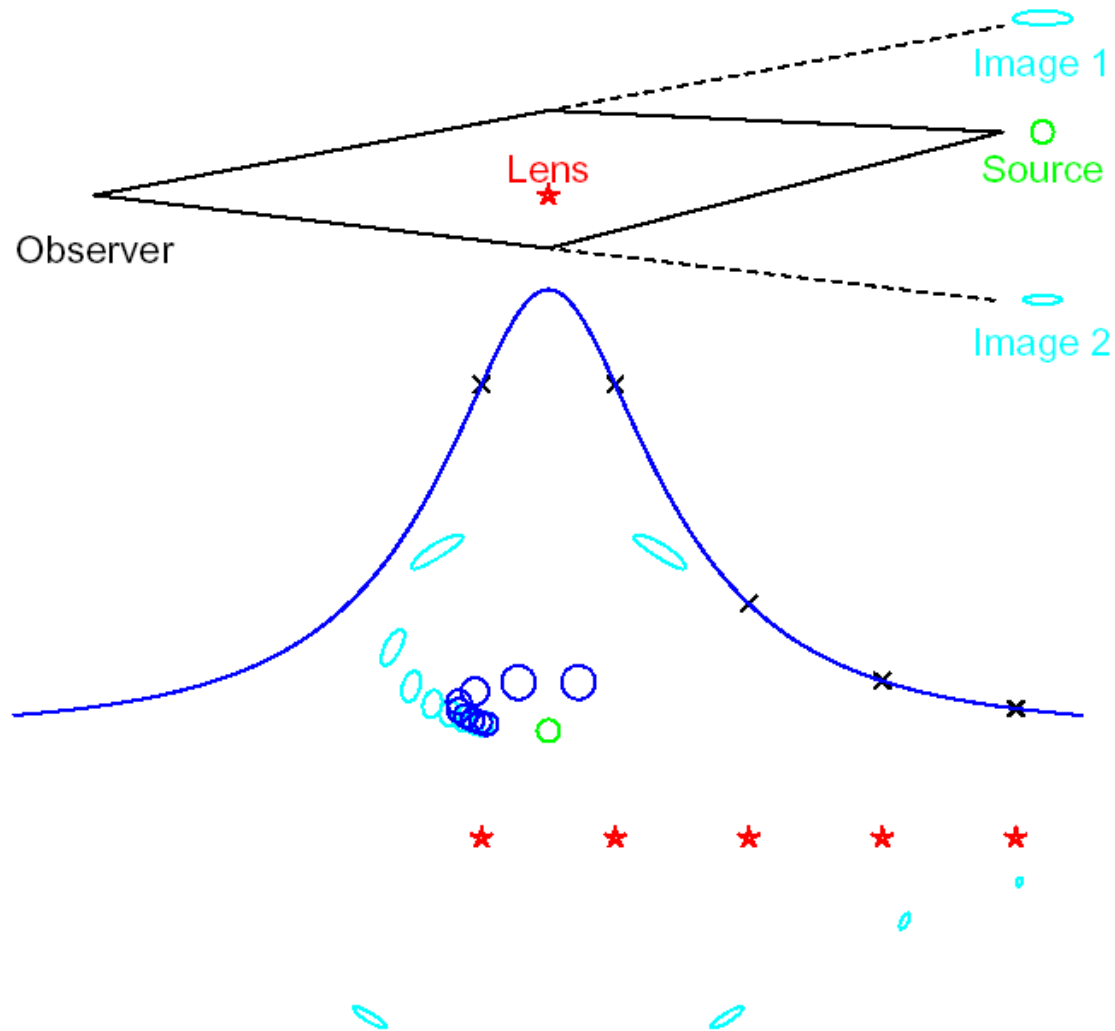


# CMDs, Einstein Radii, and Parallaxes in Planetary Microlensing Andy Gould (Ohio State)



# Mao & Paczynski

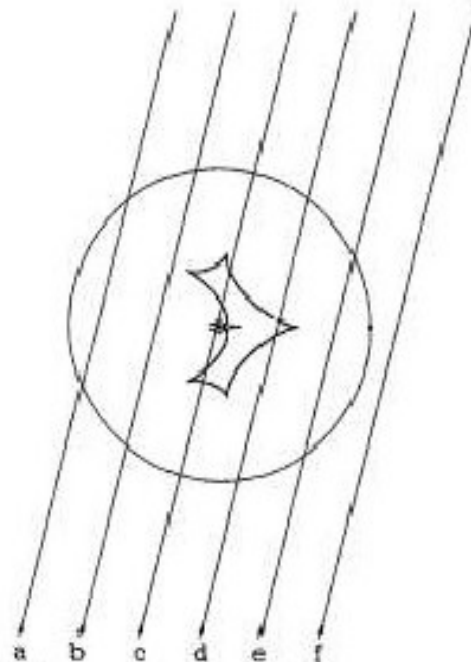
## Central Caustics

GRAVITATIONAL MICROLENSING BY DOUBLE STARS AND PLANETARY SYSTEMS

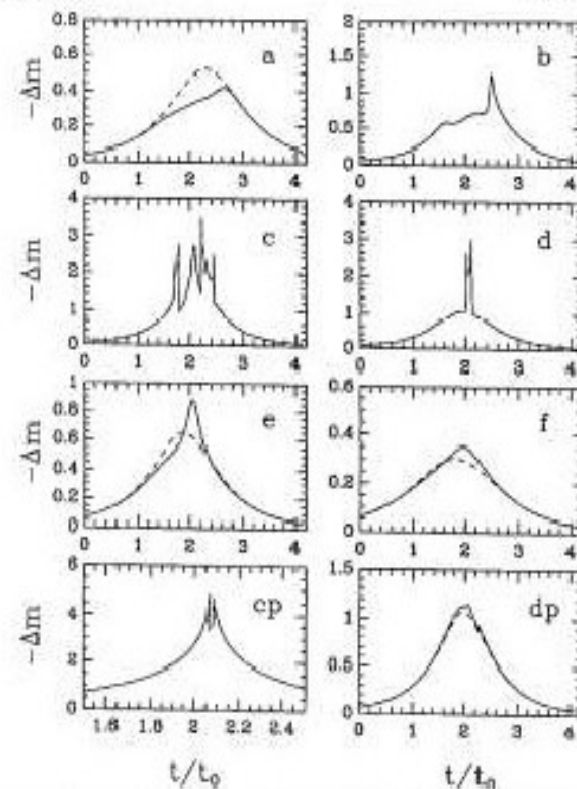
SHUDE MAO AND BOHDAN PACZYŃSKI

Princeton University Observatory, Princeton, NJ 08544

Received 1991 March 12; accepted 1991 April 2



1.—Geometry of microlensing by a binary, as seen in the sky. The  $\gamma$  star of  $1 M_{\odot}$  is located at the center of the figure, and the secondary of  $0.001 M_{\odot}$  is located on the right, on the Einstein ring of the  $\gamma$ . The radius of the ring is 1.0 mas for a source located at a distance of 8 kpc. The two complicated shapes around the primary are



the lens. The effect is strong even if the companion is a planet. A massive search for microlensing of the Galactic bulge stars may lead to a discovery of the first extrasolar planetary systems.

# Gould & Loeb

## Planetary Caustics

DISCOVERING PLANETARY SYSTEMS THROUGH GRAVITATIONAL MICROLENSSES

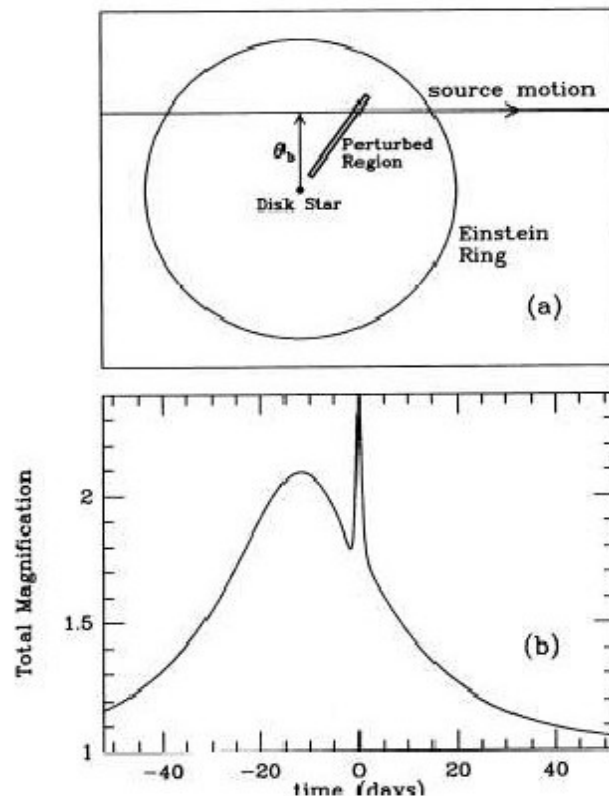
ANDREW GOULD AND ABRAHAM LOEB  
Institute for Advanced Study, Princeton, NJ 08540  
*Received 1991 December 26; accepted 1992 March 9*

### 5. OBSERVATIONAL REQUIREMENTS

Two distinct steps are required to observe a planetary system by microlensing. First, one must single out a disk star which happens to be microlensing a bulge star. Second, one must observe this star often enough to catch the deviation in the light curve due to the planet. The first step involves the observation of millions of bulge stars on the order of once per day. The second step involves the observation of a handful of stars many times per day. In the following we give a rough outline of what is required for each of these steps.

While observations from one site would be useful, there are advantages to be gained by observing from several sites. First,

two telescopes that were totally committed. Third, in view of the fleeting nature of the events, it would seem prudent to build in some redundancy in case of bad weather at a particular site. Thus, the optimal scheme would employ, say, a dozen telescopes. Each of these would be committed to carry out two observations per night. During the near-December season,



# 6 Features

# & 6 Parameters

- Time of Peak
- Height of Peak
- Width of Peak
- Time of Perturbation
- Height of Perturbation
- Width of Perturbation
- $t_0$
- $u_0$
- $t_E$
- Trajectory angle:  $\alpha$
- Planet-star separation:  $b$
- Planet/star mass ratio:  $q$

# Clear Division Between

## Known Knowns & Known Unknowns

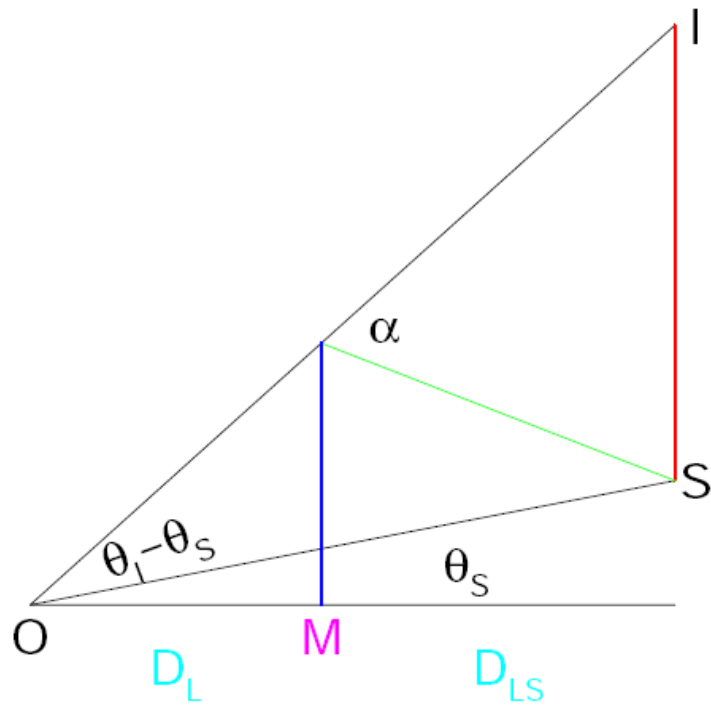
- Planet/star mass ratio
- Separation in units of Einstein radius
- Planet mass
- System distance
- Planet/star projected physical separation

And inevitably: **Unknown Unknowns**

- **Planet/star 3-D separation**
- **Planet Orbital Motion**

# What Happened?

- Most planets (so far) are in high-mag events
- High-mag events: richer and harder to interpret
- Planetary events are more susceptible to higher-order effect than regular events

