Astronomy 1142, Assignment 3: White Dwarfs, Neutron Stars, and Black Holes

This assignment is due at the beginning of class on Friday, 11/5. It should be turned in on paper, and you should staple or paper clip all sheets together. It's your responsibility to write clearly enough that we can grade your answers. Write your name on the assignment!

If you are unable to attend class on 11/5, please turn in your assignment to my mailbox in 4055 McPherson before class. Late assignments will be marked down 15 points, and no assignments will be accepted past 4 pm on Tuesday, 11/9.

You may consult with others in the class when you are working on the homework, but you should make a first attempt at everything on your own before talking to others, *and you must write up your eventual answers independently.*

You are welcome to come to my office hours or TA John Bredall's office hours for advice. This will almost certainly be helpful. Please spend some time working on the assignment before you come to office hours so that you know what your questions are.

In-person office hours, 4030 McPherson Laboratory (4th floor, SW corner)
Thursday 11/3, 11:30am-12:45pm, with David Weinberg
Friday, 11/4, 11:30am-12:45pm, with John Bredall

Virtual office hours, Zoom 827 776 2849, Passcode 1420
Thursday 11/4, 4:15pm-5:30pm, with John Bredall
Friday 11/5, 9:15am-10:30am, with David Weinberg

You can also ask me questions after class on Monday or Wednesday.

Part I:

We have a lot of reading to do this week, so the majority of this week's homework assignment is a "guided reading" with questions to answer along the way. Answer each of the numbered questions below. One or two sentences is sufficient. Each question is worth 5 points.

Chapter 3

In 1916, Karl Schwarzschild discovered the first exact solution to Einstein's equations. This solution describes the spacetime outside a spherical star. If the star is sufficiently compact, Schwarzschild's solution shows that there should be an "event horizon" (though that particular term was not coined until much later), i.e., a region from which light would be unable to escape.

1. What was Einstein's reaction to this idea?

Chapter 4: pp. 153-157 are optional reading

The 1920s saw the first observational evidence for white dwarfs, stars whose mass is comparable the sun's but whose radius is comparable to the earth's. (White dwarfs are now understood to be the end state of stars like the sun, after they have exhausted their nuclear fuel, but this understanding took several more decades to develop.) R. H. Fowler proposed (correctly) that white dwarfs are supported against gravitational collapse by the pressure of *degenerate electrons*, electrons that are moving erratically at high speed because they are squeezed into a small volume. In 1930, while on a boat from India to England, *Subrahmanyan Chandrasekhar* showed that if the gravity of the white dwarf is so strong that the electrons are moving close to the speed of light, then the degenerate electrons would be less “springy,” i.e., providing less resistance to compression.

2. From this deduction, what did Chandrasekhar conclude about white dwarfs?

3. Why is this conclusion relevant to the existence of black holes?

4. What was the reaction of Sir Arthur Eddington, the most eminent British astronomer of his day, to Chandrasekhar’s suggestion?

Chapter 5: Box 5.1 and Figure 5.1 are both important. pp. 179-190 and 197-206 are optional.

Fritz Zwicky and Walter Baade identified the existence of *supernovae*, enormously powerful stellar explosions. Zwicky also proposed the existence of *neutron stars.* A neutron star is a ball of neutrons with the density of an atomic nucleus, but held together by gravity rather than the nuclear force (which holds normal atomic nuclei together). A neutron star is roughly the mass of the sun (slightly more) but only about 10 km in radius.

5. According to Zwicky (who turned out to be correct), what is the source of energy that powers a supernova explosion?

6. What did *J. Robert Oppenheimer* and his student George Volkoff show about neutron stars, analogous to Chandrasekhar’s finding for white dwarfs? (Hint: see pp. 191-192.) Why is their finding relevant to the existence of black holes?

Part II

Read

Chapter 6: pp. 245-257 are the most important, pp. 219-235 are optional

With particular attention to pp. 254-257, write a paragraph that answers the following questions. This 1-paragraph essay is worth 20 points.

What different points of view were conveyed by the terms “frozen star” (used mainly in the Soviet Union) and “collapsed star” (used mainly in the West)? Why were these terms unsatisfactory? What is the term that *John Archibald Wheeler* coined for these objects? What ideas does this term convey?

(For the last question, you can rely on your own reaction to this term.)

Part III

What is the most surprising thing you learned in reading chapters 3-6?

Part IV

Each part of the question is worth 5 points, except parts (e) and (f) which are worth 10 points.

(a) In Section 3 of the course, we obtained the equation $M = \frac{v^2 r}{G}$, which relates the orbital speed v and orbital radius r for an object in a circular orbit to the mass that it is orbiting around. Combine this equation with the value I have given you for the Schwarzschild radius of the sun, $R_{\text{Sch},\odot} = \frac{2GM_\odot}{c^2} = 3\,\text{km}$, to show that the orbital speed divided by the speed of light is

$$\frac{v}{c} = \sqrt{\frac{1.5\,\text{km}}{r}} \times \sqrt{\frac{M}{M_\odot}}.$$

(b) The earth is 300,000 times less massive than the sun. Using the result from (a), show that a satellite orbiting at a distance $r = 7000\,\text{km}$ from the center of the earth (about 600 km above the earth's surface) will orbit the earth in about 5,500 seconds, or about 90 minutes. Remember that $c = 300,000\,\text{km/s}$.

(c) A typical white dwarf star has a radius similar to the earth but a mass $M = 0.6M_\odot$. How long would it take a satellite to orbit such a star at a distance $r = 7000\,\text{km}$?

(d) A typical neutron star has a mass $M = 1.4M_\odot$ but a radius of only 10 km. How long would it take a satellite to orbit such a neutron star at a distance $r = 7000\,\text{km}$? How long would it take to orbit at a distance $r = 12\,\text{km}$?

(e) Returning to the equations $M = \frac{v^2 r}{G}$ and $R_{\text{Sch}} = \frac{2GM}{c^2}$, give a mathematical argument (a couple of lines of equations and some words to explain them) that light should be able to orbit a black hole in a circle at a distance $r = \frac{1}{2}R_{\text{Sch}}$.

I told you (correctly) in class that light can orbit a black hole in a circle at a distance $r = \frac{3}{2}R_{\text{Sch}}$, so three times larger than your calculation implies. Why do you think this calculation gives the wrong answer?

(f) The gravitational energy released when a stellar core implodes to form a neutron star is approximately $E_{\text{grav}} = GM^2/R$, where M is the mass of the neutron star and R is its radius. In Assignment 2, we saw that the total energy that the sun will release over its entire lifetime as a normal ("main sequence") star is roughly $E_\odot = 0.001M_\odot c^2$, since about 10% of the sun's mass will fuse from hydrogen to helium and each kg of hydrogen produces about 0.99 kg of helium.

For a neutron star mass $M = 1.4M_\odot$ and radius $R = 10\,\text{km}$, what is the ratio of the gravitational energy E_{grav} released by the implosion that forms the neutron star to the total energy $E_\odot$ released by the sun during its lifetime?

Hint: Use $2GM_\odot/c^2 = 3\,\text{km}$.

(Note that the energy from the sun is produced over 10 billion years while the energy of the neutron star implosion is released in a few seconds!)