11. CMB Anisotropy

Reading: Chapter 8, sections 8.4 and 8.5

Gravitational instability and structure formation

Today's universe shows structure on scales from individual galaxies to galaxy groups and clusters up to superclusters of galaxies that can extend to 100 Mpc or more.

If the universe is smooth but not perfectly smooth, then gravity will cause structure to grow, by drawing matter into regions that start slightly above the average density (and out of regions that start slightly below the average density).

For this process to work, there must have been small-amplitude inhomogeneities present in the early universe, the "seeds" for the gravitational growth of structure.

These inhomogeneities should cause slight non-uniformities in the CMB.

Expected magnitude of fluctuations

We can describe fluctuations by the fractional density contrast

$$\delta(\mathbf{x}, t) = \frac{\rho(\mathbf{x}, t) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

At early times or on large scales, $|\delta| \ll 1$ and one can compute the growth of δ using linear perturbation theory.

For a flat universe with $\Omega_m = 1$, the prediction is $\delta(\mathbf{x}, t) \propto a(t)$.

In the simplest models with only baryonic matter, the predicted flucuations in the CMB are roughly one part in 10^3.

With non-baryonic dark matter, it is possible to lower the level of CMB anisotropies to $\sim 10^{-5}$ and remain consistent with gravitational growth of today's structure. The fluctuations in dark matter are $\sim 10^{-3}$, but the baryons and photons are smoother.

Measurement of CMB anisotropy

The "dipole" anisotropy of the CMB, which is just the reflex of the earth's peculiar velocity (which is largely due to the motion of the Milky Way), was detected in the 1970s.

The first detection of the *intrinsic* anisotropies of the CMB, reflecting structure in the universe at $z_{\rm rec}$ rather than the motion of the earth today, was obtained by the COBE satellite in 1992.

The variations are only a part in 10^5, so measuring them requires extremely high sensitivity and extremely good control of systematic errors, like contamination of the signal from other sources.

Since COBE, there have been a number of measurements of CMB anisotropy with higher resolution and sensitivity, from the ground, balloons, and satellites.

The Wilkinson Microwave Anisotropy Probe (WMAP) produced much higher resolution and more sensitive maps of CMB anisotropy, from 2003-2012.

Today's state-of-the-art CMB map is from the European Planck satellite (roughly 2010-2020), which is more sensitive and higher resolution than WMAP.

Ground-based experiments like the South Pole Telescope (SPT) and Atacama Cosmology Telescope (ACT) get to smaller angular scales because they use larger telescopes. These and future ground-based experiments are especially valuable for measuring polarization of CMB fluctuations.

Characterizing the CMB anisotropy

Suppose that we smooth a CMB map over an angular scale θ. (Any real map will automatically be smoothed at some minimum angular scale determined by the diffraction limit of the telescope used to make it, and we can subsequently smooth the map over larger scales.)

We can then plot a histogram of the fractional temperature variations $\Delta T/T$ in the smoothed map and measure the root-mean-square width of this histogram.

To characterize the structure in the map, we can plot $(\Delta T/T)_{\rm rms}$ against the smoothing scale θ.

If the map consisted of randomly placed hot and cold spots, we would expect $(\Delta T/T)_{\rm rms}$ to decline in proportion to the square-root of the sky area ($\propto 1/\theta$) because of $\sqrt{N}$ averaging of random variables.

In fact, hot and cold regions are correlated over large scales, and $(\Delta T/T)_{\rm rms}$ declines much more slowly than $1/\theta$.

The most widely used statistical measure of structure in CMB maps is the angular power spectrum, C_l, derived from a decomposition of the map into spherical harmonics.

Roughly speaking, $l(l+1)C_l$ is the square of $(\Delta T/T)_{\rm rms}$ on the scale $\theta = 200/l$ degrees.