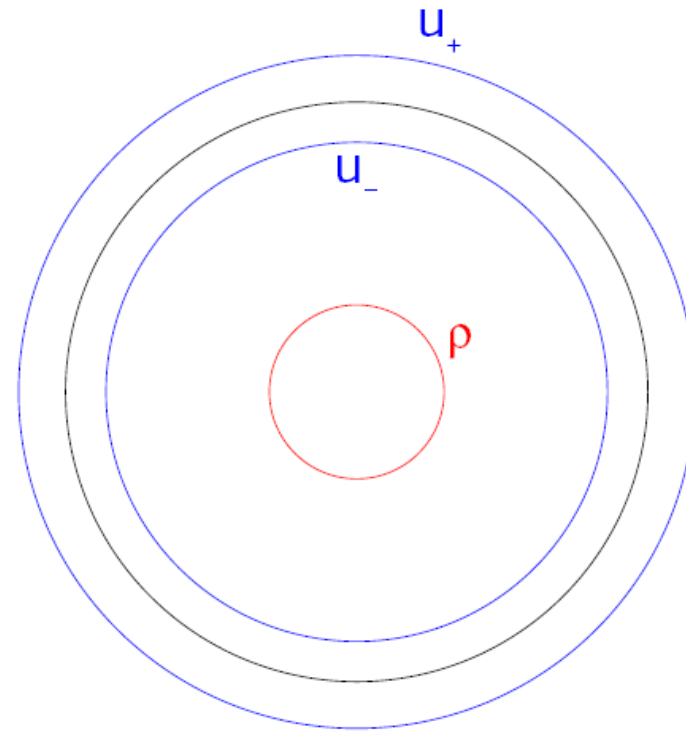


$$(\theta_l - \theta_s) D_S = \alpha D_{LS}$$

$$\alpha = 4GM / (D_L \theta_l c^2)$$

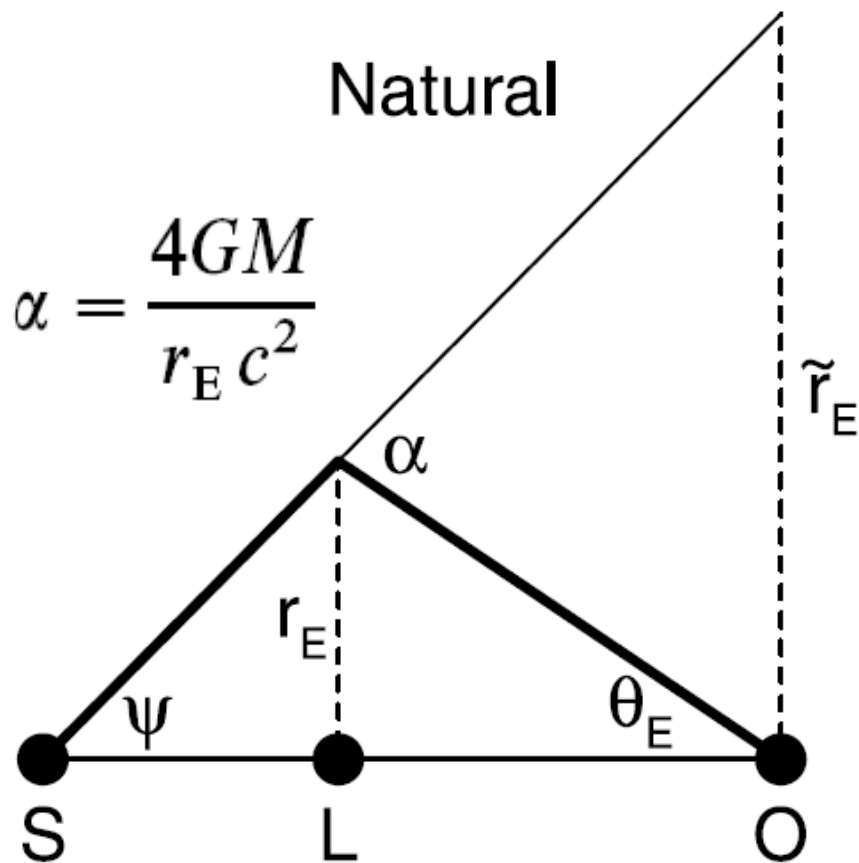
$$(\theta_l - \theta_s) \theta_l = \theta_E^2 = (4GM/c^2) (D_{LS} / D_L D_S)$$

$$\theta_l / \theta_E = [u \pm (u^2 + 4)^{1/2}] / 2; \quad u = \theta_S / \theta_E$$





# Relation of **Mass** and **Distance** to **Lensing Observables**



$$\alpha / \tilde{r}_E = \theta_E / r_E$$

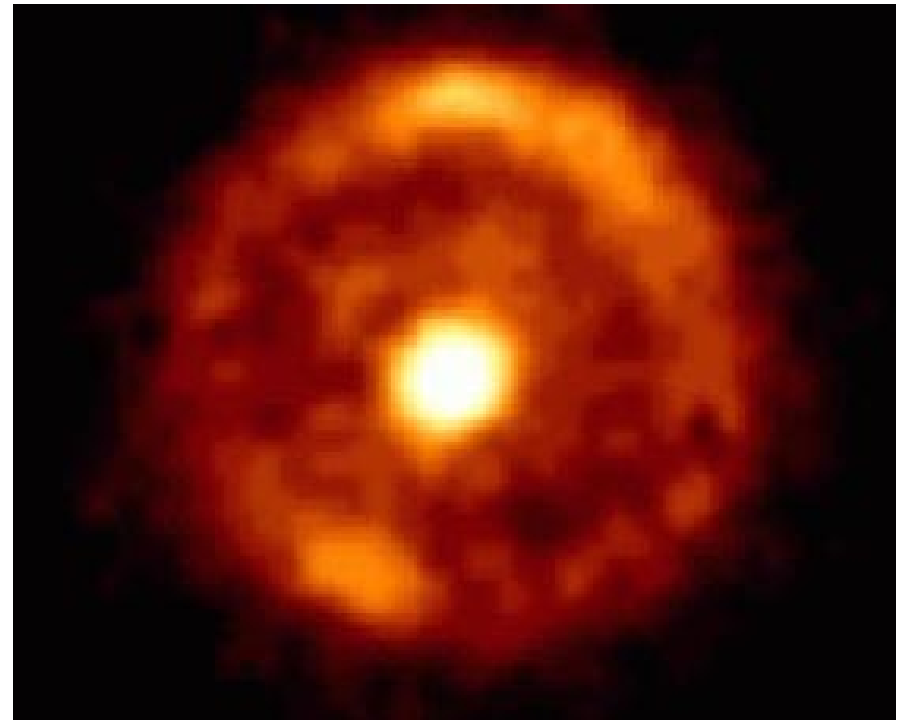
$$\theta_E \tilde{r}_E = \alpha r_E = \frac{4GM}{c^2}$$

$$\theta_E = \alpha - \psi = \frac{\tilde{r}_E}{D_l} - \frac{\tilde{r}_E}{D_s} = \frac{\tilde{r}_E}{D_{\text{rel}}}$$

$$\tilde{r}_E = \sqrt{\frac{4GM D_{\text{rel}}}{c^2}}$$

$$\theta_E = \sqrt{\frac{4GM}{D_{\text{rel}} c^2}}$$

# Some (Macro) Einstein Rings

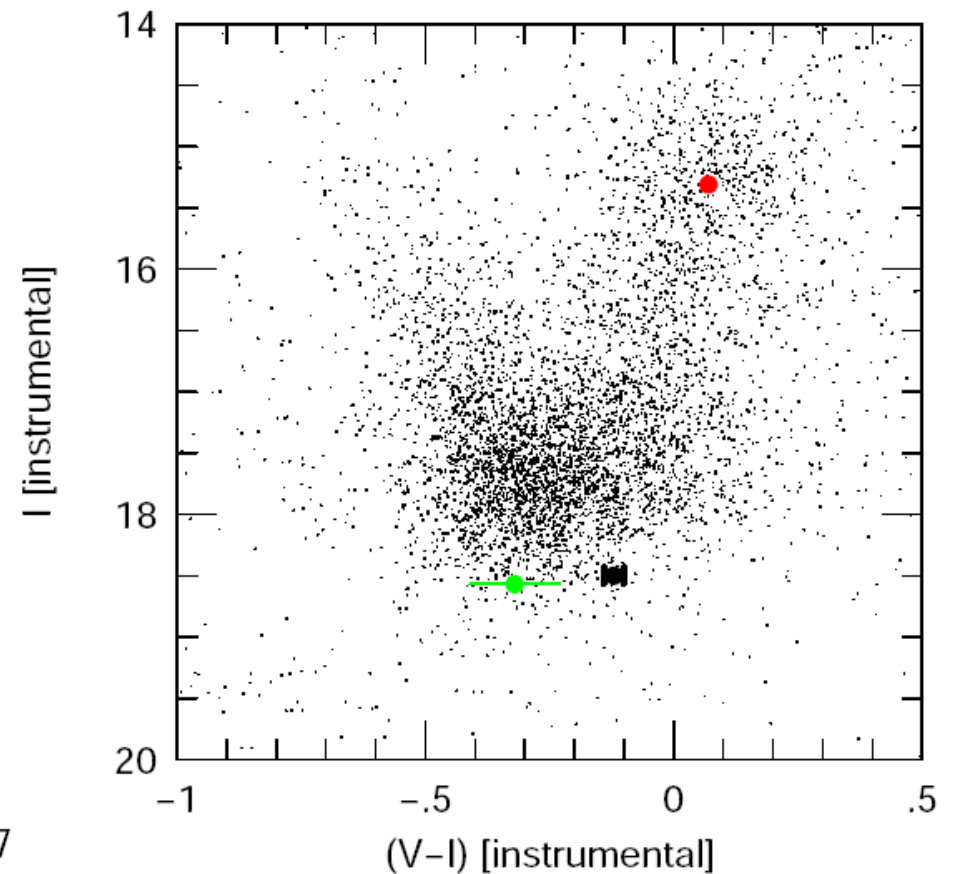
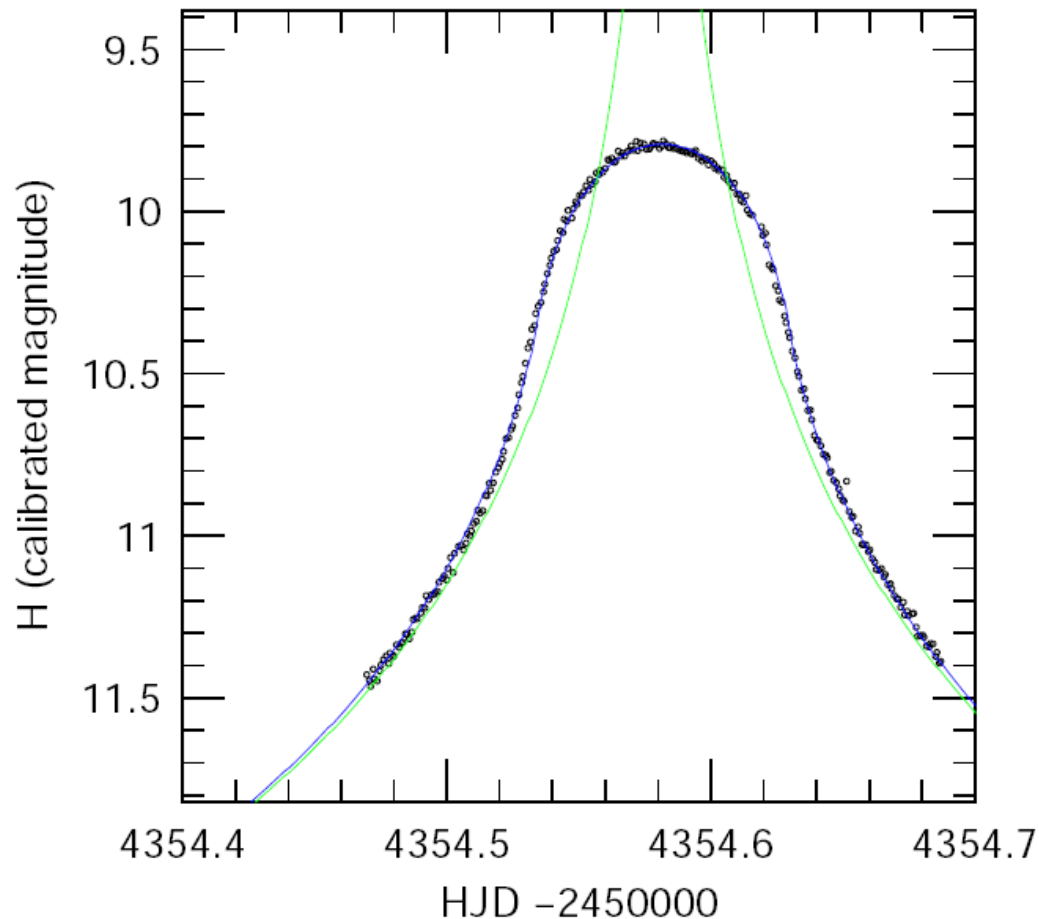


