9. Big Bang Nucleosynthesis (BBN)

Reading: Chapter 9. Skim 8.0-8.3 for background.

Using General Relativity, the assumption of a homogeneous and isotropic universe, and the standard model of nuclear and particle physics, we can predict what should happen during the first three minutes of cosmic history.

While we do not observe this epoch directly, we do observe its residue, the cosmic abundances of ^{4}He , D , ^{3}He , and ^{7}Li .

These allow us to test important aspects of this story.

The baryon-to-photon ratio

At temperature T, the number density of photons in a blackbody distribution is

$$n_\gamma \approx 0.244 \left(\frac{kT}{\hbar c}\right)^3 .$$

With the present day temperature $T_{\rm CMB} = 2.73\,{\rm K}$, one can show that the ratio of baryons to photons is

$$\eta \equiv n_b/n_\gamma = 6.0 \times 10^{-10} \left(\frac{\Omega_b h^2}{0.0222}\right) = 6 \times 10^{-10} \left(\frac{\Omega_b h_{70}^2}{0.045}\right),$$

where Ω_b is the ratio of the mean density of baryons to the critical density and I have written the Hubble constant as $h \equiv H_0/100\,{\rm km\ s^{-1}\ Mpc^{-1}} = 0.7h_{70}$ and scaled $\Omega_b h^2$ to the current observational estimate.

The photon density itself is 413 photons$\,{\rm cm}^{-3}$.

Roughly speaking, there is one baryon (proton or neutron) for every billion CMB photons.

We may discuss possible reasons for this surprising fact later, but for now we will just accept it.

Age vs. Temperature

Recall our earlier equations: in the early, radiation-dominated universe, the relation between age and temperature is

$$T(t) \approx 10^{10}{\rm K} \left(\frac{t}{1{\rm s}}\right)^{-1/2}$$
$$kT(t) \approx 1{\rm MeV} \left(\frac{t}{1{\rm s}}\right)^{-1/2} .$$

It is interesting to note that for a present-day baryon density of 2×10^{-7} atoms ${\rm cm}^{-3}$, the number density of baryons when the universe was 1 second old was $\sim (10^{10})^3 \times 2 \times 10^{-7} \sim 10^{23}{\rm cm}^{-3}$, very roughly the density of water (recall that the proton mass is 1.67×10^{-24} g).

The *energy density* is much higher than that of water, because it is dominated by radiation, not by matter. A typical photon energy is $1{\rm MeV} \sim 10^{-3} m_p c^2$, but with $n_\gamma/n_b \approx 10^9$ we get $\epsilon_\gamma \sim 10^6 \rho_b c^2$.

The neutron-to-proton ratio

When the universe is about one second old, the particle species present are photons, neutrinos, and (at an abundance smaller by a factor $\sim 10^9$) protons, neutrons, and electrons.

Presumably there are also dark matter particles, but they do not matter for BBN.

The universe is still dense and hot enough that weak interactions involving neutrinos can convert neutrons to protons and vice versa.

Since neutrons are more massive than protons, they are less abundant — conversion of a neutron to a proton is less probable than conversion of a proton to a neutron.

In thermal equilibrium

$$\frac{n_n}{n_p} = e^{-Q/kT}, \quad Q \equiv (m_n - m_p)c^2 = 1.2934\,\text{MeV}.$$

The conversion reactions become slow compared to the age of the universe at $t \sim 3$ seconds, $kT \sim 0.7$ MeV.

Since the interaction rate is dropping quickly as the density and temperature of the universe decline, there are no subsequent conversions.

The neutron-to-proton ratio "freezes in" at

$$\frac{n_n}{n_p} \approx e^{-1.2934/0.7} \approx 1/6.$$

Deuterium synthesis

Synthesis of elements heavier than hydrogen has to start with deuterium formation ($n + p \to \text{D} + \gamma$).

The binding energy of deuterium is $B_D = 2.22$MeV.

Naively one expects deuterium synthesis to begin when the temperature falls to $kT \sim B_D$.

However, the baryon-to-photon ratio is $\eta \sim 5 \times 10^{-10}$, so the exponential tail of the blackbody distribution can still dissociate deuterium even when kT is significantly below B_D.

The frequency distribution of photons in a blackbody distribution is

$$\frac{dN}{d\nu} \propto \frac{\nu^2}{e^{h\nu/kT} - 1} \approx \nu^2 e^{-h\nu/kT}$$

where the last approximation is for $h\nu/kT \gg 1$.

Synthesis of deuterium actually begins when $kT \sim B_D/-\ln\eta \sim 0.1$ MeV, at time $t \sim$2 minutes.

