Astronomy 5682
Problem Set 1
Due Monday, Jan 23, Start of Class

Homework assignments will usually be due on Thursday, but this one has an unusual schedule because of the MLK holiday on Jan 16. I encourage you to start early. Please turn in your completed assignment at the start of class, or you can turn it in early to my mailbox (next to my office, which is inside the main astronomy office in 4055 McPherson).

As noted on the syllabus, you may consult with your classmates when working on prolem sets; this is often a good way to learn. However, you must write up your solution independently.

Please write your name on the first page and staple multiple sheets together.

Question 1

(a) The typical luminosity of a bright galaxy is $L \sim 10^{11} L_\odot$. The average space density of such galaxies is $n \sim 0.001$ galaxies/Mpc^3. If the universe is 15 billion years old, what is the total flux $F_{\rm gal}$ received at earth from all of the external galaxies in the universe? (Ignore the contribution from stars in the Milky Way.)

Express your answer in $\rm erg\,s^{-1}\,cm^{-2}$, remembering that the luminosity of the sun is $L_\odot = 3.9 \times 10^{33}\,\rm erg\,s^{-1}$ and $1\,\rm Mpc = 3.1 \times 10^{24}\,cm = 3.26$ million light years.

(b) How does $F_{\rm gal}$ compare to the flux $F_\odot$ received from the sun at earth (i.e., what is the ratio $F_\odot/F_{\rm gal}$)?

Ignore all of the obvious potential complications, such as redshifting of light and evolution of the galaxy population. Just do the "naive" calculation for a static, non-evolving universe. Don't worry much about factors of two; aim for a simple calculation with order-of-magnitude accuracy. If you need guidance, see §2.1.

Question 2

Hubble's law says that a galaxy at distance d is moving away from us at a speed $v = H_0 d$.

A modern estimate of the expansion rate is $H_0 = 70\ \rm km\,s^{-1}\,Mpc^{-1}$.

(a) If you assume that galaxies have always been moving at a constant speed, what do you infer for the age of the universe? (Explain your answer, and give a value in Gyr, where 1 Gyr = 1 billion years. You can find relevant unit conversions in the back of the book if you need them.)

(b) Suppose that the expansion of the universe has been slowing down over time because of the decelerating effects of gravity. Is the implied age of the universe larger or smaller than the one you gave in part (a)? Explain.

(c) The inferred ages of the oldest star clusters are 13-14 Gyr. Given your answers to (a) and (b), is this cause for concern? Why or why not?

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Question 3

Assume that the light in the galaxies of Question 1 is produced entirely by stars like the sun. This is a poor assumption in detail, but it gives roughly the right answer for the calculation in this problem.

(a) What is the total mass M, in $M_\odot$ ($= 2.0 \times 10^{30}$ kg), contained in a sphere of radius 100 Mpc?

(b) If the Hubble constant is $H_0 = 70\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, what is the recession speed v of an object at a distance of 100 Mpc?

(c) For a test mass m at a distance $R = 100$ Mpc, what is the ratio α of the kinetic energy $mv^2/2$ to the gravitational potential energy GMm/R, according to the numbers you just derived?

(It is not immediately obvious that you can ignore the gravitational effects of all matter *outside* the 100 Mpc sphere, but it turns out that you can, for reasons we will discuss in class.)

(d) What is the value of α at $R = 200$ Mpc? (If you think carefully about this part, it will be easy.)

(e) Show that the general result is

$$\alpha = \frac{3H_0^2}{8\pi G\bar{\rho}} \ ,$$

where $\bar{\rho}$ is the average density of matter in the universe.

(f) Based on your results, do you expect the expansion of the universe to continue forever, or do you expect gravity to eventually halt and reverse this expansion? Explain your reasoning.

(Note: We have ignored two important components of the real universe, dark matter and dark energy, so the answer based on your calculation here will not necessarily be the same as the answer for the real universe.)