# To measure angular Einstein radius:

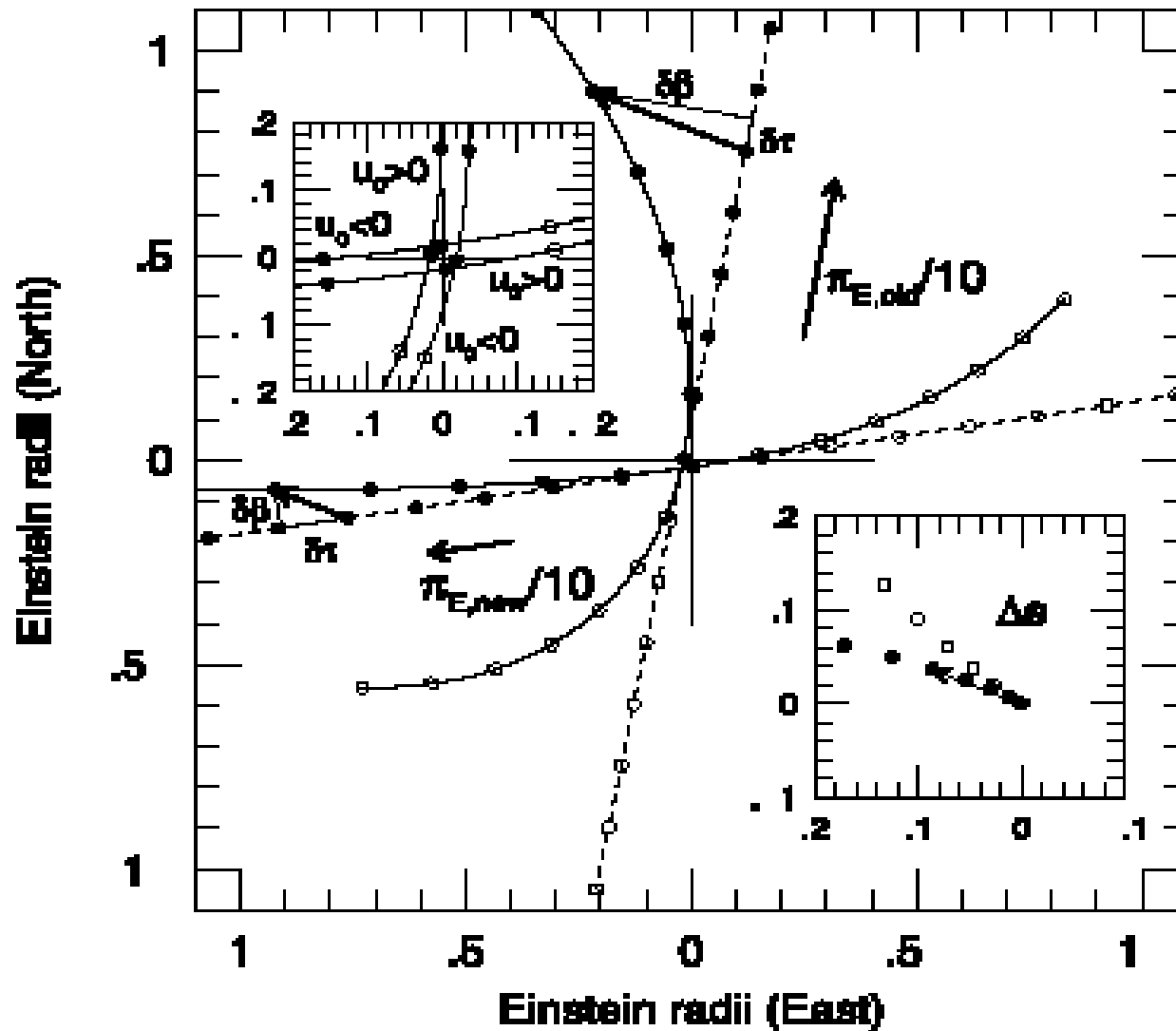
$t_{*}=0.047$  day;  $t_E=14.4$  day;  $\theta_{*}=1.05 \mu\text{as}$

$$\theta_E = \theta_{*}(t_E/t_{*}) = 322 \mu\text{as}$$

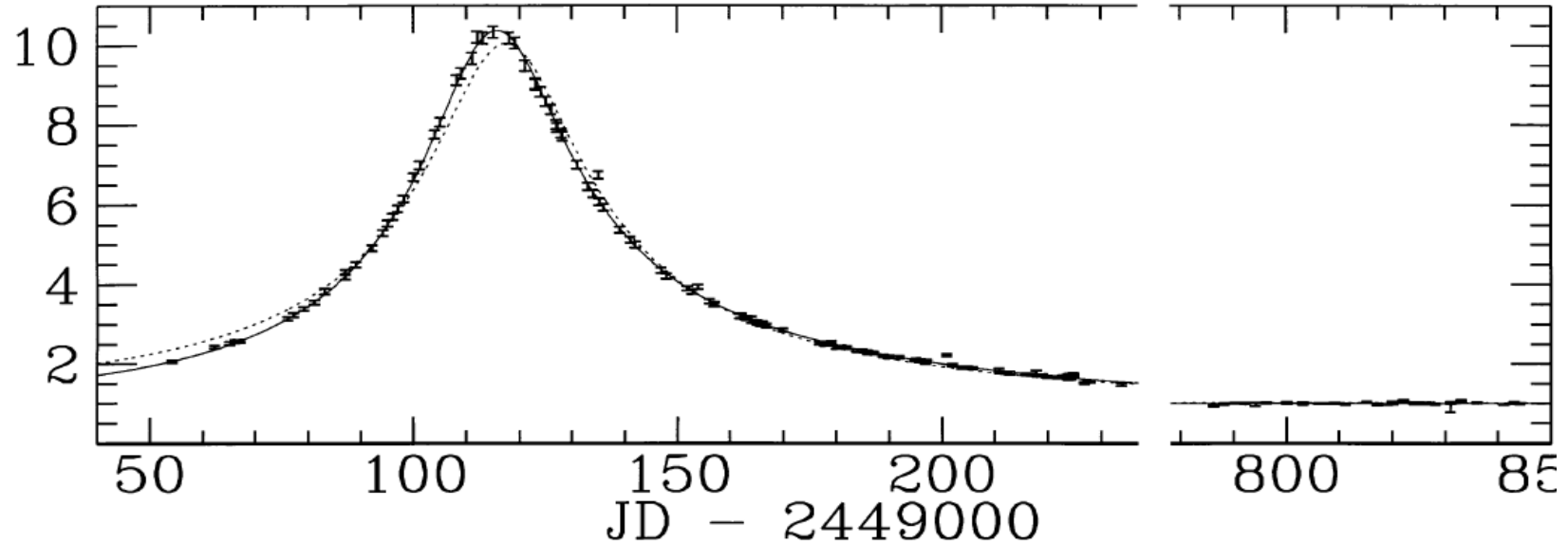
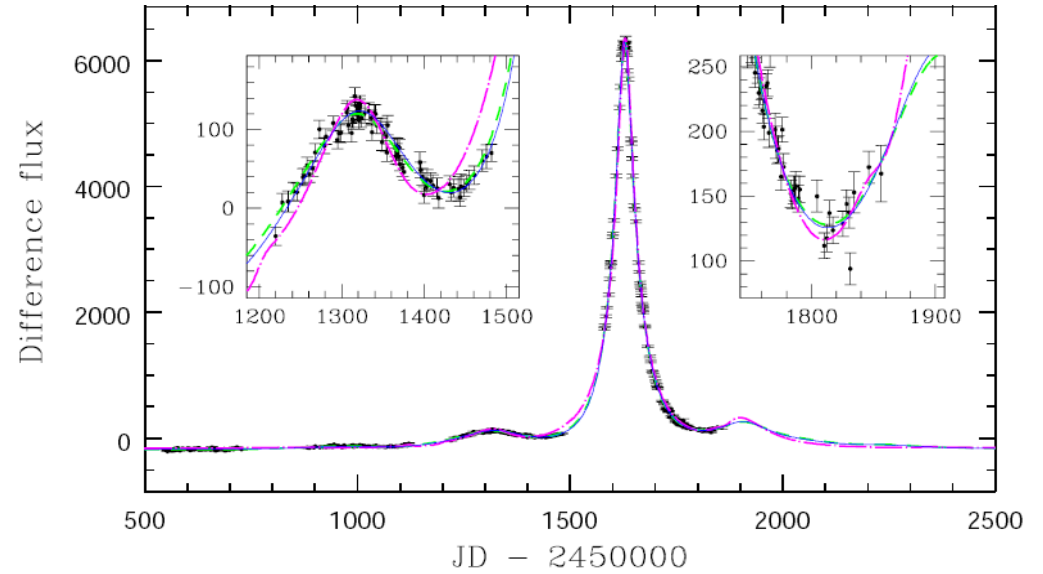
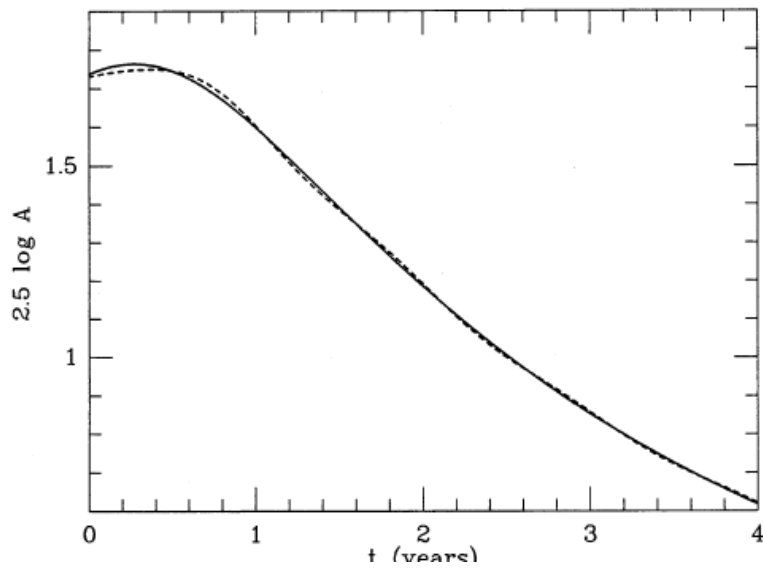
MOA-2007-BLG-400



# Accelerated motion of Earth induces non-linear motion in Einstein ring



# To determine microlens parallax: Measure Resulting Lightcurve Distortion



# Another Crackpot Idea: Terrestrial Microlens Parallaxes

PHOTON STATISTICS LIMITS FOR EARTH-BASED PARALLAX MEASUREMENTS OF  
MACHO EVENTS

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Enrico Fermi Institute and Department of Physics, University of Chicago, 5640 S. Ellis Avenue, Chicago, IL 60637-1433

*Received 1995 March 8; accepted 1996 January 11*

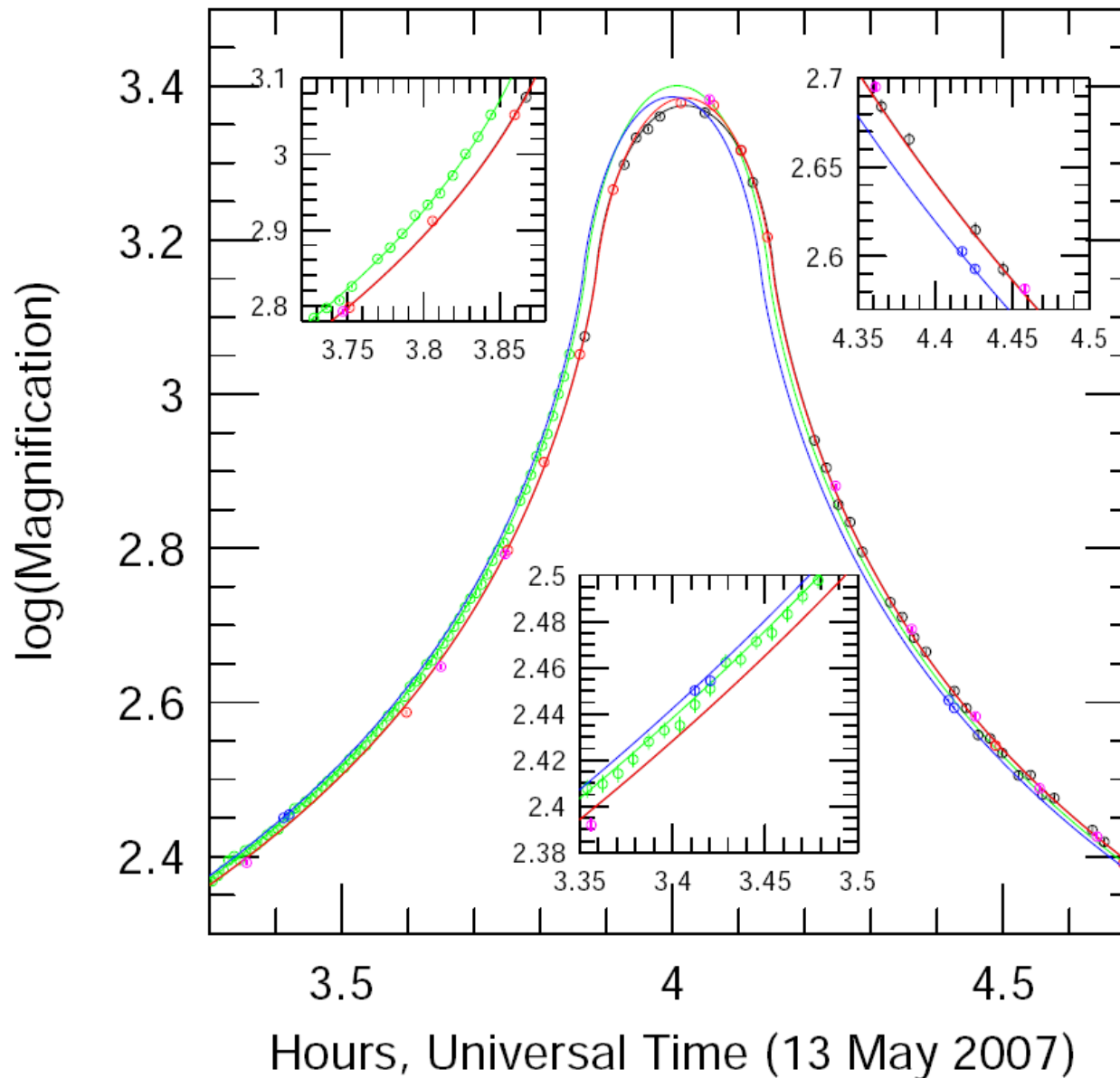
## ABSTRACT

We analyze the limitations imposed by photon-counting statistics on extracting useful information about MACHOs from Earth-based parallax observations of microlensing events. We find that if one or more large (say 2.5 m) telescopes are dedicated to observing a MACHO event for several nights near maximum amplification, then it is possible, in principle, to measure the velocity of the MACHO well

issues. We thank Andrew Gould for pointing out an error in the original version of this manuscript. This research was

