6. General Relativity

Parachute story: "It doesn't hurt when you fall, only when you land."
Monkey-and-hunter puzzle.
See *Thorne Fig. 2.2.*

Weightlessness

FIGURE 3: ALBERT FALLS FROM THE ROOF

In 1907, while preparing a review article on relativity, Einstein has "der glücklichste Gedanke meines Lebens," "the happiest thought of my life."
Sounds good.
The thought: "If a person falls freely he will not feel his own weight."
At greater length: "For an observer falling freely from the roof of a house there exists — at least in his immediate surroundings — no gravitational field."

What does this mean?
Since all objects fall at the same rate in a gravitational field, they have no relative acceleration.
This phenomenon of weightlessness is easier to understand now, since we have seen films of, e.g., astronauts in spaceships, or been on free-fall amusement park rides. It must have required quite a breakthrough to see it in 1907.
Einstein: Free-fall is the "natural" state, being held up by the earth is the unnatural one. Analogous to Galileo figuring out inertia.

The equivalence principle

FIGURE 4: STEEL BOX TOWED BY ROCKET SHIP

I'm in a big, windowless steel box, on the surface of the earth.
I drop a bowling ball and a baseball simultaneously. What happens?
Both accelerate to floor, with acceleration

$$g_E = \frac{GM_E}{R_E^2} = 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}.$$

Now the box and I are taken to outer space, far from any planets or stars, and set adrift in some direction, moving at constant velocity.
I drop the balls. What happens? They float. (And I need magnetic boots to stay attached to the floor.)

Now the top of the box is attached to a rocket ship, which *accelerates* at g.
I drop the balls. What happens? They accelerate to the floor at g.
What do I feel? "Gravitational" pull towards the floor.
If $g = g_E$, how does this compare to being on the ground? Same.

Based on this reasoning, Einstein introduced
The Equivalence Principle: A reference frame accelerating at g is exactly equivalent to a static reference frame in a uniform gravitational field of acceleration g.

This principle is demonstrably true in Newtonian mechanics.

Einstein conjectured that it holds true for all physical processes, including light propagation. (Analogous to what he did for the principle of relativity, which states the equivalence of all inertial reference frames.)

Since going into free fall eliminates the effects of a uniform gravitational acceleration, this principle also implies that a freely falling reference frame in a uniform gravitational field is equivalent to a reference frame moving at constant velocity in gravity-free space, in which special relativity applies.

Opens route to extending principle of relativity to accelerating frames, by bringing in gravity.

Inertial and Gravitational Mass

Newton's laws introduce mass in two different places:

inertial mass = resistance to acceleration ($a = F/m$)

gravitational mass = ability to exert gravitational force ($F = GMm/r^2$)

All objects accelerate at the same rate because of a seemingly "coincidental" cancellation between inertial and gravitational mass.

From the point of view of General Relativity, equivalence of inertial and gravitational mass is just a necessary consequence of the equivalence principle.

Bending of light

Route from equivalence principle to full theory of gravity is still long and difficult. Took Einstein 8 years.

For the moment, let's look at some consequences of the principle of equivalence alone, which Einstein figured out early on.

FIGURE 5: STEEL BOX, WITH LIGHT BEAM CROSSING

Back to steel box, of size l, on earth.

A beam of light passes through the box, starting parallel to the floor.

Does gravity bend the light beam?

Principle of equivalence tells us yes.

Imagine the box accelerating upward at g_E.

Light goes almost straight across, takes time $t = l/c$ to cross the box.

In this time, what happens to the box? Goes up a height

$$h = \frac{1}{2}g_E t^2 = \frac{1}{2}g_E \left(\frac{l}{c}\right)^2 .$$

Relative to floor, light drops, hits the far wall lower by this amount.

By the principle of equivalence, the same must happen in a box sitting on the earth: gravity bends light.

For $l = 100m$, the drop is

$$\frac{1}{2}\left(10\frac{\mathrm{m}}{\mathrm{s}^2}\right)\left(\frac{100\mathrm{m}}{3\times 10^8\mathrm{m/s}}\right)^2 \approx 5\times 10^{-13}\mathrm{m}$$

about 1/100 the size of a hydrogen atom.

