scientists led by Andrea Ghez and Reinhard Genzel, who were awarded the 2020 Nobel Prize in Physics for their work.

The angular diameter of the semi-major axis of S2's orbit is 0.125 arc-seconds, equal to 6.25×10^{-7} radians.

The period of its orbit is 16 years.

For this problem, it is convenient to use AU as the unit of length. The distance to the Galactic center is about 1.6×10^9 AU.

1. Using $\theta = L/D$, what is the length of the semi-major axis of the orbit of S2, in AU?

2. Using $(M/M_\odot) = (r/1\,\mathrm{AU})^3 \times (1\,\mathrm{yr}/P)^2$, what mass do you infer for the black hole, in solar masses?

In-Class Questions, 10/29/21

1. Which of the following is true?

(a) At the center of the sun is a black hole with a radius of about 3 km.

(b) There is no black hole in the sun because most of its mass is outside 3 km.

(c) The central 3 km of the sun will become a black hole once the gas cools.

(d) The sun is powered by gravitational accretion onto a "neutron core" as originally proposed by Lev Landau and described in Thorne's book.

2. Which two of the following are true?

(a) We do not observe strong light bending by the sun because the closest a light ray can pass is much, much larger than R_{Sch}.

(b) If the sun collapsed to make a black hole, light rays could orbit around it in a circle.

(c) If the sun collapsed to make a black hole, the earth would spiral towards it and eventually be swallowed.

(d) If the sun collapsed to make a black hole, the earth's orbit would change in a way that made the advance of perihelion much larger (more than what we currently observe for Mercury)

3. The radius of the sun is about 700,000 km. Suppose that evil aliens managed to replace the sun with a black hole that has a Schwarzschild radius $R_{\mathrm{Sch}} = 700,000\,\mathrm{km}$. Which of the following would be true? Choose all that apply.

(a) The orbit of the earth would be unchanged, though we would get very cold.

(b) The earth would spiral in towards the black hole and eventually be swallowed by it.

(c) The earth could still orbit in a circle, but the orbital period would be shortened from one year to about 18 hours.

(d) Tides on the earth would be drastically larger, with waves much bigger than the largest tsunamis ever seen.

X-ray Iron Lines from a Black Hole Accretion Disk

For a non-spinning black hole that is being fed with gas from a companion, the inner edge of the accretion disk is at $R = 3R_{\rm Sch}$. Our Newtonian formula $v^2 = GM/R$ for the speed in a circular orbit is somewhat inaccurate this close to a black hole, but it's not too far off. As shown in class, it predicts (approximately) that at $R = 3R_{\rm Sch}$, $v/c = 0.4$.

We previously wrote the Doppler shift formula in terms of wavelength, but for our current purpose it is better to write in terms of energy:

$$E_o = \frac{E_e}{\left(1 + \frac{v}{c}\right)} \ .$$

Here E_e is the energy of a photon emitted by an atom near the black hole, and E_o is the energy that we observe for that photon when we detect it far from the black hole. Remember that for atoms moving *away* from you v/c is positive (so energy is reduced, redshift) and for atoms moving *towards* you v/c is negative (so energy is increased, blueshift).

Highly ionized iron atoms emit X-ray photons with an energy $E_e = 6.4\,\text{keV}$. (For our purposes, you just need to know that a keV is a unit of energy.) Suppose that we use an X-ray telescope to detect the iron emission from a black hole with an accretion disk.

1. Considering *just* the effects of Doppler shifts, what should be the energies of the highest energy photons that we detect?

(a) 12.8 keV

(b) 10.67 keV

(c) 7 keV

(d) 6.4 keV

(e) 4.6 keV

2. Considering *just* the effects of Doppler shifts, what should be the energies of the lowest energy photons that we detect?

(a) 10.67 keV

(b) 7 keV

(c) 6.4 keV

(d) 6 keV

(e) 4.6 keV

There is an additional effect we have to consider, namely gravitational redshift. For photons emitted at a distance R from a non-spinning black hole, we should multiply the energy we computed using the Doppler formula by another factor

$$f = \sqrt{1 - \frac{R_{\rm Sch}}{R}} \ .$$

For $R = 3R_{\rm Sch}$, this factor is $\sqrt{2/3} = 0.82$.