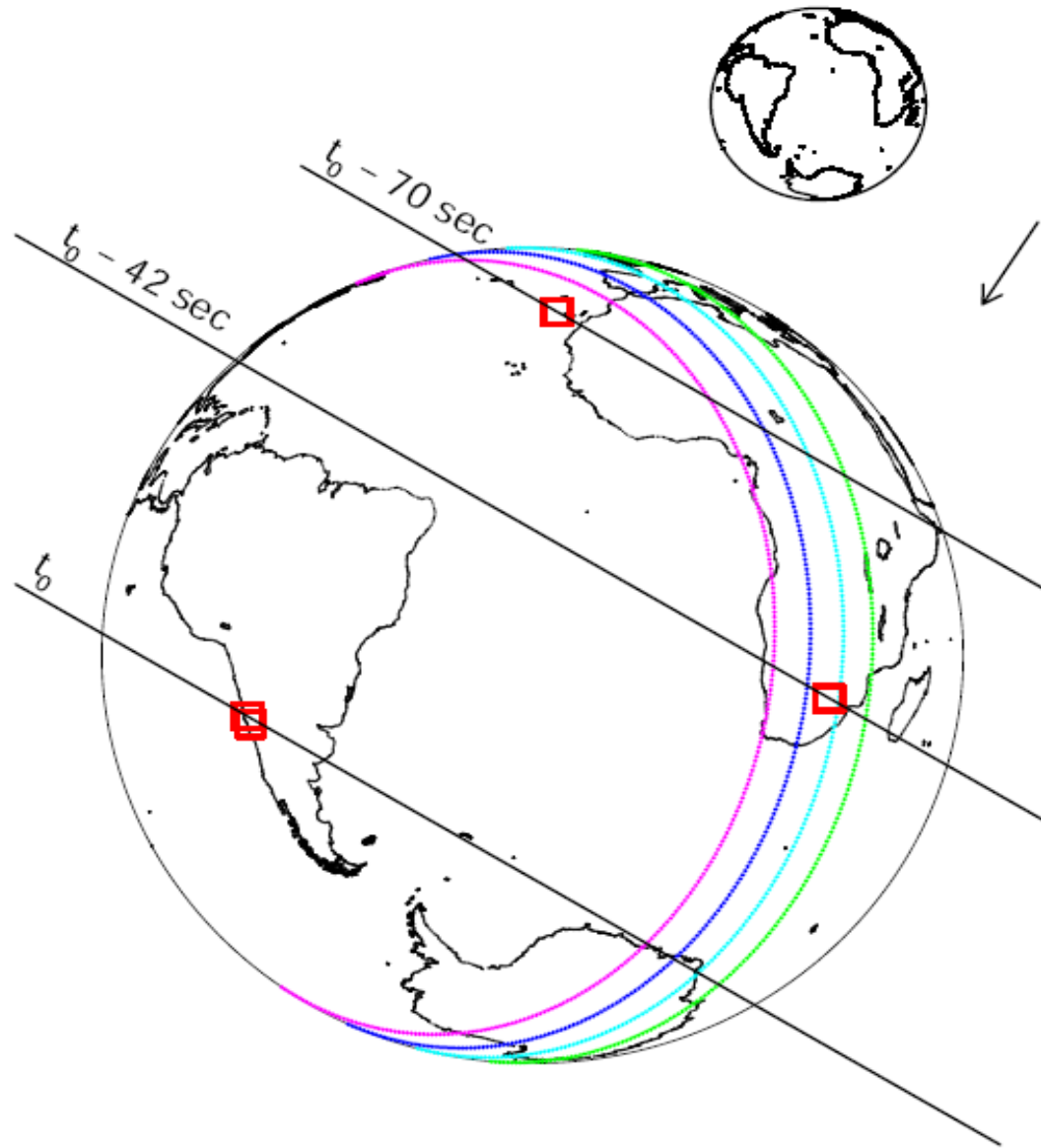
# OGLE-2007-BLG-224

Canaries South Africa Chile



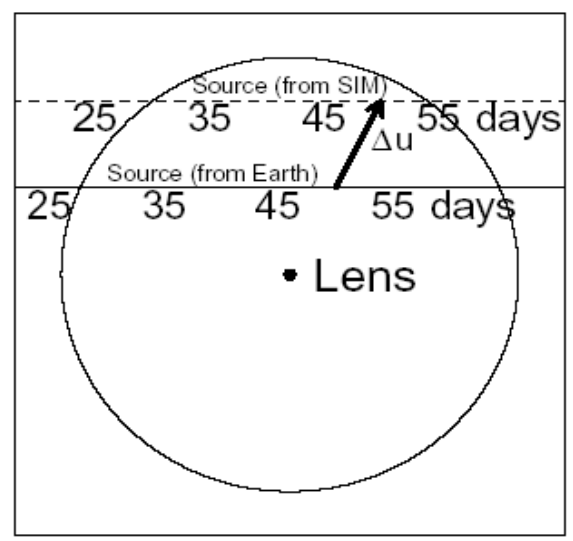
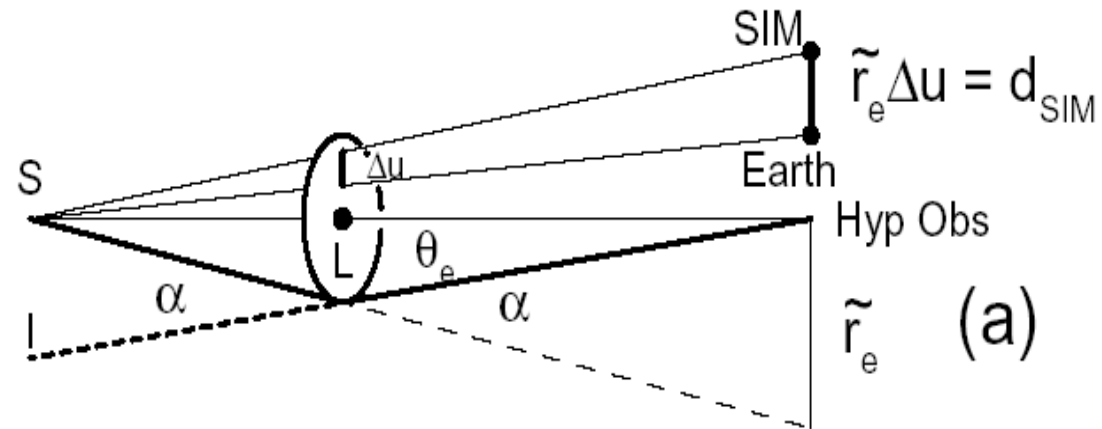
# Terrestrial Parallax:

## Simultaneous Observations on Earth



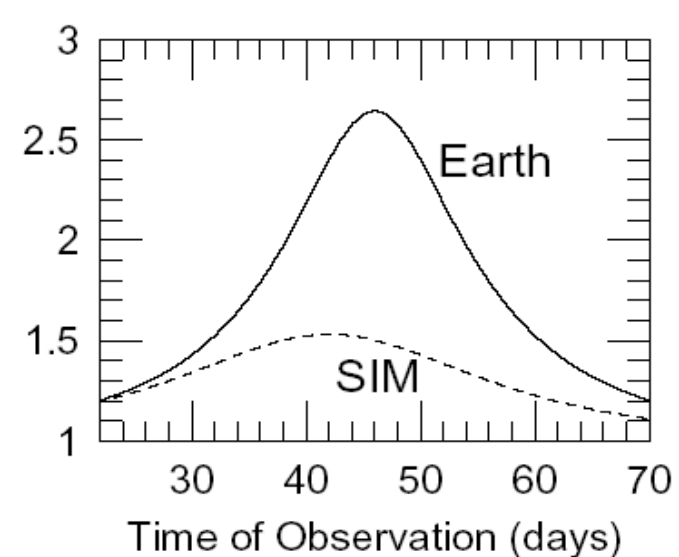
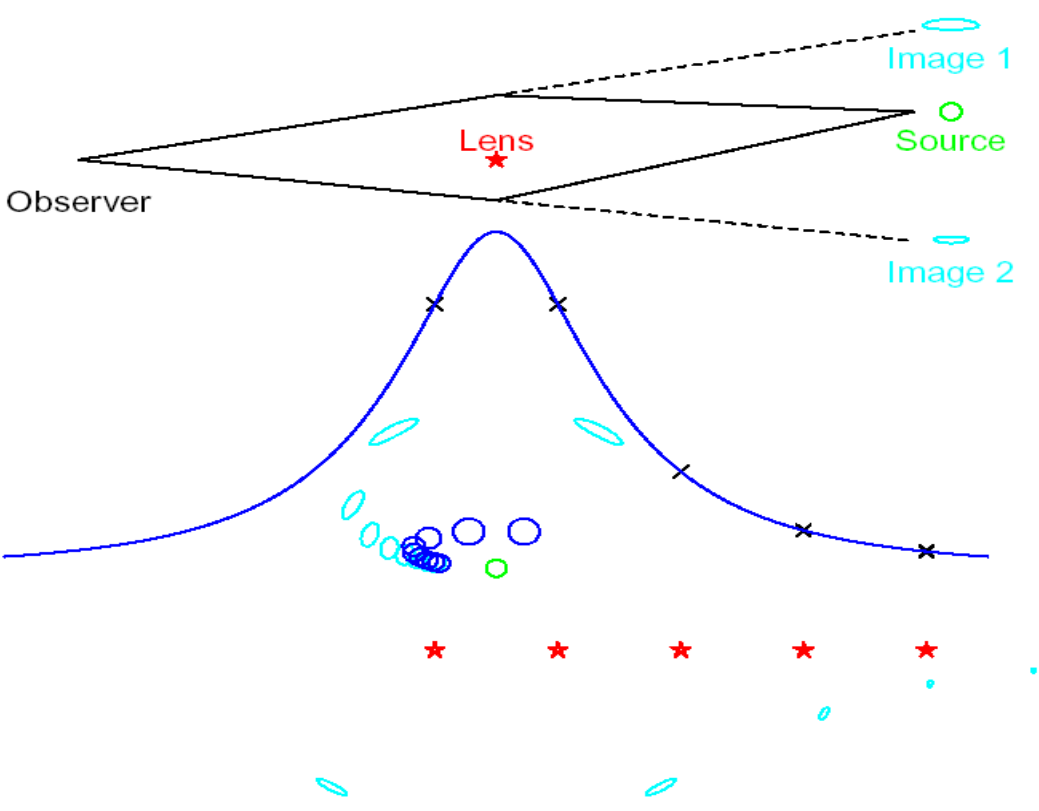