Clearly too small to measure in a laboratory.
Can you think of a way the bending of light by gravity could be measured?

Gravitational Redshift

Doppler shift: EM radiation of wavelength λ received from an object moving away at velocity v is shifted to a longer wavelength

$$\lambda_o = \lambda_e \times \left(1 + \frac{v}{c}\right).$$

This is called a redshift, since the shift is to longer, "redder" (in the case of visible light) wavelengths.
If the object is moving towards at velocity v, the radiation is shifted to shorter wavelength (just change v to $-v$ in the above formula). This is a blueshift.
This phenomenon is straightforward to understand, and it affects all sorts of waves, including sound waves, ocean waves, etc. See *Thorne Box 2.3* for explanation.

FIGURE 6: STEEL BOX, WITH LIGHT BEAM GOING FROM BOTTOM TO TOP

Consider a box of height l that is floating in space.
Shine light beam from bottom of box to top.
How long does it take? $t = l/c$.
If the light is emitted with wavelength λ_e, what wavelength is it received at at the top of the box? λ_e.
Consider a second reference frame that is moving upward at constant velocity $v \ll c$. In that reference frame, what wavelength is the light observed at?

$$\lambda_o = \lambda_e \times (1 + \frac{v}{c}).$$

Redshifted, because the source is moving away from the receiver.
Now consider a third reference frame that is *accelerating* at a rate g, equal to the gravitational acceleration on earth.
After time t the frame is moving upward with velocity $v = gt = gl/c$. At that moment, it is just like the second reference frame.
What does this velocity do to the light? Doppler shift.
Red shift or blue shift? Red shift, light source is moving away.
Observed wavelength is

$$\lambda_o = \lambda_e \times \left(1 + \frac{v}{c}\right) = \lambda_e \times \left(1 + \frac{gl}{c^2}\right).$$

By equivalence principle, same thing happens in box sitting on earth: gravity shifts the frequency of light.
Phenomenon usually called gravitational redshift (though falling light is blueshifted).

For $l = 100$m, the frequency changes by one part in 10^{14}.
Hard to measure in laboratory, first done in 1959.
Shift due to sun's gravity also detected.
Strong gravitational redshifts seen in some extreme astrophysical sources, believed to be neutron stars or black holes.

Effect of gravity on clocks

I generate radio waves by shaking electrons at frequency ν_e at bottom of box.

At top of box, you receive it at a different frequency ν_o.

In 1 sec, I send out ν_e cycles, but in 1 sec you receive only ν_o cycles, with $\nu_o < \nu_c$.

How is this possible? Your clock must run at a different rate.

The extra cycles don't disappear, but they arrive late.

Same number of cycles takes more time to arrive $\Longrightarrow$ your clock runs faster.

Closer to a gravitating object ("deeper" in the gravitational field), time passes more slowly. One second at the surface of the earth corresponds to more than one second at high altitude.

In general relativity, rate at which time flows depends on gravity as well as on velocity.

Explains why you can never see someone fall into a black hole from afar; their clock gets infinitely out of sync with yours.

See *Thorne Box 2.4*.

Special and General Relativity

Principle of relativity (1905): The laws of physics are the same for any observer moving at uniform velocity. No absolute standard of rest.

Principle of relativity (1915): The laws of physics are the same for any freely falling observer. No absolute standard of acceleration.

(This is not how Einstein phrased it.)

Jump from special to general made possible by principle of equivalence: effects of uniform acceleration are same as effects of uniform gravitational field.

"Uniform gravitational field" means a gravitational field that produces the same acceleration (direction and magnitude) everywhere.

Original theory of relativity emerges as a special case of general relativity, with no gravitational fields and no accelerating observers.

Tidal Acceleration

FIGURE 7: BIG STEEL BOX ON EARTH

Go back to the box on the earth, but now make it big, a noticeable fraction of earth radius.

Can I do experiments to tell difference between this and a box being accelerated by a rocket ship?