3. Considering *both* the effects of Doppler shifts and gravitational redshift, what should be the energies of the highest energy photons that we detect?

(a) 13.0 keV

(b) 10.67 keV

(c) 8.7 keV

(d) 6.4 keV

(e) 5.3 keV

4. Considering *both* the effects of Doppler shifts and gravitational redshift, what should be the energies of the lowest energy photons that we detect?

(a) 8.7 keV

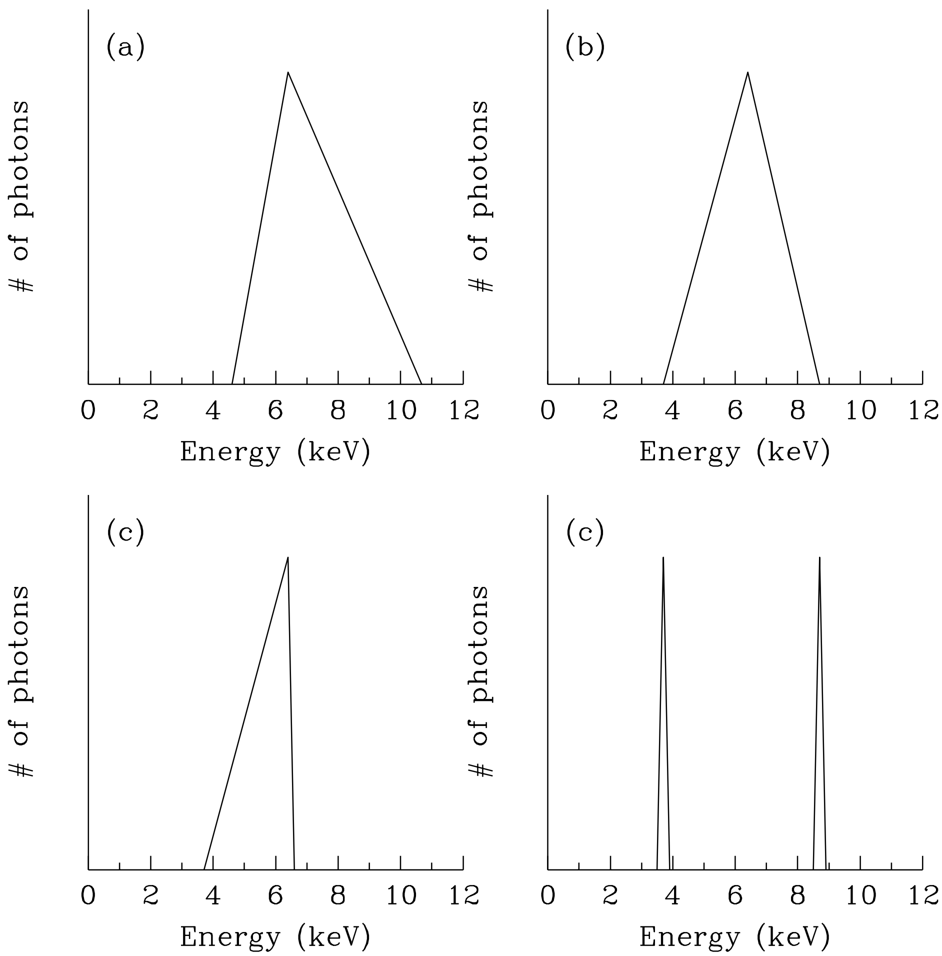
(b) 6.4 keV

(c) 5.3 keV

(d) 4.6 keV

(e) 3.7 keV

5. A plot of the distribution of photon energies from the accretion disk should most closely resemble which of the examples below?



In-Class Questions, 11/5/2021

In Chapters 2-8 of Black Holes & Time Warps we have encountered a number of scientists who made important contributions to the understanding of black holes, including: Albert Einstein, Karl Schwarzschild, Subrahmanyan Chandrasekhar, Arthur Eddington, Fritz Zwicky, Lev Landau, J. Robert Oppenheimer, John Wheeler, Yakov Zeldovich, Igor Novikov

For three of these scientists, write down (in one or two sentences) a surprising or memorable fact that you learned about them from your reading.