# Space-Based Parallaxes & Einstein Radii : SIM



$$\tilde{r}_e = \frac{d_{SIM}}{\Delta u}$$

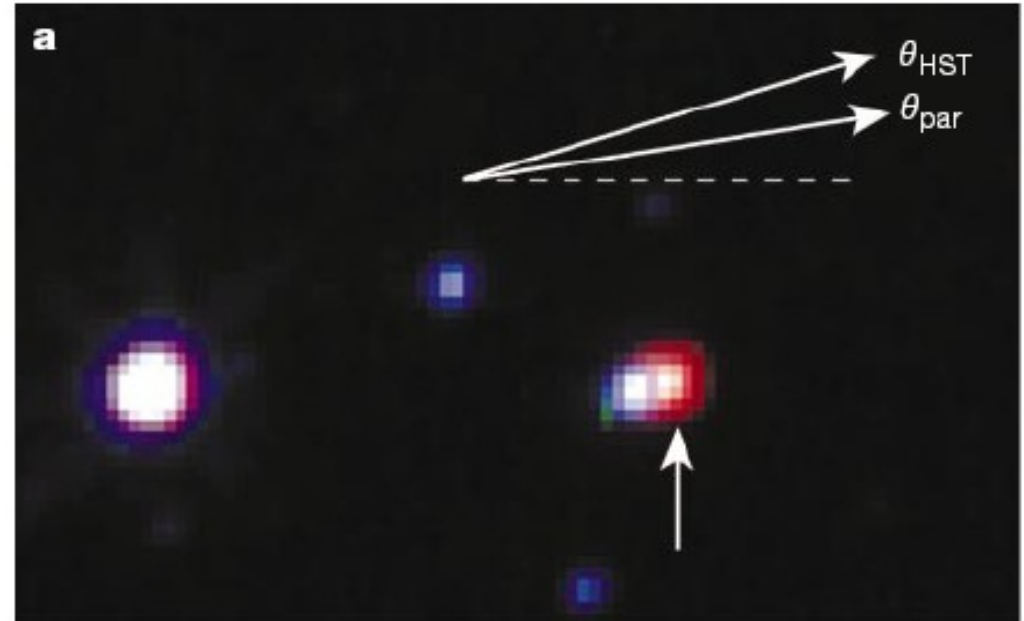
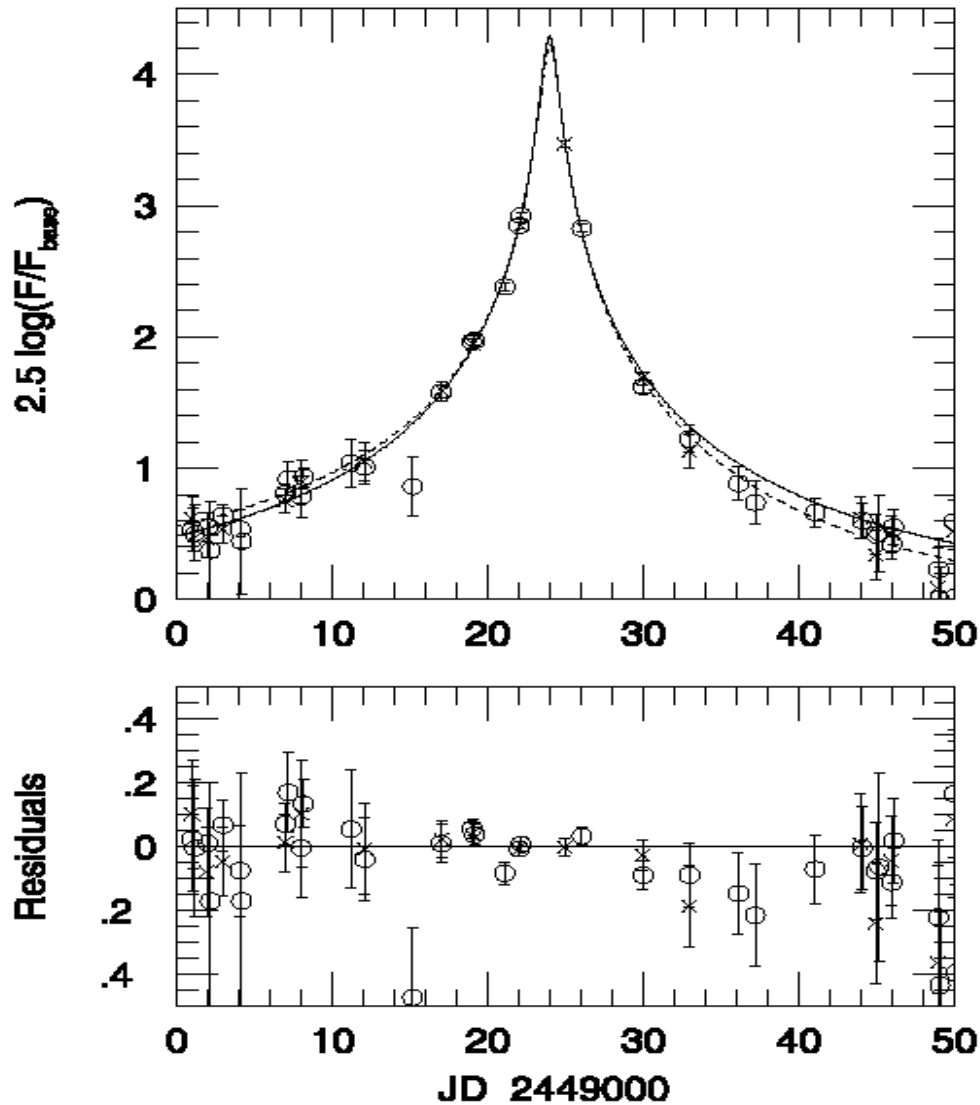
(b)



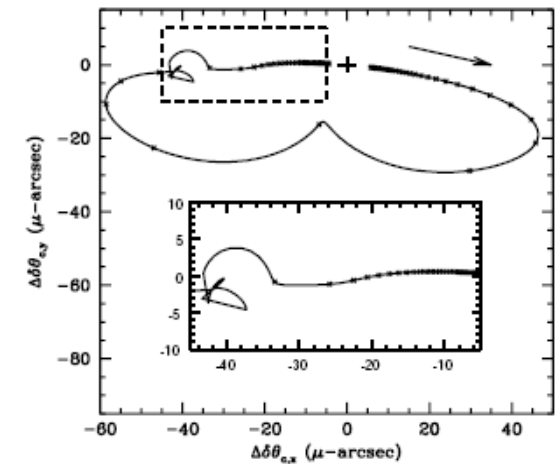
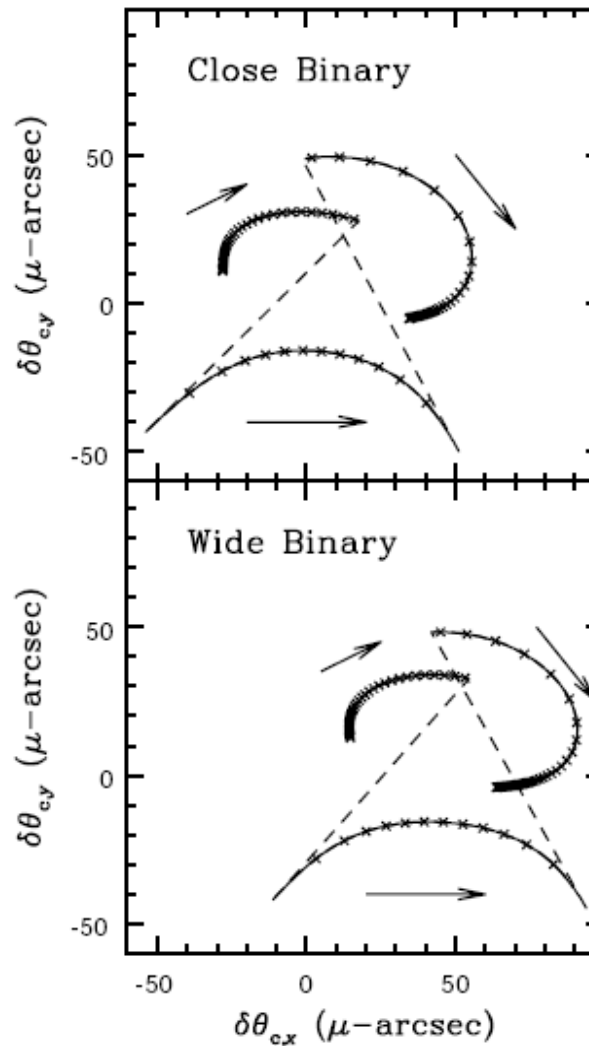
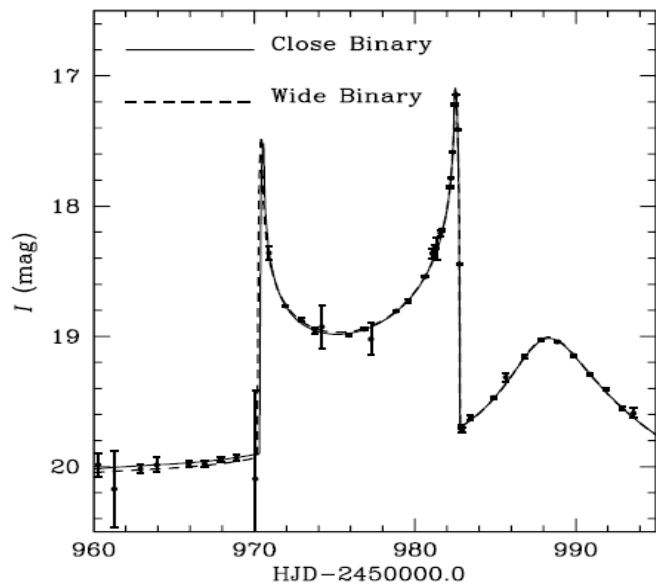
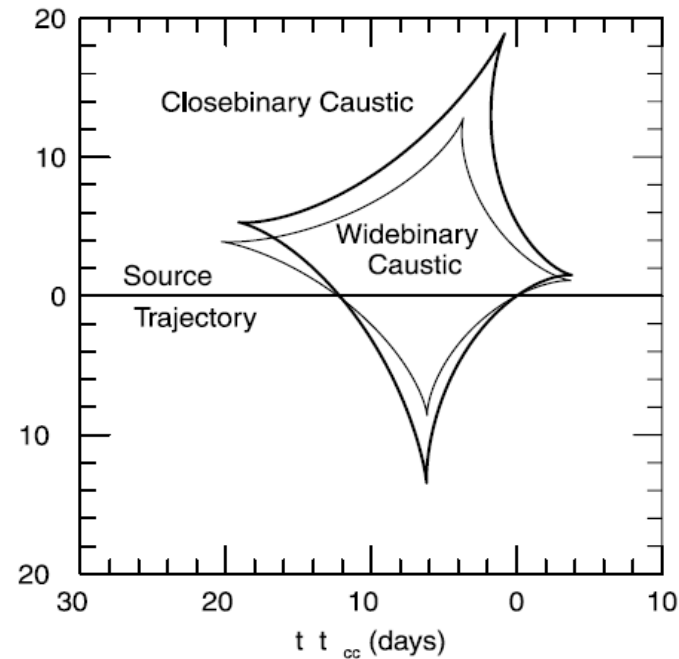
(c)