Yes. Balls dropped at two sides of box converge as they fall. Balls dropped at top and bottom of box accelerate at different rates.

Even a freely falling observer sees balls change speeds or direction because gravitational field is not uniform.

A non-uniform gravitational field produces *relative* accelerations of freely falling objects, so it can't be mimicked by uniform acceleration.

For a freely falling observer, effects of gravity vanish nearby, but not over scales where gravitational acceleration changes magnitude or direction.

Even in small box, could measure these effects with very high precision measurements.

See *Thorne Fig. 2.3*.

See *Thorne Box 2.1*.

Spacetime paths of freely falling objects

According to Newton, and to our everyday intuition, space and time are absolute.

Einstein showed that they are not: observers moving relative to one another measure different time separations and spatial separations between the same pair of events.

But we can still think of events as occurring at set locations in a four-dimensional *spacetime*, a concept introduced in 1908 by Hermann Minkowski, who had once been Einstein's college math professor.

In a uniform gravitational field:

A static observer sees freely falling objects accelerate, change direction ⟶ follow curved paths through spacetime.

Note: Even a ball falling straight down follows a curved path in space*time*. See FIGURE 8.

A freely falling observer sees other freely falling objects move at constant speed and direction, following *straight* paths in spacetime.

In a non-uniform gravitational field:

Even a freely falling observer sees other freely falling objects follow curved paths.

Einstein's hypothesis: Even in a non-uniform gravitational field, freely falling objects follow the "straightest possible" paths, those that give the shortest "interval" between spacetime points.

How can "straight" paths through spacetime appear curved? If spacetime itself is curved.

Shortest paths in curved surfaces or spaces or spacetimes are called *geodesics*, studied in non-Euclidean geometry.

Etymology: geodesic ← divide the earth

An example

Two astronauts in outer space, initially at rest.

Gravity pulls them together, they converge, then pass each other.

Eventually come to a stop, and the process repeats.

Spacetime diagram shows sinusoidal path.

On a sphere, geodesics are great circles, arcs whose center is the center of the sphere.

Follow two geodesics, starting at equator. They start parallel, converge, cross, diverge, reach maximum separation, converge again. Just like the two astronauts.

Following spacetime geodesics can produce the gravitational effects that we are familiar with.

Two astronauts example also resembles edge-on view of the earth's orbit around the sun.

Note that if time axis is marked in years and position axis in light years (as we should do to put them on the "same footing"), then the paths are nearly straight (curvature is small).

See *Thorne Figs. 2.4, 2.5, 2.6.*

Einstein's Theory of Gravity

Summary of Newtonian Gravity:

1. Matter tells gravity how to exert force.

Force determined by gravitational force equation, $F_g = GMm/r^2$.

2. Force tells matter how to move.

Freely falling objects accelerate at $a = F_g/m$.

Summary of General Relativity:

1. Matter tells spacetime how to curve.

Curvature governed by Einstein Field Equation. (See *Thorne Box 2.6.*)

2. Curved spacetime tells matter how to move.

Freely falling objects follow geodesic paths. Paths governed by the geodesic equation.

Mathematical description of curvature is complicated and difficult. Developing GR and finding the field equation took Einstein a long time, and GR calculations are usually very difficult — technically, and sometimes conceptually.

General Relativity and Newtonian Gravity

In the limit where

- velocities are much smaller than c
- gravity is "weak" (light-bending angles are small)

GR gives almost the same results as Newtonian gravity.

It had to, or it would have been ruled out before it started because of the great success of Newtonian gravity in describing the solar system.

Both GR and Newtonian gravity predict the earth's orbit around the sun, but they explain it very differently:

- Newton: The sun exerts a gravitational force on the earth. The earth follows an ellipse around the sun because of the acceleration produced by that force.
- The sun curves spacetime around it. The earth follows an ellipse around the sun because both the earth and the sun follow geodesic paths in curved spacetime.

Empirical Evidence for GR

The First Tests

GR predicts very slight departures from elliptical orbits.

