Lecture 11: The Internal Structure of Stars
Reading: Section 18-2

Key Ideas:

- Observational Clues to Stellar Structure
  - H-R Diagram
  - Mass-Luminosity Relation
- Hydrostatic Equilibrium
  - Balance between Gravity and Pressure
- Laws of Stellar Structure
  - Ideal Gas Law
  - Law of Gravity
- Core-Envelope Structure of Stars
- Degeneracy Pressure

From Stellar Properties to Stellar Structure

Any theory of stellar structure must explain the observed properties of stars.

Seek clues in <u>correlations</u> among the observed properties, in particular

- Mass
- Luminosity
- Radius
- Temperature

The Hertzsprung-Russell Diagram

The Main Sequence

Strong correlation between Luminosity and Temperature

- Holds for 85% of nearby stars including the Sun

All other stars differ in <u>size</u> from main sequence stars

<u>Giants & Supergiants</u>

- Very large radii, but same masses as main-sequence stars

<u>White Dwarfs</u>

- Very compact (about 1 earth radius) and Masses about $0.6M_{sun}$

Mass-Luminosity Relationship

For Main-Sequence stars

$$\left(\frac{L}{L_{sun}}\right) = \left(\frac{M}{M_{sun}}\right)^4$$

More massive main-sequence stars are more luminous. The fact that the mass is raised to the fourth power means that a small increase in mass equals a large increase in luminosity.

Not true of giants, supergiants or white dwarfs

Mean Stellar Density

Mean Density =Mass / Volume
Main Sequence: small range of mean density

Sun (G2V): ~1.6 g/cc
O5V Star: -0.005 g/cc
M0V~5 g/cc

Giants: $\sim 10^{-7}$ g/cc
Supergiants: $\sim 10^{-9}$ g/cc
White Dwarfs: $\sim 10^{5}$ g/cc

Note that for giants and supergiants in particular, the mean density averages out extremely high density (in the centers) and extremely low density (in the outer layers)

Interpreting the Observations:

Main-Sequence Stars:

Strong L-T relationship on H-R diagram
Strong M-L relationship

Implies they have similar internal structure & governing laws

Giants, Supergiants, & White Dwarfs

Must have very different internal structure than main-sequence stars of similar mass.

Basic Observational Fact:

The Sun's Radius is not changing as far as we can see. But we know that the force of gravity is pulling the Sun together. *Pressure* is resisting the force of gravity.

Hydrostatic Equilibrium

Gravity makes a star contract

Pressure makes a star expand

<u>Counteract each other:</u>

Gravity confines the gas against pressure
Pressure supports the star against gravity

Exact balance=<u>hydrostatic equilibrium</u>

Means the star neither expands nor contracts.

Laws of Stellar Structure 1: The Ideal Gas Law

Most stars obey the Ideal Gas Law

Pressure=Density x Temperature

In words:

Compressing a gas results in higher Pressure
Expanding a gas results in lower Pressure
Heating a gas results in higher Pressure
Cooling a gas results in lower Pressure

Laws of Stellar Structure 2: The Law of Gravity

Stars are very massive & bound together by their <u>Self-Gravity</u>

As star contracts, gets more gravitationally bound
As star expands, gets less gravitationally bound
Contraction releases energy

Expansion requires energy

As gas compresses, density increases, and gravitational energy is released so T increases, and the pressure goes up for two reasons.

Central Pressure and Temperature in the Sun

The observation that the Sun is in hydrostatic equilibrium (along with the mass and radius) gives us enough information to estimate the central pressure and temperature of the Sun.

We can use the mass and the radius to estimate how much the force of gravity is pressing down on the center of the Sun. Then we know that the pressure has to be sufficient to keep the center from collapsing further, but not big enough to cause it to expand.

P at center=350 billion atmospheres (1 atmosphere is the air pressure at the surface of the Earth)

Then use
Pressure = constant x density x temperature
To estimate the temperature

T=15 million K

Note that this temperature is determined by the hydrostatic equilibrium condition.

Core-Envelope Structure

Outer layers press down on inner layers
    The deeper you go into a star, the greater the pressure
The Gas Law says
    Greater pressure from hotter, denser gas
Consequence:
    Hot, dense, compact CORE
    Cooler, lower density, extended ENVELOPE

Example: The Sun

<u>Core</u>

Radius=0.25 R_{sun}
T=15 Million K
Density=150 g/cc

<u>Envelope</u>

Radius=R_{sun} (700,000 km)
T=5800 K (at surface)
Density=10^{-7} g/cc

The Essential Tension

The life of a star is a constant tug-of-war between Gravity and Pressure.

Tip the internal balance either way, and it will change the star's outward appearance (most obviously, it will expand or contract)

Internal Changes have *External* Consequences, which is helpful for figuring out what is going on inside of stars.

Degeneracy Pressure (see page 472)

The Pauli Exclusion Principle says that no two particles in a quantum cell (a very, very small volume of space) can have the same energy. Adding more particles at the same energy (=degenerate particles) is not allowed. If there are any more particles in the box, they must be moving.

Particles moving quickly=high pressure

Different kind of pressure than ideal gas pressure

Pressure depends *only* on density
Particles move because of quantum mechanical effects, not because of temperature.

Maximum pressure occurs when particles approach the speed of light.

<u>Bottom line</u>

Degeneracy pressure is important at high densities

Degeneracy pressure depends only on the density and is independent of the temperature

There is a maximum pressure that degenerate particles (electrons, neutrons) can exert.

Fermions and Bosons

Particles that obey the Pauli exclusion principle are called fermions.
Fermions include electrons, protons and neutrons.

Particles that don’t are called bosons. The most famous bosons are photons.