# MACHO-LMC-5

## Angular Einstein radius from proper motion

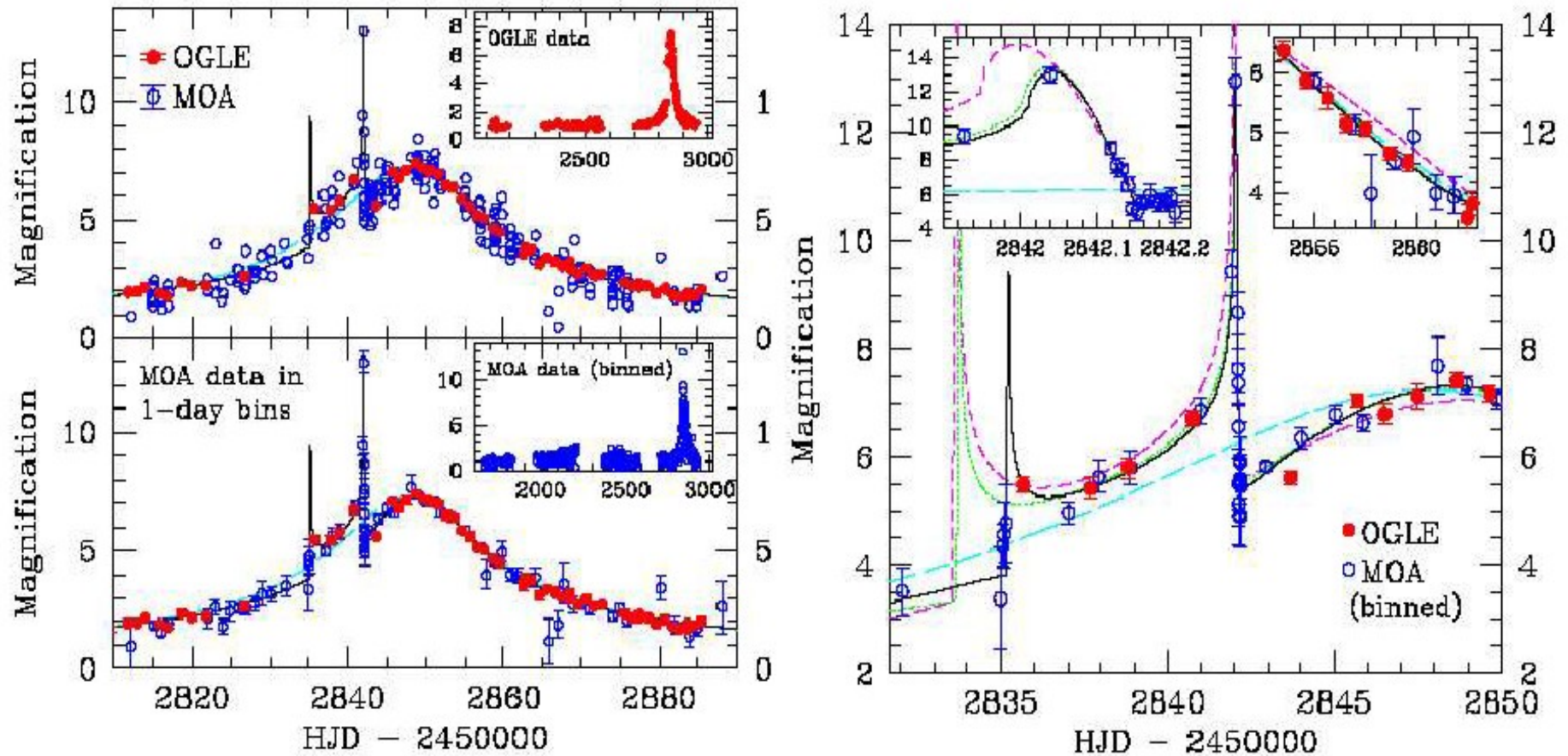


# Geometry, Photometry, Astrometry

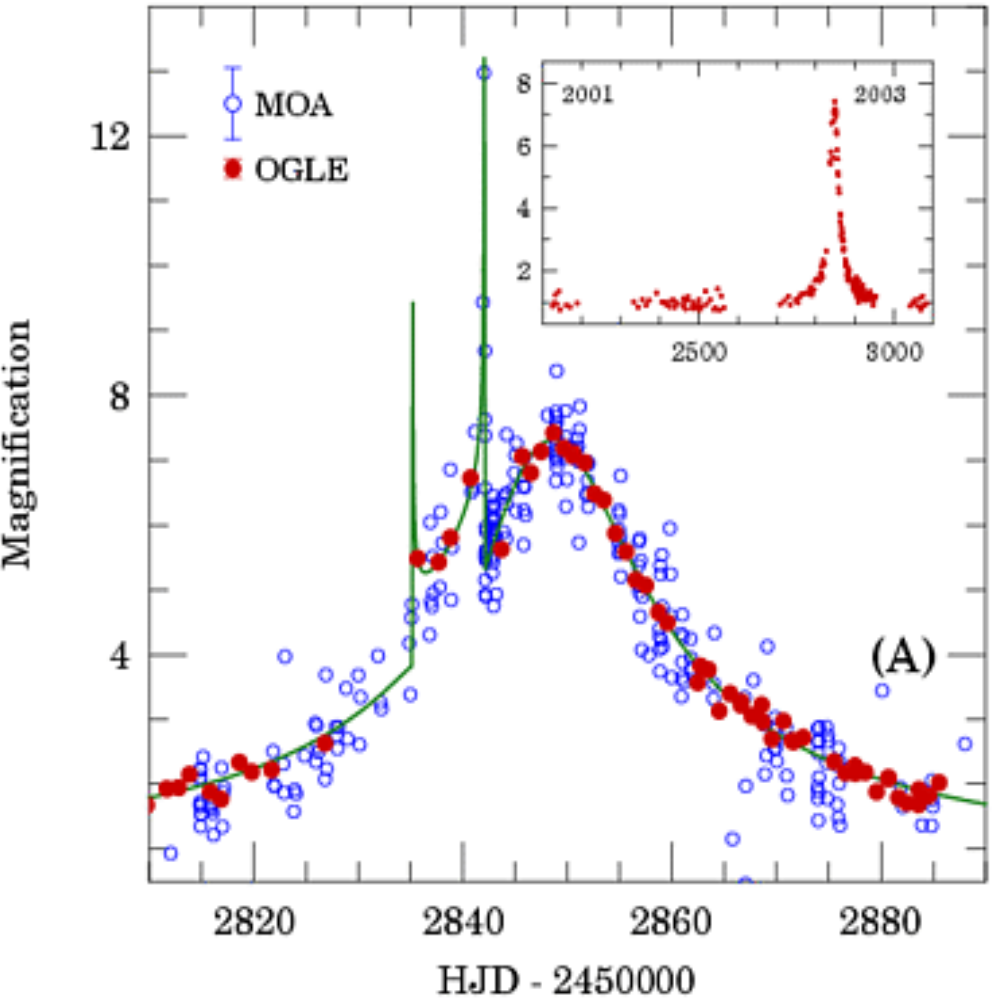
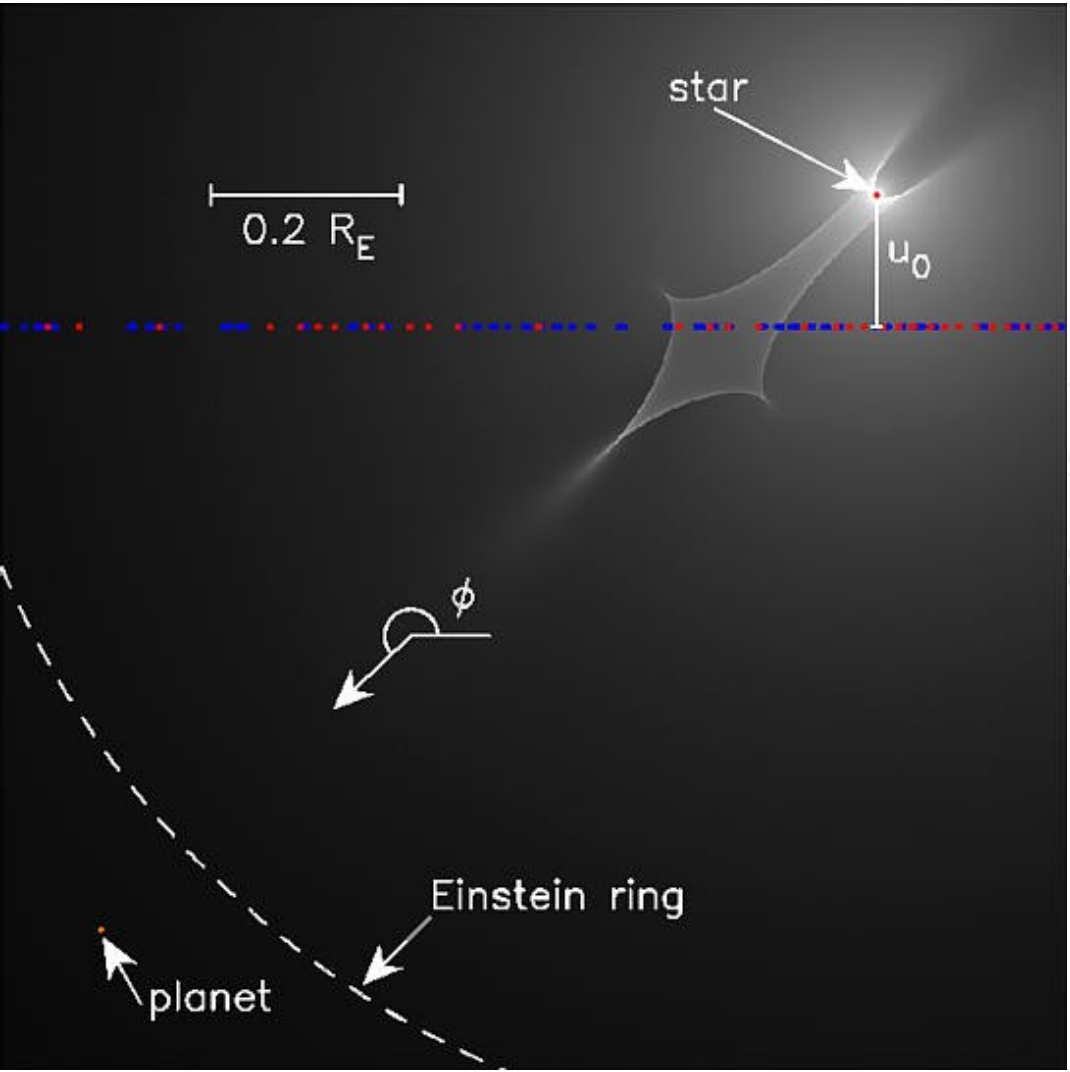


# OGLE-2003-BLG-235/MOA-2003-BLG-53

## “Pure-Survey” Detection

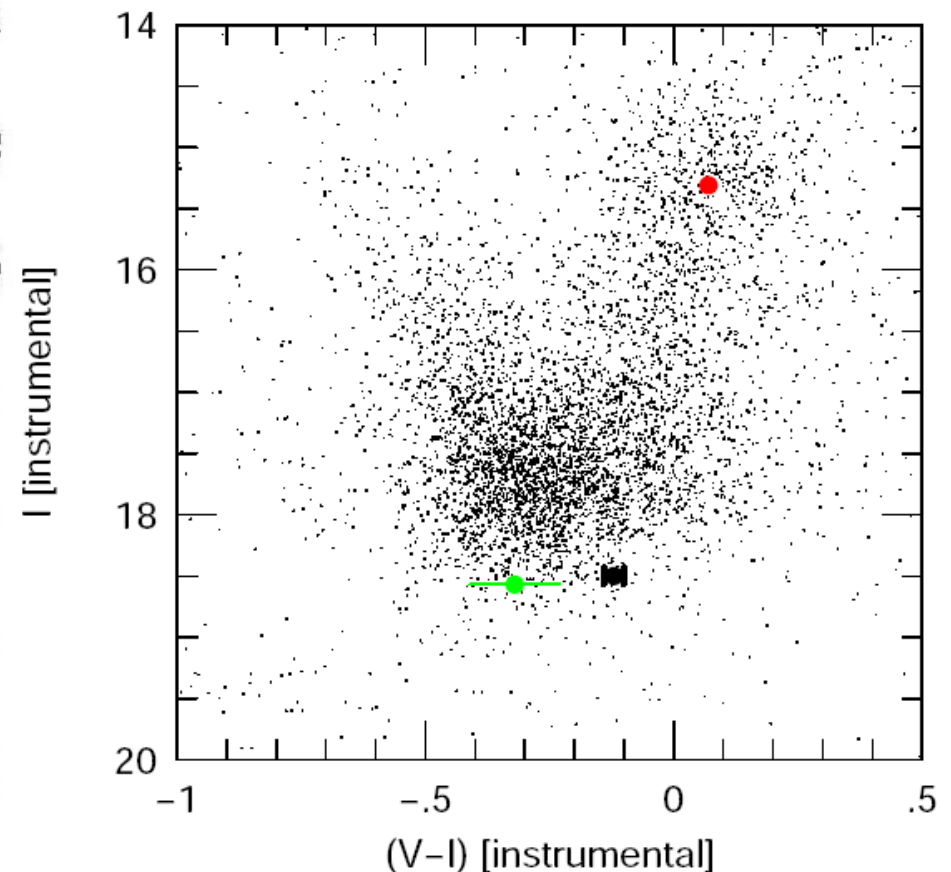
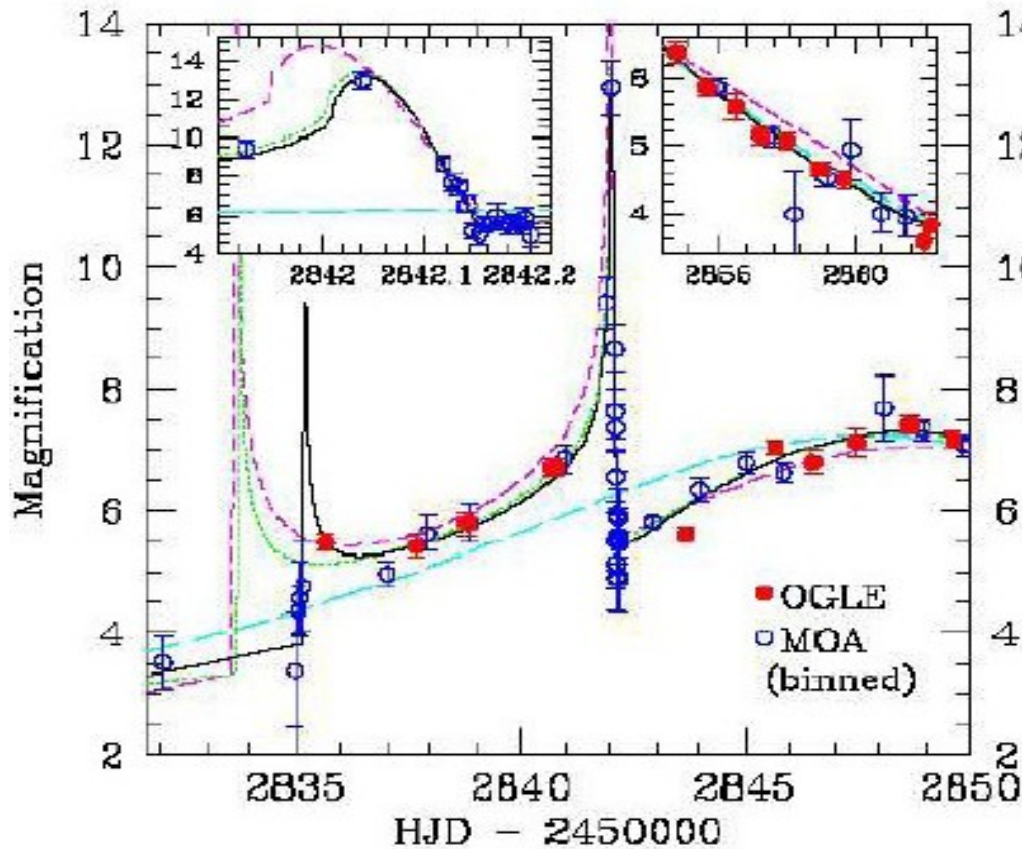


