6. Measuring Cosmological Parameters via Expansion

Reading: Chapter 6. Note that the textbook emphasizes the quantity q_0, but for reasons that will become clear, I do not consider this a useful parameter.

Cepheids and Type Ia supernovae as standard candles

If you observe the apparent flux f of an object of known luminosity L, you can infer its distance from the relation

$$f = \frac{L}{4\pi d^2}.$$

Cepheid variable stars exhibit a tight correlation between period and luminosity. Measure P, infer L.

Type Ia supernovae (supernovae without hydrogen absorption lines, produced by thermonuclear explosions of white dwarfs) rise and fall over the course of about a month.

Their peak luminosities (luminosity at maximum) are similar, with a scatter of about 40% from one supernova to another.

In the early 1990s, it was recognized that supernovae that rise and fall slower are more luminous, and vice versa.

If you measure the shape of the light curve (the time required to rise and fall), you can "standardize" the Type Ia "candle," leaving residual scatter of 10-15% in peak luminosity (0.1-0.15 magnitudes).

This makes Type Ia supernovae powerful tools for cosmology.

Measuring H_0

At first glance, measuring H_0 looks easy: find an object of known distance d, measure its redshift, and infer $H_0 = v/d$.

The first step is the hard one. You have to *calibrate* your standard candles.

In rough outline, the best direct route to H_0 is currently:

- Measure the distance to the LMC, using stars that can be matched to stars in the Milky Way.

- With the known distance to the LMC, calibrate the period-luminosity relation for Cepheids.

- Find Cepheids in somewhat more distant galaxies (few Mpc) that have also hosted Type Ia supernovae. Use the periods and fluxes of the Cepheids to infer the distance to the galaxies, and thereby calibrate the luminosity of Type Ia supernovae.

- Find Type Ia supernovae in galaxies that are far enough away (50-200 Mpc) that one can ignore their peculiar velocities. Use the peak fluxes of the supernovae to get the distances to the galaxies, and measure the galaxy redshifts.

- Infer $H_0 = d/v$.

The LMC step can be circumvented by measuring Cepheid distances via parallax, but there are relatively few Cepheids close enough to have precisely measured parallaxes. These are usually shorter period Cepheids (which are more common) relative to the more luminous ones we observe in other galaxies.

Observations from the *Gaia* satellite and *Hubble Space Telescope* are slowly changing this situation.

Comoving distance

As asserted at the end of Section 5, we can calculate the comoving distance to an object at redshift $z = a^{-1} - 1$ as

$$r = \int_a^1 dr = \int_a^1 \frac{c\,dt}{a(t)} = \int_a^1 \frac{c\,da}{a^2 H} = cH_0^{-1} \int_a^1 \frac{da}{\left[\Omega_{r,0}a^{-4} + \Omega_{m,0}a^{-3} + \Omega_{\Lambda,0} + (1-\Omega_0)a^{-2}\right]^{1/2}} .$$

The argument is is similar to the one leading to equation 5.83 in the textbook, combined with the relation

$$dr = \frac{c\,dt}{a(t)} = \frac{c}{H}\frac{da}{a^2} = \frac{c}{H_0}\frac{H_0}{H}\frac{da}{a^2}.$$

One can use $a \equiv (1+z)^{-1} \Rightarrow da = -dz(1+z)^{-2} = -a^2 dz$ to write this formula as

$$r = \frac{c}{H_0}\int_0^z \frac{dz'}{\left[\Omega_{r,0}(1+z')^4 + \Omega_{m,0}(1+z')^3 + \Omega_{\Lambda,0} + (1-\Omega_0)(1+z')^2\right]^{1/2}},$$

with $\Omega_0 = \Omega_{r,0} + \Omega_{m,0} + \Omega_{\Lambda,0}$.

This formula allows us to compute the comoving distance to an object at redshift z if we specify H_0, $\Omega_{r,0}$, $\Omega_{m,0}$, and $\Omega_{\Lambda,0}$.

Note that if $z \ll z_{\rm eq} \sim 3600$, the $\Omega_{r,0}(1+z')^4$ term is negligible compared to the $\Omega_{m,0}(1+z')^3$ term.

IMPORTANT: For forms of dark energy that are not a cosmological constant, one needs to make the substitution

$$\Omega_{\Lambda,0} \longrightarrow \Omega_{\rm DE,0}\frac{\rho_{\rm DE}(z)}{\rho_{\rm DE,0}} = \Omega_{\rm DE,0}(1+z)^{3(1+w)},$$

where the last equality holds for dark energy with $p = w\epsilon$.

