X-ray Iron Lines from a Black Hole Accretion Disk

For a non-spinning black hole that is being fed with gas from a companion, the inner edge of the accretion disk is at $R = 3R_{\text{Sch}}$. Our Newtonian formula $v^2 = GM/R$ for the speed in a circular orbit is somewhat inaccurate this close to a black hole, but it's not too far off. As shown in class, it predicts (approximately) that at $R = 3R_{\text{Sch}}$, $v/c = 0.4$.

We previously wrote the Doppler shift formula in terms of wavelength, but for our current purpose it is better to write in terms of energy:

$$E_o = \frac{E_e}{\left(1 + \frac{v}{c}\right)} \ .$$

Here E_e is the energy of a photon emitted by an atom near the black hole, and E_o is the energy that we observe for that photon when we detect it far from the black hole. Remember that for atoms moving *away* from you v/c is positive (so energy is reduced, redshift) and for atoms moving *towards* you v/c is negative (so energy is increased, blueshift).

Highly ionized iron atoms emit X-ray photons with an energy $E_e = 6.4\,\text{keV}$. (For our purposes, you just need to know that a keV is a unit of energy.) Suppose that we use an X-ray telescope to detect the iron emission from a black hole with an accretion disk.

1. Considering *just* the effects of Doppler shifts, what should be the energies of the highest energy photons that we detect?
(a) 12.8 keV
(b) 10.67 keV
(c) 7 keV
(d) 6.4 keV
(e) 4.6 keV

2. Considering *just* the effects of Doppler shifts, what should be the energies of the lowest energy photons that we detect?
(a) 10.67 keV
(b) 7 keV
(c) 6.4 keV
(d) 6 keV
(e) 4.6 keV

There is an additional effect we have to consider, namely gravitational redshift. For photons emitted at a distance R from a non-spinning black hole, we should multiply the energy we computed using the Doppler formula by another factor

$$f = \sqrt{1 - \frac{R_{\text{Sch}}}{R}} \ .$$

For $R = 3R_{\text{Sch}}$, this factor is $\sqrt{2/3} = 0.82$.

3. Considering *both* the effects of Doppler shifts and gravitational redshift, what should be the energies of the highest energy photons that we detect?
(a) 13.0 keV
(b) 10.67 keV
(c) 8.7 keV
(d) 6.4 keV
(e) 5.3 keV

4. Considering *both* the effects of Doppler shifts and gravitational redshift, what should be the energies of the lowest energy photons that we detect?
(a) 8.7 keV
(b) 6.4 keV
(c) 5.3 keV
(d) 4.6 keV
(e) 3.7 keV

5. A plot of the distribution of photon energies from the accretion disk should most closely resemble which of the examples below?