Bond et al. 2004, ApJ, 606, L155



# Resolving Degeneracies I: The Angular Einstein Radius ( $\Rightarrow$ proper motion)

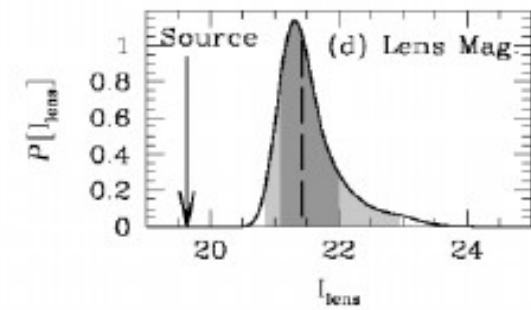
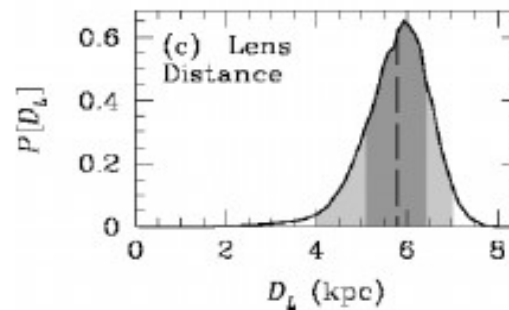
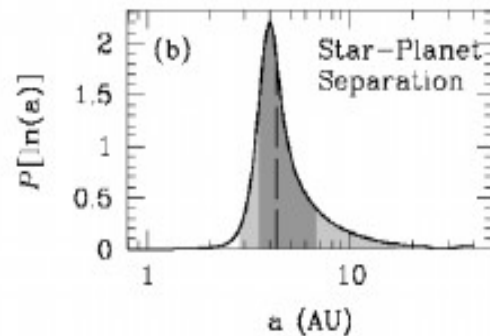
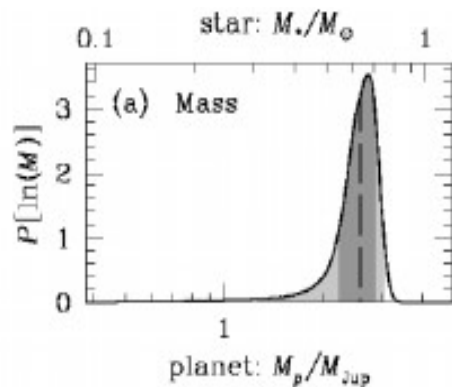
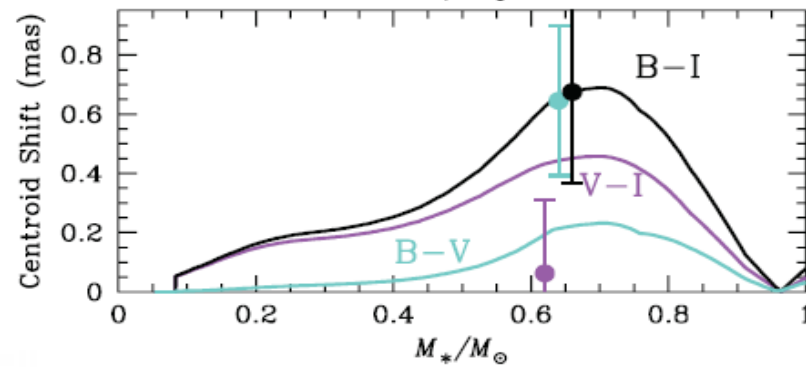
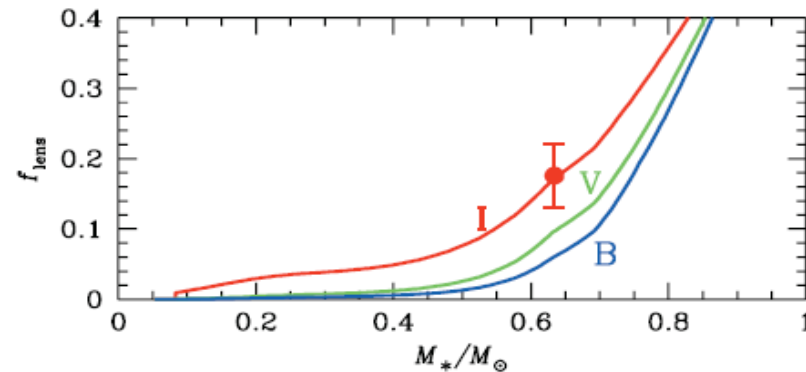
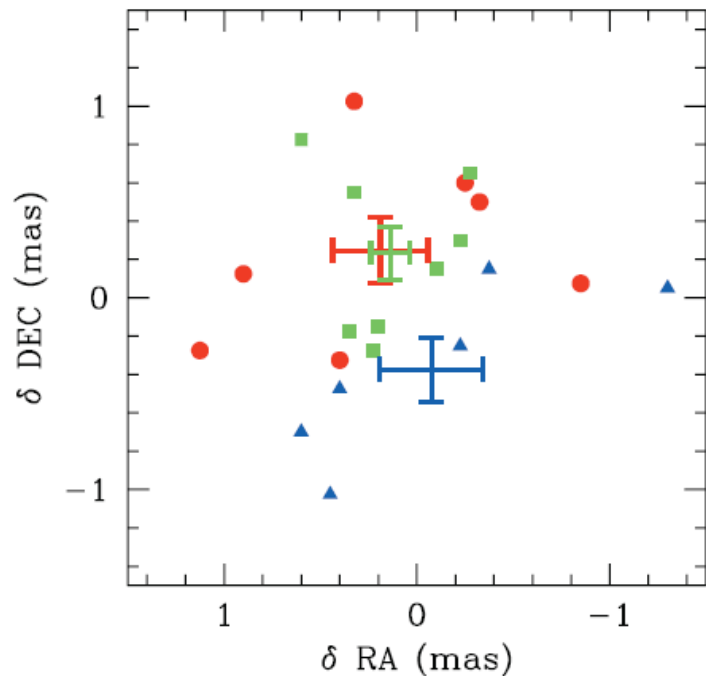
$$\theta_E = \sqrt{\frac{4GM}{D_{\text{rel}} c^2}}$$



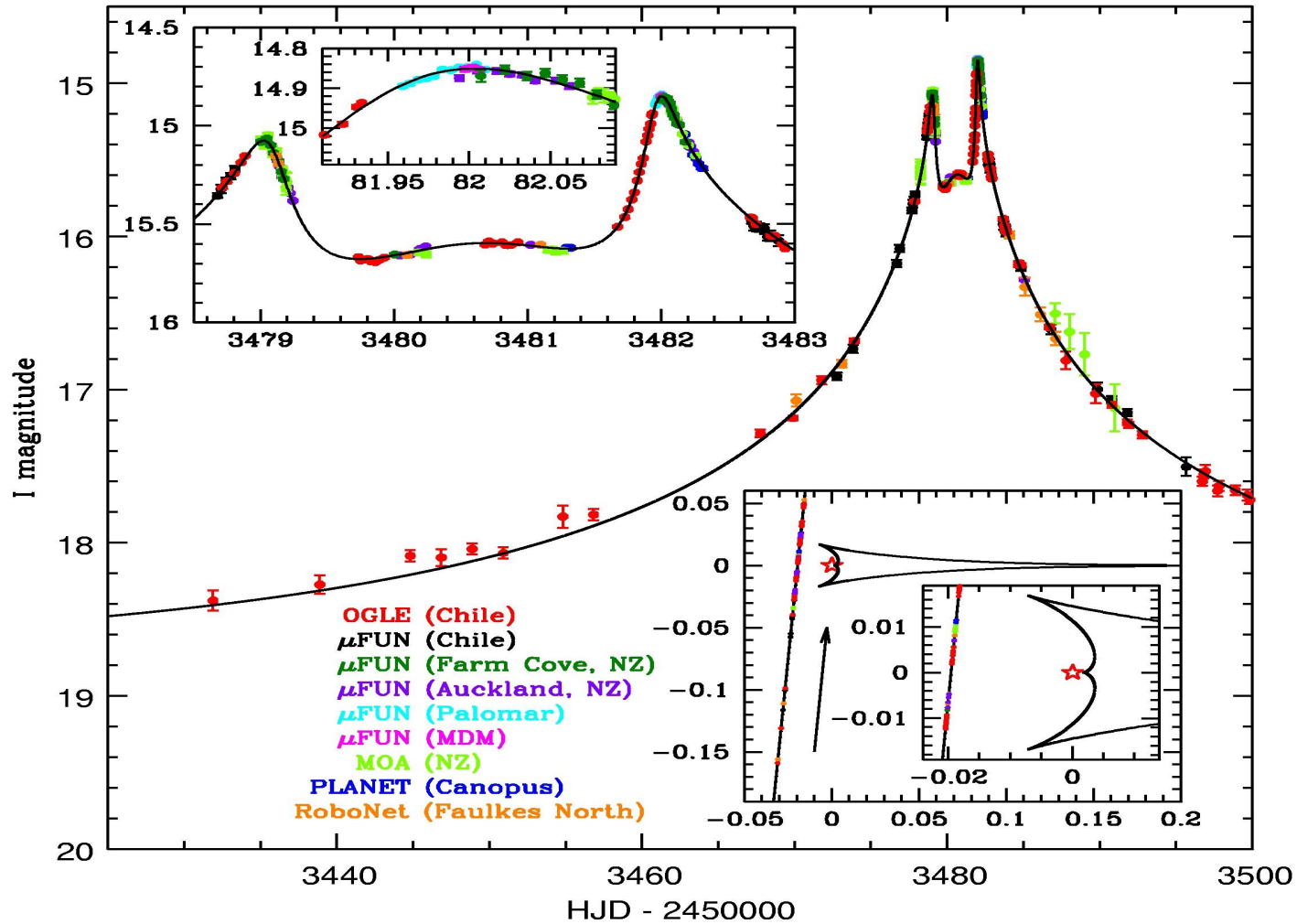
# Resolving Degeneracies II:

## Centroid Motion

(using known proper motion)



# 2<sup>nd</sup> Microlensing Planet

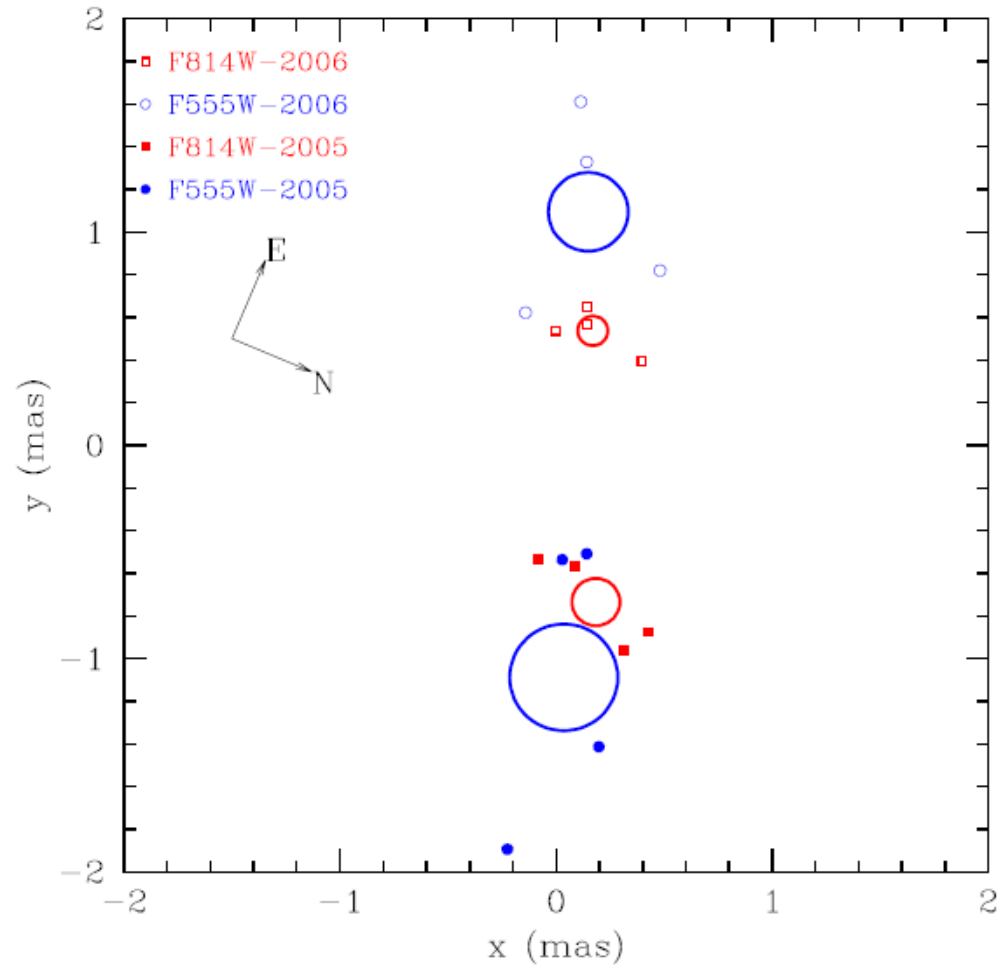
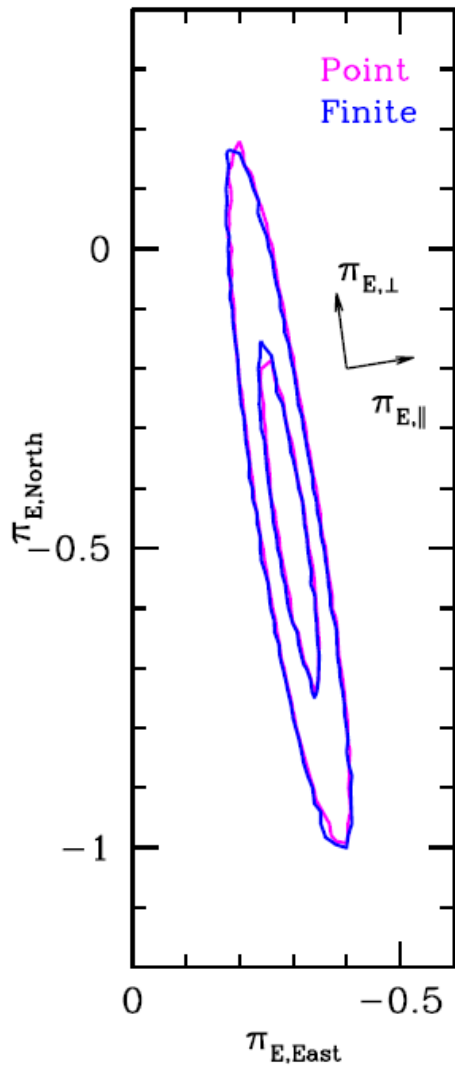


