Astronomy 5682
Problem Set 5: Primordial Nucleosynthesis
Due Thursday, March 30

(a) The baryon-to-photon ratio

For a present-day microwave background temperature $T_{\rm CMB} = 2.73$ K, *show that the ratio* of baryons to photons is

$$\eta \equiv n_B/n_\gamma = 2.7 \times 10^{-8}\,\Omega_B h^2,$$

where $\Omega_B = \rho_B/\rho_c$ is the baryon density parameter with $\rho_c = 1.87 \times 10^{-29} h^2 \,{\rm g\,cm^{-3}}$ and $h \equiv H_0/(100\ {\rm km\,s^{-1}\,Mpc^{-1}})$. *What is the baryon-to-photon ratio* at a redshift $(1+z) = 10^5$? (Think carefully.)

It will help to remember that the number density of photons in a blackbody background of temperature T is

$$n_\gamma = \int_0^\infty \frac{1}{\pi^2 c^3}\frac{\omega^2 d\omega}{e^{\hbar\omega/kT}-1} = \left(\frac{kT}{\hbar c}\right)^3 \frac{1}{\pi^2}\int_0^\infty \frac{x^2 dx}{e^x - 1} \approx 0.244\left(\frac{kT}{\hbar c}\right)^3.$$

(b) The age-temperature relation

The relation between the photon temperature T and the radiation energy density in the early universe is

$$\epsilon_r(T) = 1.68 a_{\rm SB} T^4,$$

where the factor 1.68 accounts for the contribution of neutrinos.

Use this result and the Friedmann equation to *show that the relation* between age and temperature in the early universe is

$$t \approx 1.31 \left(\frac{1\,{\rm MeV}}{kT}\right)^2\ {\rm s}.$$

For your convenience: $a_{\rm SB} = 7.56 \times 10^{-15}\,{\rm erg\,cm^{-3}\,K^{-4}}$, $k = 8.62 \times 10^{-5}\,{\rm eV\,K^{-1}}$.

(c) Neutron freeze-out

When the universe is sufficiently hot and dense, the ratio of neutrons to protons is maintained at the thermal equilibrium value

$$\frac{n}{p} = e^{-Q/kT}, \quad Q \equiv (m_n - m_p)c^2 = 1.2934\,{\rm MeV},$$

by reactions like $n + \nu \longrightarrow p + e^-$ that convert neutrons to protons or vice versa. However, in an expanding, cooling universe, the reaction rate is a rapidly decreasing function of time, and once the time for a particle to experience a conversion reaction gets longer than the age of the universe, conversion stops, and the n/p ratio "freezes out." An approximate expression for the conversion time per particle in the relevant temperature range is

$$t_c \approx 0.5 \left(\frac{1\,{\rm MeV}}{kT}\right)^5\ {\rm s}.$$

By setting this equal to the expansion timescale t from part (b), *derive values* for the freeze-out temperature T_F and the corresponding freeze-out value of the n/p ratio.

(d) Time of deuterium synthesis

All usable routes to the synthesis of heavier elements start with the synthesis of deuterium ($n{+}p \longrightarrow D + \gamma$). Deuterium is the most weakly bound nucleus, with a binding energy $B_D = 2.22\,\mathrm{MeV}$. Naively, one might expect that deuterium would form when kT falls to $\sim B_D$, since the typical photon would no longer be able to dissociate a deuterium nucleus. However, as shown in part (a), the number of photons per baryon is enormous, so rare photons on the high-energy tail of the Planck distribution may be abundant enough to dissociate deuterium even when kT is substantially smaller than B_D. A crude approximation that gives about the right answer is to say that deuterium is synthesized rapidly when $e^{-B_D/kT} \approx \eta$, where η is the baryon-to-photon ratio. Using this approximation and a baryon density $\Omega_B h^2 = 0.02$, *deduce the temperature* T_D *and the time* t_D at which deuterium synthesis occurs.

(e) Neutron abundance at deuterium synthesis

After the freeze-out described in (c), the neutron abundance drops slowly because of free neutron decay, which has an e-folding time $\tau_n \approx 900\,\mathrm{s}$ (half-life 624 s). *What is the* n/p *ratio* at the time of deuterium synthesis t_D? (Remember that each neutron decay produces a proton.)

(f) The helium mass fraction

At the density and temperature prevailing at t_D, the reactions that produce deuterium and the reactions that convert deuterium to ^{4}He are very fast, but almost no ^{4}He is processed to heavier elements. Thus, to a first approximation (one that is good enough to get the ^{4}He and H abundances but not the abundances of the other light elements), *all* of the neutrons existing at t_D are processed into helium. From the calculation in (e), *what is the primordial helium mass fraction* Y (the fraction of all baryonic mass that is in helium)? *What is the hydrogen mass fraction* X? *Give results to two decimal places.*

The remaining three questions require some serious thought about the physical processes that determine the helium and deuterium abundances, and in some cases about the relative importance of effects that go in opposite directions. Since you only need the net sign of the effects, you can rely on physical argument without redoing your calculations. However, for (h) it may be useful to think about how your calculation would change as a check on your reasoning. You can also check (g) and (h) for consistency with Figure 9.5 of the textbook.

(g) The deuterium abundance and $\Omega_B h^2$

To a second approximation, a residual deuterium abundance "freezes out" when the timescale of reactions that process deuterium to heavier elements becomes longer than the age of the universe. For $\Omega_B h^2 = 0.02$, numerical calculations predict a deuterium-to-hydrogen ratio $D/H \approx 3 \times 10^{-5}$. If $\Omega_B h^2 = 0.1$, *would the predicted deuterium abundance be higher or lower?* Explain your answer.

It is believed that processing in stars can reduce the abundance of deuterium below the primordial value but cannot increase it. If an observation yielded a secure value of $D/H = 10^{-4}$ in some astronomical system, *what would the implication be* for $\Omega_B h^2$?

(h) The helium abundance and $\Omega_B h^2$

Would raising $\Omega_B h^2$ to 0.1 raise or lower the predicted helium abundance? Explain your answer. (Hint: think about what happens to t_D.)

(i) The helium abundance and the number of neutrino species

Suppose that an additional neutrino species were discovered, implying that the radiation energy density is $\epsilon_r(T) = 1.91 a_{\mathrm{SB}} T^4$ instead of $\epsilon_r(T) = 1.68 a_{\mathrm{SB}} T^4$. *Would this raise or lower* the predicted helium abundance? Explain your answer.