Physics of CMB anisotropy

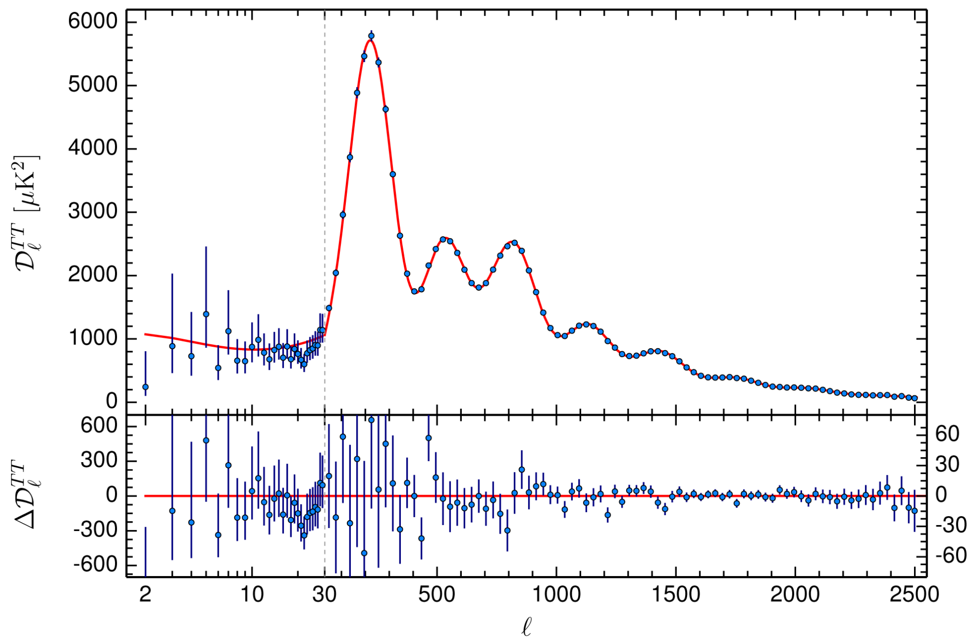
It is not surprising that if the density of the universe is slightly inhomogeneous at $z_{\rm rec}$, then the CMB will be slightly non-uniform.

There are several effects that link the inhomogeneities of the matter distribution to the CMB anisotropies:

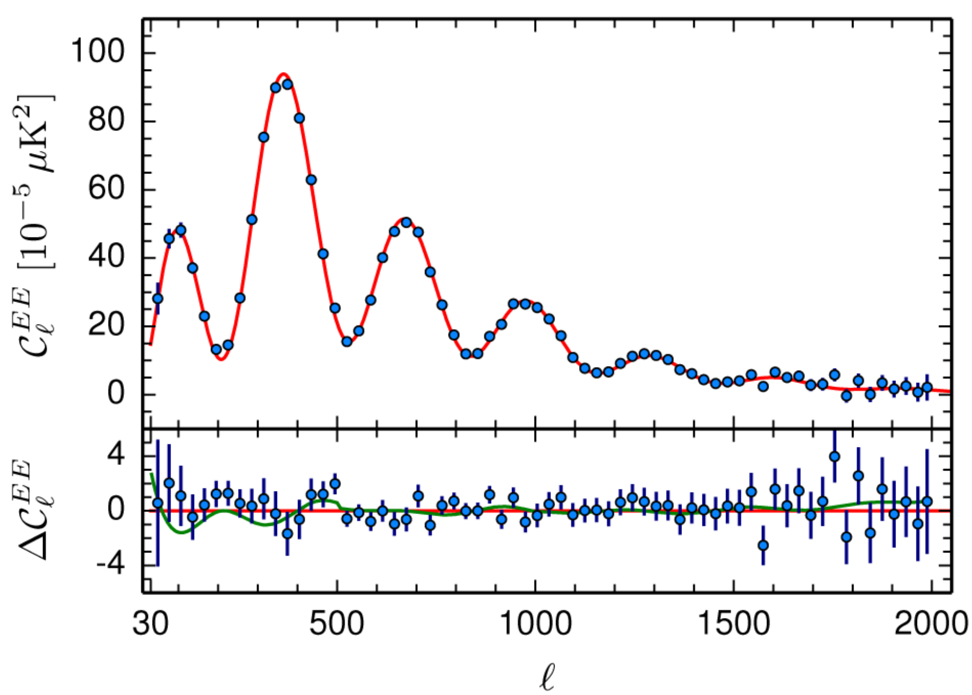
- Adiabatic temperature fluctuations. Where the density is higher, the photon temperature is hotter.
- Doppler shifts. The gravitational perturbations caused by the density fluctuations induce peculiar velocities, and photons are blueshifted or redshifted if they last scattered off electrons with peculiar velocities toward us or away from us, respectively.
- Gravitational redshifts, as the photons climb out of the dark matter potential wells, or blueshifts as they fall off of potential hills. This effect makes the photons coming out of denser regions redder (cooler temperature).

Different processes dominate on different scales, so if we can measure CMB anisotropy over a wide range of scales we can separate them to some degree.

A small fraction of CMB photons get scattered by electrons after the universe becomes reionized at $z < 8$, which imprints some additional fluctuations on small angular scales.



Planck satellite measurements of the angular power spectrum of temperature fluctuations in the CMB.



Planck satellite measurements of the angular power spectrum of polarization fluctuations in the CMB.

Acoustic oscillations

As long as the universe is ionized, so that photons are tightly coupled to the electrons and baryons, the photons and baryons behave like a single, high-pressure fluid with a "sound speed" of approximately $c/\sqrt{3}$.

An overdense region may start to collapse under its own gravity, but eventually the buildup of pressure will halt and reverse the expansion.

If there is enough time, a perturbation can contract and re-expand several times, a cycle of "acoustic oscillations."