Udalski et al. 2005, ApJ, 628, L109

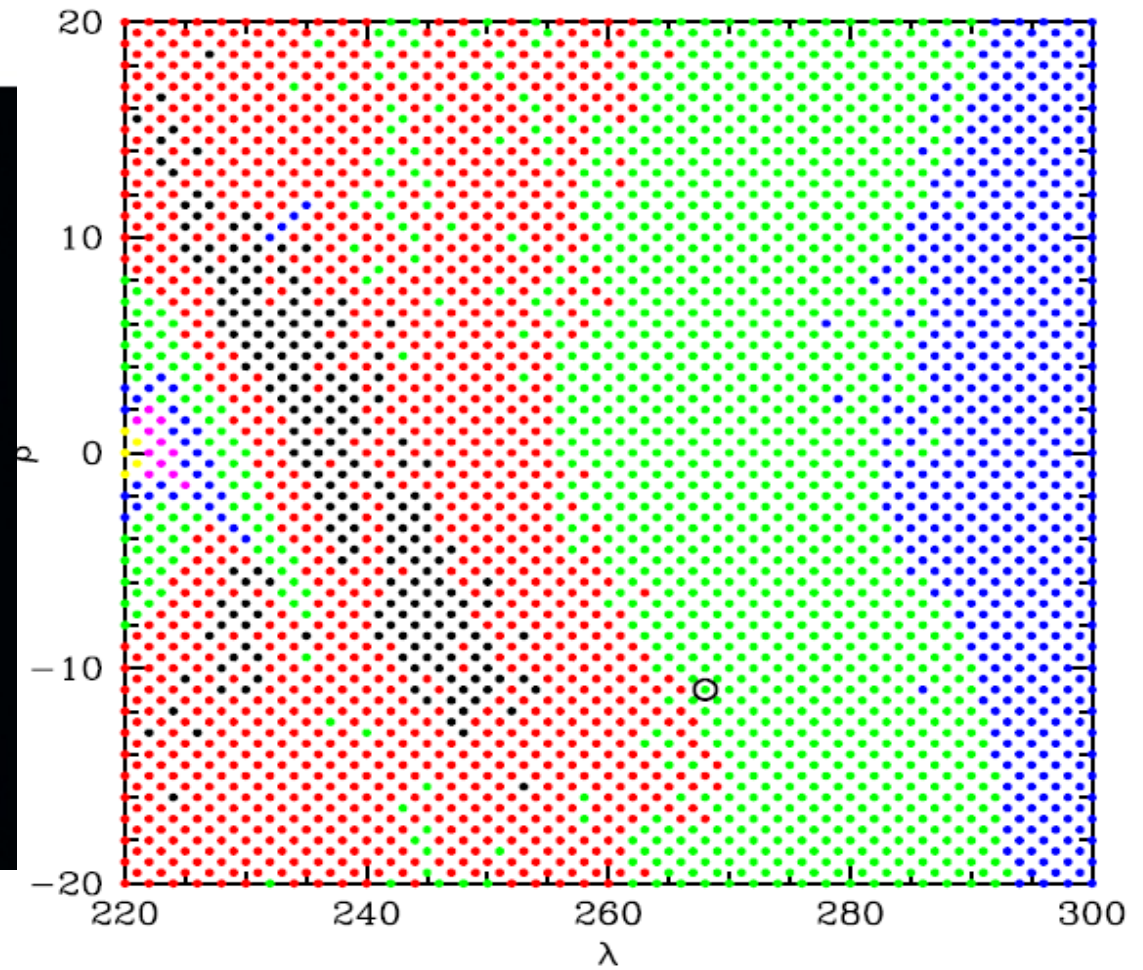


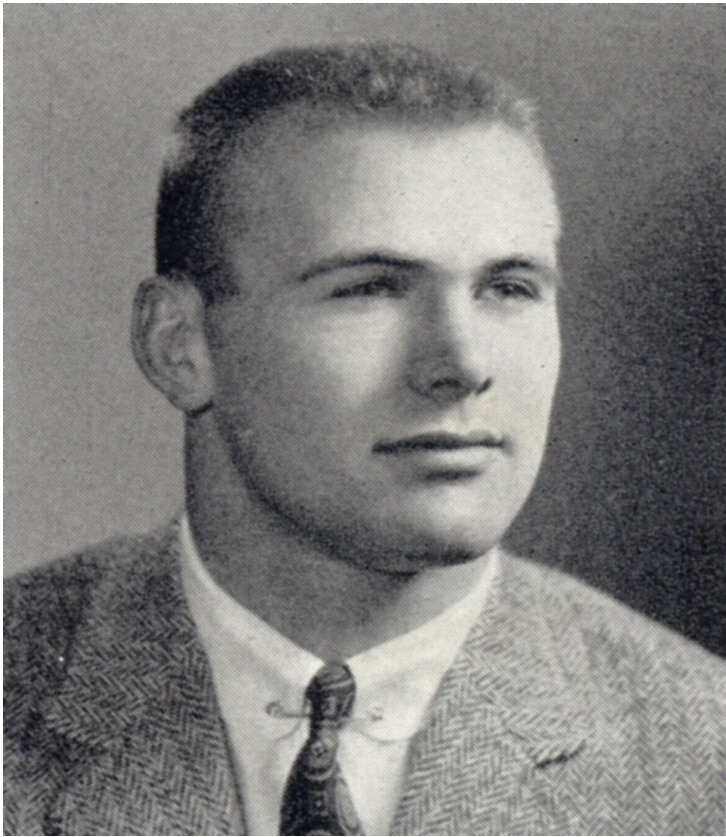
# Resolving Degeneracies III:

## Parallax



# Xallarap vs. Parallax



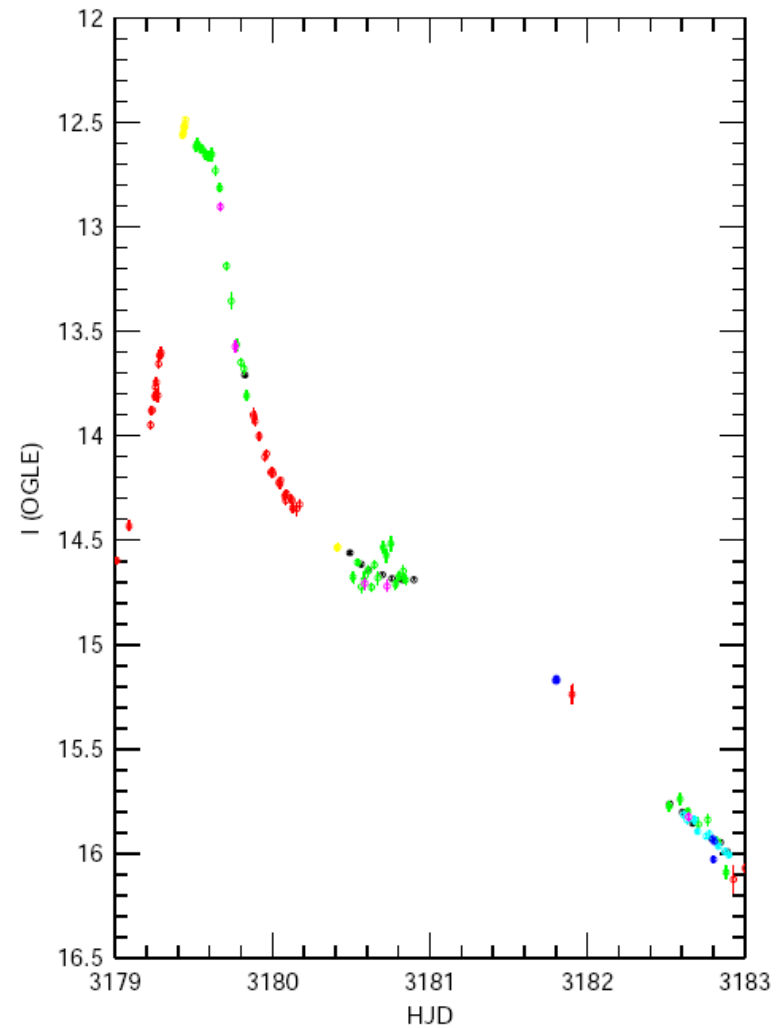
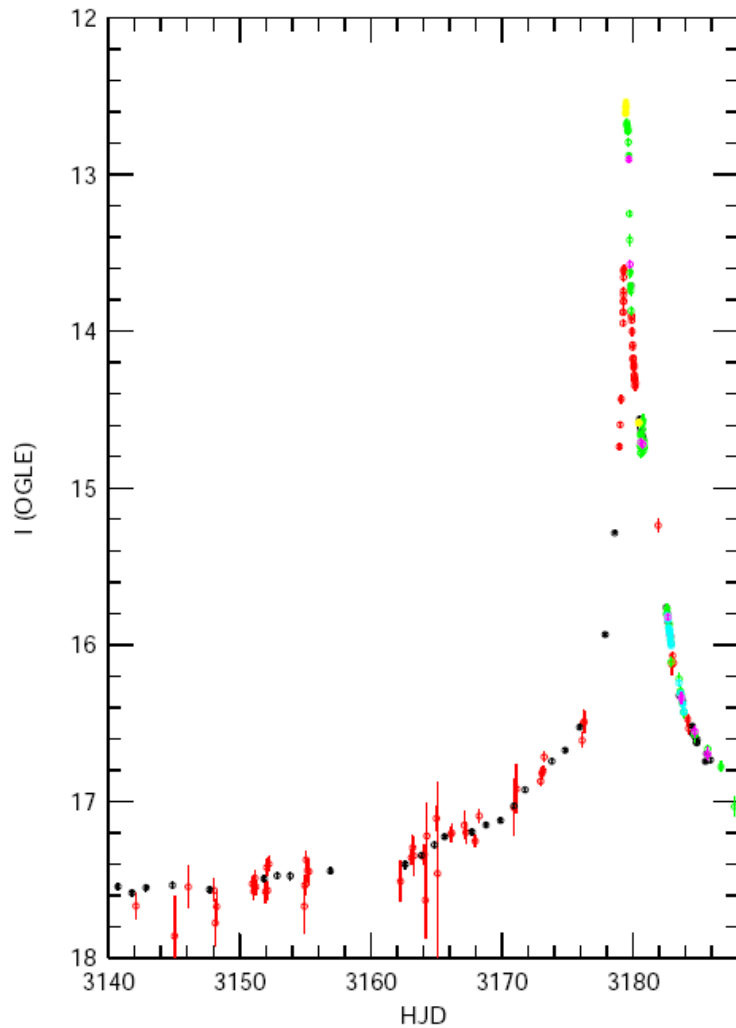


Princeton '54



# Unknown Knowns

## MOA-2004-BLG-033



# Conclusions

- Contrary to original expectation, we can often measure the angular Einstein radius, and sometimes measure the microlens parallax
- When both are measured, we get the host and planet masses and distances