The decay time for free neutrons is ~ 900 sec, so in two minutes a small but non-negligible fraction of the neutrons left over from "freeze-out" have decayed.

The neutron-to-proton ratio at the time of deuterium synthesis is $n/p \sim 1/7$.

The ^{4}He fraction

The reaction rate for $n + p \rightarrow \mathrm{D} + \gamma$ is fast, so when the temperature falls below 0.1 MeV, all neutrons are quickly processed into deuterium.

The reactions that process D into ^{4}He are also fast, and to first order all neutrons go into ^{4}He .

A robust prediction of the standard big bang model is that $24 \pm 1\%$ of the baryonic mass of the universe is in ^{4}He , with almost all of the rest in hydrogen.

There is a weak dependence of the predicted helium fraction on η, and uncertainty in η is the main contribution to the $\pm 1\%$ uncertainty.

Just as the mass fraction of heavy elements is usually written Z, the mass fraction of helium is usually written Y (and hydrogen X).

The *primordial* helium abundance, which is the mass fraction of helium produced in the big bang, not including later contributions from stars, is written Y_P.

The Observations: The helium fraction in the interstellar gas of the most metal-poor galaxies is $Y = 0.24 \pm 0.01$. It is difficult to reduce the systematic uncertainties in the observations below 0.01.

Deuterium

A small fraction ($\sim 10^{-4} - 10^{-5}$) of D "escapes" and is never processed into heavier nuclei.

This fraction is sensitive to η. Higher $\eta \Longrightarrow$ less chance of escape, hence *smaller* deuterium abundance.

The primordial deuterium abundance is therefore a good way to determine the mean baryon density of the universe.

Stars can destroy deuterium by fusing it into heavier elements, but as far as we know they cannot make it because there are no conditions under which they will make deuterium and not fuse it into heavier elements.

The Observations: The measured deuterium-to-hydrogen ratio in the interstellar medium of distant, metal-poor gas absorbing the light of background quasars is (D /H) $= (2.6 \pm 0.5) \times 10^{-5}$. Taking this to be the primordial ratio implies $\Omega_b h^2 = 0.021 \pm 0.002$.

^{3}He and ^{7}Li

Incomplete processing leaves a ^{3}He fraction similar to that of D ($\sim 10^{-5}$).

A small amount ($\sim 10^{-10}$) of ^{7}Li is produced by $^4\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^7\mathrm{Li}$.

^{3}He and ^{7}Li can both be destroyed and produced in stars, which makes it challenging to determine the primordial value.

The Observations: Accounting for the uncertainties in stellar production and destruction, which are a factor of several, measurements are consistent with the BBN (Big Bang Nucleosynthesis) predictions.

The Roadblock

There are no stable elements of atomic number 5 or 8.

Starting with protons and neutrons, there is no way to bridge the gap at atomic number 8 to build heavier nuclei.

Stars do it by the "triple-α" reaction: $^4\mathrm{He} + {}^4\mathrm{He} + {}^4\mathrm{He} \longrightarrow {}^{12}\mathrm{C}^*$, but this requires high temperature *and* density.

Consequence: only light elements are made in the early universe. All elements heavier than ^{7}Li are made in stars.

The BBN Bottom Line

The standard big bang model, which assumes GR, a homogeneous and isotropic universe, and the standard model of nuclear and particle physics, makes quantitative predictions for the abundances of ^{4}He , D , ^{3}He , and ^{7}Li produced in the early universe.

In the simplest formulation, this theory has one free parameter, the baryon-to-photon ratio η, which has small impact on the predicted ^{4}He abundance and substantial impact on the other predictions.

There are uncertainties in measuring chemical abundances and in correcting the present-day abundances to primordial values.

Within these uncertainties, the observationally inferred primordial abundances of ^{4}He , D , ^{3}He , and ^{7}Li are consistent with the predictions of standard BBN, for $\eta \approx 5 \times 10^{-10}$.

The ^{4}He test has a precision of $\sim 10\%$, while the others have a precision of a factor of several.

This agreement between predictions and observations is strong evidence that the standard big bang model applies back to at least $t \sim 1$ s and $T \sim 10^{10}$K.

The primordial D /H ratio constrains the baryon density to $\Omega_b h^2 = 0.021 \pm 0.002$, assuming standard BBN.

Recently, CMB anisotropies have provided an entirely independent way to constrain the baryon density, yielding $\Omega_b h^2 = 0.022 \pm 0.001$.

The agreement of these two independent determinations is further strong evidence for the standard model.

This agreement rules out many possible variations on the standard model of particle physics, such as extra light neutrino species, or time-variation of the fundamental constants that control the strength of the nuclear, electromagnetic, and weak forces.