It had been known since 1860 that the long axis of Mercury's orbit advances by 0.10 arc-seconds per orbit, or 43 arc-seconds per century.

Called *advance of perihelion.*

There are 3600 arc-seconds in one degree, so this is a very small angle. A full cycle would take 13 million Mercury orbits, or 3 million years.

Various suggestions: Venus more massive, unseen asteroids, oblate sun.

November 1915: Einstein finds gravitational field equation, shows that GR precisely explains the shift in Mercury's orbit.

First empirical confirmation of GR. Life-changing experience for Einstein. Motivated to pursue GR by theoretical considerations, but success with Mercury shows that it describes the real universe.

Somewhat analogous to interaction of Saturn and Jupiter as evidence for universal gravity.

Next prediction, bending of light by sun, confirmed in 1919 solar eclipse expedition. Made Einstein world famous.

Gravitational redshift confirmed by laboratory experiment in 1959.

Subsequent Tests

Orbits

Many high-precision tests in solar system, using precise measurements of planetary motions (e.g., with radar ranging) and space probes (which would completely miss their targets if only Newtonian gravity were used to predict their orbits).

We can precisely measure the orbits of some neutron stars (which we'll say more about later in the course) in binary systems (i.e., orbiting another neutron star or white dwarf).

In some cases these have orbital periods of hours and show much stronger advance of perihelion, in excellent agreement with GR predictions.

Light Bending

Precise measurements of light (or radio wave) bending by the Sun.

Gravitational lenses, which appear when the light-bending by an intervening galaxy produces multiple images (sometimes nearly circular arcs) of a background galaxy or quasar.

Gravitational redshift and time dilation

Precise laboratory measurements.

Gravitational redshift of light from the sun (changing wavelengths by less than 0.001%).

Stronger gravitational redshifts (0.05%) from white dwarf stars, which are compact and have stronger gravity.

Clocks on satellites run faster than clocks on earth surface – small effect, but crucial for accuracy of GPS, missile guidance, etc. Also confirmed in the 1971 airplane experiment.

Gravitational time delay as radio signals pass by neutron stars in binary systems.

Gravitational waves

GR predicts the existence of gravitational waves.

Indirect evidence from binary neutron star discovered in 1974. GR predicts the neutron stars should move closer together as gravitational waves carry off energy.

Measured changes in orbit agree perfectly with GR predictions, with uncertainty of less than 1%.

First direct detection of gravitational waves in 2015, by the Laser Interferometer Gravitational-Wave Observatory (LIGO). By now LIGO has detected roughly 100 gravitational wave signals, from mergers of black holes (mostly) or neutron stars (a few cases) in very distant galaxies.

The properties of the gravitational waves agree well with GR predictions.

Black holes

Although it took time to recognize it, GR predicts the existence of black holes, completely collapsed objects that trap light and have event horizons rather than solid surfaces.

There is a lot of strong circumstantial evidence for black holes with the properties predicted by GR, as we will discuss in the rest of the course.

Final comments

GR provides a successful unified description of mechanics (motions, accelerations, etc.), electromagnetic phenomena, and gravity.

GR also provides excellent account of the expansion and growth of structure in the universe, spanning an enormous range of length and time. However, this success depends on the existence of two new components of the universe, known as "dark matter" and "dark energy," which we have never detected directly (but whose gravitational effects we measure).

While GR is clearly "more right" than Newtonian gravity, Newtonian calculations are almost perfectly accurate in most circumstances. Newtonian gravity is easier to work with technically and conceptually, so physicists use it when possible.

At present, there are no clear experimental failures of GR. We expect GR to break down at extremely small length and timescales, where quantum effects become important, but we do not know of ways to probe this regime experimentally.