Thus, the comoving distance to an object will be different if dark energy has $w \neq -1$, even if $\Omega_{m,0}$ and Ω_0 are unchanged.

Luminosity distance

Suppose we observe a standard candle of known luminosity L and redshift z.

From the metric

$$ds^2 = -c^2dt^2 + a^2(t)\left[dr^2 + S_k^2(r)d\Omega^2\right]$$

we can see that photons emitted isotropically by a source at comoving distance r are today (at t_0, with $a_0 = 1$) spread over a sphere of proper surface area

$$A_p(t_0) = 4\pi S_k^2(r),$$

since the solid angle is $\int d\Omega^2 = 4\pi$ steradians.

Recall that the comoving distance is defined so that it is equal to the proper distance at $t = t_0$, so for a flat universe with $S_k(r) = r$ this is just the usual Euclidean result.

For a positively curved space, the photons are spread over a smaller surface area than they would be in a flat universe, and for a negatively curved space they are spread over a larger surface area.

You might expect that the relation between flux, luminosity, and distance in an FRW universe would be

$$f = \frac{L}{4\pi S_k^2(r)} \qquad \text{(Warning: Incorrect Equation)}$$

However, there are two additional effects.

First, the energy of photons drops by a factor of $(1+z)$, and since f is an *energy* flux not a photon number flux, this reduces f by a factor of $(1+z)$.

Second, there is time dilation between the emitted frame and observed frame, so that photons emitted in a time interval Δt are received in a time interval $\Delta t(1+z)$. This reduces the flux by an additional factor of $(1+z)$.

Bottom line: The relation between flux, luminosity, and distance is

$$f = \frac{L}{4\pi S_k^2(r)(1+z)^2}.$$

The quantity

$$d_L = S_k(r)(1+z)$$

is often called the *luminosity distance*, because

$$f = \frac{L}{4\pi d_L^2}.$$

Cosmologists also often refer to the angular diameter distance d_A, for which the angular size of an object of physical length l is $\theta = l/d_A$.

If you find yourself in need of formulas for cosmological distance measures, a good general reference is Hogg (1999, arXiv:astro-ph/9905116).

Supernova cosmology

If we measure the redshift of an object like a supernova, we can calculate its luminosity distance *if* we specify the values of H_0, $\Omega_{m,0}$, $\Omega_{r,0}$, and $\Omega_{\Lambda,0}$.

Using the (approximately) known luminosity of a Type Ia supernova, we can *measure* the luminosity distance if we measure the apparent flux.

By comparing expected and measured values of the luminosity distance at a variety of redshifts, we can pin down the values of $\Omega_{m,0}$, and $\Omega_{\Lambda,0}$.

If we take the *ratio* of fluxes at different redshifts, the value of H_0 cancels out, so in practice this way of studying expansion history does not depend on H_0 or Cepheid calibration.

Improvements in digital detector technology in the 1990s made it feasible to start searching large areas of sky for high-redshift supernovae.

Two groups set out to do this, with the goal of measuring the deceleration of the universe and determining. $\Omega_{m,0}$.

Instead they found an accelerating universe and showed that $\Omega_{\Lambda,0} > 0$.

Over the last 25 years, multiple groups have carried out much larger surveys using bigger and better digital cameras, discovering and measuring light curves of many hundreds (now more than 1000) of Type Ia supernovae to $z = 1$ and beyond.

The goal is to make highly precise measurements of distance vs. redshift to test whether "dark energy" really is a cosmological constant, with $w = -1$, or a new kind of field with a different value of w.

A big challenge is controlling observational uncertainties (e.g., photometric calibration) and astrophysical uncertainties (e.g., dust extinction, redshift evolution of supernova luminosities) at accuracy of 1% or better.

One goal of the *Nancy Grace Roman Space Telescope* is to carry out a survey of several thousand supernovae discovered and measured with a huge infrared digital camera on a Hubble-sized space telescope designed for high accuracy.

Baryon acoustic oscillations (BAO) offer a different way of doing the same kinds of investigations, using the angular diameter of a "standard ruler" instead of the apparent flux of a "standard candle."

Final Remark

It has been common practice for decades to summarize the impact of cosmological parameters in the quantity

$$q_0 \equiv -\left(\frac{\ddot{a}a}{\dot{a}^2}\right)_{t_0} ,$$

which is a dimensionless measure of the current deceleration rate of the universe.

Standard formulas for the luminosity or angular size distance are expressed in terms of q_0.

In my view, this practice is no longer useful, because there is no unique relation between the single parameter q_0 and the two parameters $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$, which are the quantities of physical interest.