Because of this effect, there is a preferred scale on which the temperature fluctuations are largest. This is the scale for which an overdense region has had just enough time to collapse to maximum compression when the universe recombines.

On very large scales there has been insufficient time for contraction to occur, and gravitational redshifts produce approximately "scale-invariant" fluctuations.

The *physical* size of the acoustic scale can be computed from first principles, once the densities of dark matter, baryons, and radiation are specified. The acoustic scale is a "standard ruler."

The *angular* size of the acoustic scale depends on the comoving distance to the last scattering surface and on geometry (curvature).

Measuring cosmological parameters with the CMB

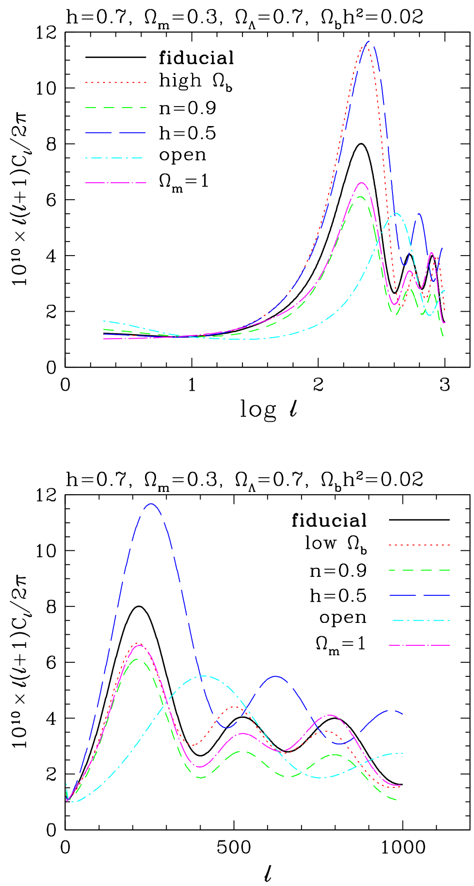
If we specify the statistical properties of the density fluctuations present at $z_{\rm rec}$, and the matter and energy contents of the universe ($\Omega_{r,0}$, $\Omega_{m,0}$, $\Omega_{\Lambda,0}$, H_0, etc.), then we can predict the full pattern of CMB anisotropy.

Model predictions can be tested against measurements, and the measurements can be used to infer the properties of the primordial fluctuations and the matter and energy contents of the universe.

Changing parameters changes the predictions. There is some degeneracy among the parameters (i.e., changes in one can be traded off against changes in another), but with good measurements over a wide range of scales these degeneracies can mostly be broken.

There is a nice illustration of these effects at
`http://space.mit.edu/home/tegmark/movies.html`

Plots on the next page convey the basic idea.



Theoretical predictions of the temperature power spectrum for a fiducial cosmological model (black solid line, parameters as indicated) and models in which one parameter is varied at a time.

Top and bottom panels show logarithmic and linear l axes, respectively.

Overall amplitude sets the initial conditions for structure formation.

Dotted red: High baryon density.

Dashed green: Change of initial conditions, from different inflation model.

Long-dashed blue: Lower Hubble constant.

Dot-dashed green: No cosmological constant, $\Omega_m = 0.3$, open universe ($k = -1$)

Dot-dashed red: No cosmological constant, $\Omega_m = 1$, flat universe ($k = 0$)

The existence of oscillations implies that the perturbed matter and photon densities started out equal, a.k.a. “adiabatic” fluctuations.

The heights of the peaks depends most strongly on the matter density and the baryon density.

The angular location of the peaks depends most strongly on curvature.

The overall “tilt” depends on inflationary initial conditions.

CMB Polarization

CMB anisotropies are polarized, i.e., they have slightly different amplitudes when measured in orthogonal polarizations.

The polarized signal is only about 10% of the full signal (which is only 10^{-5} in the first place), and the polarization from foreground contaminants is hard to remove, so these measurements are tough.

First detections in 2005, consistent with predictions from inflation.

Roughly speaking, polarization doubles (or even triples) the information content of CMB anisotropies and better pins down what is causing anisotropy on a given scale.

The usual effects that produce anisotropy can produce only particular kinds of statistical patterns, known as “E-mode” polarization.

“B-mode” polarization can be produced by gravitational lensing of the E-mode pattern by clustered matter at low redshifts.

B-mode polarization can also be produced by gravity waves created in the early universe, a.k.a. “tensor fluctuations.”

Detection of B-mode polarization, especially from gravity waves, is the current holy grail of experimental CMB research, because it would provide sharp insights into the physics (such as inflation) that created the fluctuations.

The challenge is partly one of raw sensitivity, but also controlling instrumental effects and astrophysical foregrounds (e.g., emission from dust in the Milky Way) at part-in-a-million levels.