In-Class Questions, 11/8/2021

The black hole (BH) at the center of the Milky Way (MW) is approximately $4 \times 10^6 M_\odot$.

1. What is the Eddington luminosity limit for this BH, in solar luminosities?

2. Suppose that gas were accreting onto this BH through an accretion disk, and that the BH's spin is negligible, so that the luminosity is

$$L = \frac{1}{12}\dot{M}c^2 = 1.2 \times 10^{12} L_\odot \left(\frac{\dot{M}}{1.0 M_\odot \, \mathrm{yr}^{-1}} \right).$$

What is the maximum possible accretion rate $\dot{M}$, in $M_\odot \, \mathrm{yr}^{-1}$?

3. The Milky Way contains about 10^{11} stars. Suppose that the average luminosity of a star is equal to that of the Sun. If the MW black hole were accreting at the maximum allowed rate, would its luminosity be:

(a) much less than the combined luminosity of the MW's stars

(b) roughly equal to the combined luminosity of the MW's stars

(c) much greater than the combined luminosity of the MW's stars

In-Class Questions, 11/15/21

1. The argument that quasars are powered by supermassive black holes was based partly on their prodigious energy output and partly on the fact that they can vary substantially in brightness from one day to the next. Why was the rapid variability important to the argument?

(a) It showed that the energy must arise in thermonuclear explosions.

(b) It showed that the energy was being affected by gravitational time dilation.

(c) It showed that the energy was being produced in a small volume.

(d) The rapid variability could be naturally explained as an effect of black hole spin.

2. In the equation $L = (1/12)\dot{M}c^2$, the factor of 1/12 is calculated from

(a) The efficiency of nuclear fusion in producing energy from $E = mc^2$.

(b) The gravitational energy released as gas spirals in to the innermost stable circular orbit.

(c) The rate at which the black hole is gaining mass.

(d) The rate at which black hole spin adds energy to the accreting gas.

3. One of the key developments enabling the discovery of quasars in the 1960s was

(a) Fiber optics enabling simultaneous spectroscopic observations of many objects.

(b) Launch of the first X-ray satellites.

(c) Theoretical demonstration that the maximum mass of a neutron star is three solar masses.

(d) Improved angular resolution of radio telescopes.

4. In 1963, Maarten Schmidt recognized that spectral lines from the quasi-stellar radio source 3C 273 were shifted to longer (redder) wavelengths by 16%. What was the implication?

(a) The emitting gas must be orbiting a black hole at 16% of the speed of light.

(b) The emitted light must be gravitationally redshifted by 16%, which can only happen close to the event horizon of a black hole.

(c) The light is redshifted by the expansion of the universe, so the source must be very distant and therefore very luminous.

(d) The emitting gas is being ejected from the source at 16% of the speed of light.

In-Class Questions, 11/17/21

For this question we will use the (approximate) equations

$P = 10^{-3}(M/10M_\odot)\,\mathrm{s}$

$h = (1/5)(R_{\mathrm{Sch}}/D)$

and the equation for Schwarzschild radius

$R_{\mathrm{Sch}} = 3\,\mathrm{km}(M/M_\odot)$

Suppose that two million-solar mass black holes ($10^6 M_\odot$) merged at the center of the Milky Way.

1. Would the period of the gravitational waves emitted in the last stage of the merger be closest to

(a) 0.001 sec

(b) 1 sec

(c) 100 sec

(d) 1000 sec

2. The distance to the Galactic center is about 25,000 light years, which is about $2.4 \times 10^{17}\,\mathrm{km}$. What would be the strain h of the gravitational waves received at earth?

(a) $h = 1/5$

(b) $h = 2.5 \times 10^{-9}$

(c) $h = 2.5 \times 10^{-12}$

(d) $h = 2.5 \times 10^{-18}$

3. The diameter of the earth is (very) roughly 10,000 km, or 10^7 meters. As this gravitational wave passes, what would be the change in the diameter of the earth?

(a) No change

(b) 2.5 meters

(c) 2.5 centimeters (= 0.025 meters)

(d) 2.5 millimeters (= 0.0025 meters)

(e) 0.025 millimeters (= 2.5×10^{-5} meters)

In-Class Questions, 11/22/21

What is the most surprising thing that you learned in our discussion of (or reading about) gravitational waves?