Figure 3

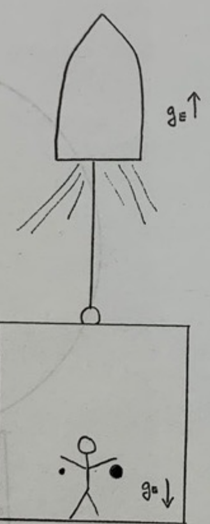


Figure 4

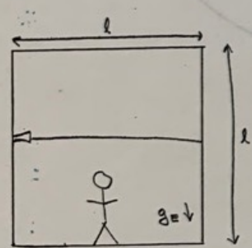


Figure 5

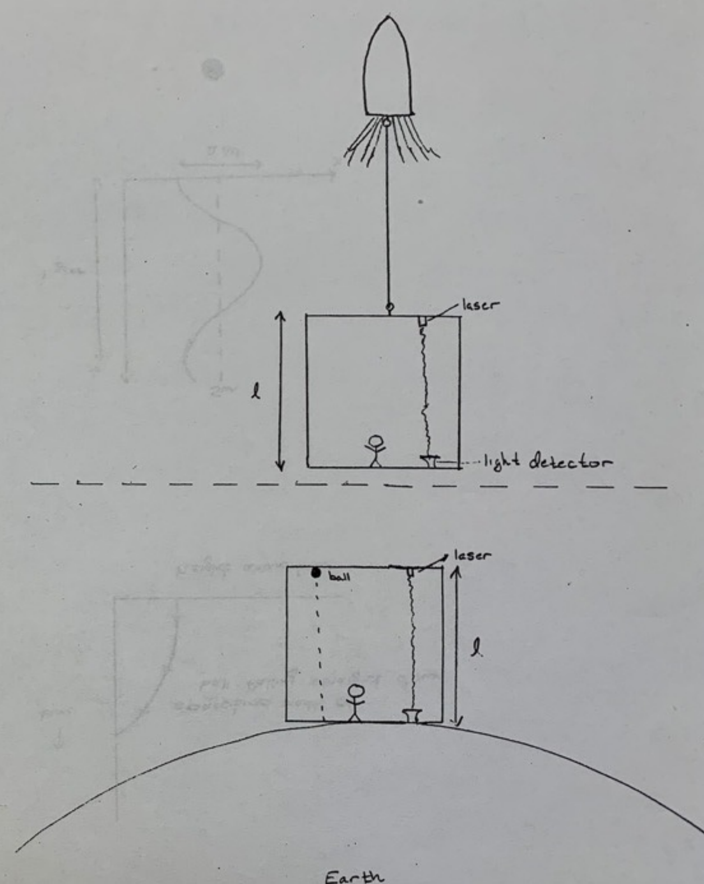


Figure 6

Figure

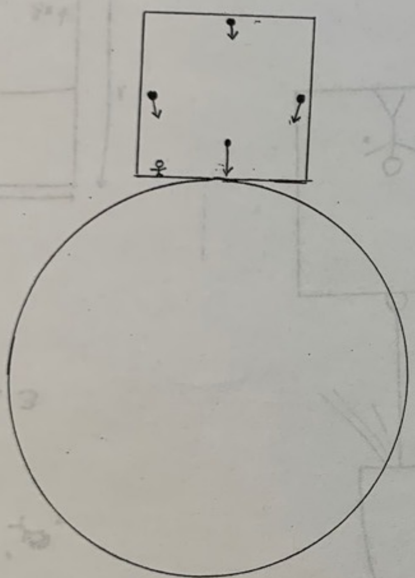
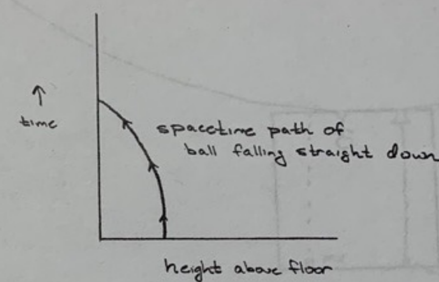


Figure 7



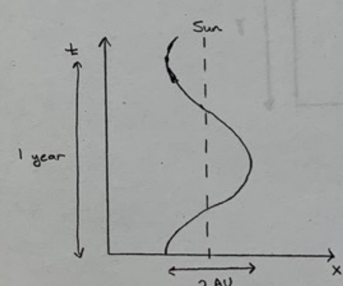


Figure 8