In-Class Questions 12/01/21

As shown in class, the expected angular diameter of the event horizon for a 6 billion Msun black hole at the distance of Messier 87 is 0.7×10^{-10} radians. The choice of 6 billion solar masses is a reasonable estimate based on the motions of stars and gas in M87 and on trends in similar galaxies, but it has significant uncertainty.

1. As discussed in class, the smallest angular scales that can be resolved by a network of radio telescopes observing at wavelength λ with distances d between the telescopes is $\theta_{\rm min} = \lambda/d$ radians. The Event Horizon Telescope observes at wavelength of about 1 mm (10^{-3} m), and its telescopes are spread all over the globe. For an approximate value of d, we can take the diameter of the earth, which is about 10,000 km, or 10^7 m.
What is the smallest angular scale that can be resolved by the EHT?
(a) 10^{-11} radians
(b) 10^{-10} radians
(c) 10^{-9} radians
(d) 10^{-7} radians

2. Calculations based on GR predict that the "photon ring" surrounding the event horizon in a black hole image should have a diameter that is about five times larger than the diameter of the event horizon itself. What is the predicted size of the photon ring for the M87 black hole?
(a) 3.5×10^{-9} radians
(b) 3.5×10^{-10} radians
(c) 0.7×10^{-10} radians
(d) 0.14×10^{-10} radians
(e) 3.5×10^{-11} radians

3. Suppose that the EHT observed a bright ring with diameter of 1.75×10^{-10} radians in M87, surrounding a dark region. Which of these would be the most plausible interpretation of that observation?
(a) General Relativity is not correctly predicting the structure of the black hole image
(b) The EHT is not correctly measuring the image of the black hole
(c) The mass of the black hole in M87 is 3 billion solar masses not 6 billion solar masses
(d) The ring being observed by the EHT is not the photon ring but something else, maybe the inner edge of the accretion disk

Hawking radiation and black hole evaporation (12/6/21)

Stephen Hawking showed that a black hole of mass M would emit radiation with wavelength $\lambda \approx R_{\rm Sch}$. Recall that $R_{\rm Sch} = 2GM/c^2$ and that the energy of a photon of wavelength λ is $E = hc/\lambda$ where h is Planck's constant.

1. Which of these is a correct (approximate) expression for the typical energy of photons emitted by a black hole of mass M?
(a) $E = 2GMh/c$
(b) $E = hc^3/2GM$
(c) $E = hc^2/2GM$
(d) $E = 2GM/hc^3$

2. The "light crossing time" of a black hole is $t = R_{\rm Sch}/c = 2GM/c^3$. Roughly speaking, in Hawking radiation a black hole emits one photon per crossing time. Which of these is a correct (approximate) expression for the *Hawking luminosity* of a black hole, i.e., the rate at which it emits energy by Hawking radiation?
(a) $L = hc^6/(2GM)^2$
(b) $L = h$
(c) $L = hc^6/(2GM)$
(d) $L = hc^3/(2GM)$

3. Given a long enough time, a black hole could radiate its entire mass-energy Mc^2 by Hawking radiation. Which of these is a correct (approximate) expression for the time required for a black hole to completely evaporate by Hawking radiation?
(a) $t_{\rm evap} = Mc^2/h$
(b) $t_{\rm evap} = hc^4/(4G^2M^3)$
(c) $t_{\rm evap} = 4G^2M^3/hc^4$
(d) $t_{\rm evap} = 4G^2M^3/hc^6$

4. The correct answer to # 3 is (c). If you look up and plug in the values of G, h, and c, you find that for a black hole of mass $M = M_\odot = 2 \times 10^{30}$ kg the evaporation time is 10^{63} years. Suppose that nature somehow made black holes with a mass $M = 2 \times 10^{12}$ kg $= 10^{-18} M_\odot$. What would the evaporation time for such black holes be?
(a) $t_{\rm evap} = 10^{45}$ yr
(b) $t_{\rm evap} = 10^{12}$ yr
(c) $t_{\rm evap} = 10^{9}$ yr
(d) $t_{\rm evap} = 1000$ yr

For reference, 10^{12} kg is roughly the mass of a mountain.