1) A glob of hot hydrogen gas, when it's not moving, produces an emission line with a wavelength $\lambda_0 = 656.3$ nanometers (nm). Using a telescope, an astronomer sees a distant interstellar gas cloud with the same hydrogen emission line at a wavelength $\lambda = 656.6$ nanometers. Is the interstellar gas cloud moving toward us or away from us? What is the radial velocity of the interstellar gas cloud?

First,

$$\Delta\lambda = \lambda - \lambda_0 = 656.5 \text{ nm} - 656.3 \text{ nm} = 0.2 \text{ nm}.$$

In words, the wavelength increased by 0.2 nm, making the emission line slightly redder (a redshift). Since $\Delta\lambda > 0$, the hydrogen glob must be **moving away** from us.

Next we must determine the radial velocity v of this hydrogen glob. The wavelength change is related to velocity by the following formula:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c},$$

where $\Delta\lambda$ is the change in wavelength, λ_0 is the wavelength at rest (in the lab), c is the speed of light, and v is radial velocity (along the line of sight). Multiplying both sides of the equation by c, we find:

$$v = c\,\frac{\Delta\lambda}{\lambda_0}\,.$$

Plugging in the numbers from above and $c = 3.0 \times 10^5$ km/s, we have:

$$v = (3.0 \times 10^5 \text{ km/s}) \left(\frac{0.2 \text{ nm}}{656.3 \text{ nm}}\right) = (3.0 \times 10^5 \text{ km/s}) \times (3.047 \times 10^{-4})$$

Note that the units of wavelength (nm) canceled out. Performing the final multiplication leaves:

$$v = 9.14 \times 10^1 \text{ km/s} = 91.4 \text{ km/s}$$

2) At its closest approach, the planet Saturn is 8 astronomical units (AU) from the Earth. When Saturn is this close, how long does it take light to travel from Saturn to the Earth? The Sun's nearest neighbor among the stars, a dim little star called Proxima Centauri, is 1.295 parsecs (pc) from the Earth. How long does it take light to travel from Proxima Centauri to the Earth?

Part I Answer:
This question asks for the light travel time to two different celestial bodies. The basic formula to remember is: $distance = speed \times time$. Unit conversion is also important. Using 1 AU $= 1.5 \times 10^8$ km and $c = 3.0 \times 10^5$ km/s, find:

$$t_{travel} = \frac{distance}{speed} = \frac{8\text{ AU}}{c} = \frac{8\text{ AU} \times (1.5 \times 10^8\text{ km/ AU})}{3.0 \times 10^5\text{ km/s}}$$

After simplifying the numerator (note that AU units cancel out), the km units and powers of 10 cancel out, so you find:

$$\frac{12 \times 10^8\text{ km}}{3.0 \times 10^5\text{ km/s}} = 4 \times 10^3\text{ s} = 4000\text{ s} \approx 66.7\text{ minutes} \approx 1.11\text{ hours}$$

This problem could also be solved by direct comparison to the Sun-Earth light travel time. Sunlight takes approximately 8.5 minutes to reach the Earth, traveling a distance of 1 AU. Light traveling from Earth to Saturn must go 8 times farther and thus takes 8 times longer.

$$time(Earth \rightarrow Saturn) = 8 \times time(Sun \rightarrow Earth) \approx 8 \times 8.5\text{ minutes} = 68\text{ minutes}$$

As you can see, both methods yield approximately the same answer.

Part II Answer:
As in the previous part, there are two ways to attack this question. Proxima Centauri is 1.295 pc away. The simplest method is to use $c = 1$ ly/yr (by definition) and 1 pc $= 3.26$ ly. Plugging these in, find:

$$t_{travel} = \frac{distance}{speed} = \frac{1.295\text{ pc}}{c} = \frac{1.295\text{ pc} \times (3.26\text{ ly/pc})}{1\text{ ly/yr}} \approx \frac{4.22\text{ ly}}{1\text{ ly/yr}} = 4.22\text{ yr}$$

You could also solve this in metric units with 1 pc $= 3.08 \times 10^{13}$ km and $c = 3.0 \times 10^5$ km/s, finding:

$$t_{travel} = \frac{1.295\text{ pc} \times (3.08 \times 10^{13}\text{ km/pc})}{3.0 \times 10^5\text{ km/s}} = \frac{3.989 \times 10^{13}\text{ km}}{3.0 \times 10^5\text{ km/s}} \approx 1.33 \times 10^8\text{ s}$$

Using 1 yr $\approx 3.15 \times 10^7$ s, again find $t_{travel} \approx 4.22$ yr.

3) The Voyager 1 spacecraft was launched in September 1877, and is now the most distant human-made object. Voyager 1 is presently 111.3 AU from the Sun, and is moving away from the Sun at a speed of 17,080 m/s. If it were traveling directly toward Proxima Centauri, and maintained its present speed for the entire journey, how long would it take to reach Proxima Centauri?

Answer:

This is very similar to the previous questions. Now, however, the traveling object is Voyager 1 instead of light. As before, use the fact that *distance* = *speed* $\times$ *time* to get the answer (after proper unit conversion). In practice, the 111.3 AU Voyager 1 has already traveled is negligible compared to the length of the total trip. As a result, you can come up with the answer quickly using: 1 pc = 3.08×10^{13} km. Keep in mind that, since these unit conversions are approximate, the exact answer can vary depending on your choice of unit conversion factors. Plugging in, find:

$$t_{travel} = \frac{distance}{speed} = \frac{1.295 \text{ pc} \times (3.08 \times 10^{13} \text{ km/pc})}{17.08 \text{ km/s}} i$$

The units of pc and km cancel, leaving:

$$t_{travel} = 2.33 \times 10^{12} \text{ s} \approx 3.89 \times 10^{10} \text{ minutes} \approx 6.49 \times 10^{8} \text{ hours} \approx 2.70 \times 10^{7} \text{ days} \approx 74,000 \text{ yr}$$

Solving this problem while accounting for the distance already traveled by Voyager 1 require more than one unit conversion. For example, I used the approximate relations 1 pc = 206,265 AU and 1 AU = 1.5×10^{8} km. In this case, you would compute the travel time using the *remaining distance* to Proxima Centauri rather than the total distance.

$$d_{remain} = d_{total} - d_{traveled} = 1.295 \text{ pc} - 111.3 \text{ AU}$$

Using the conversion ratio above, find that:

$$d_{remain} = 1.295 \text{ pc} \times (206,265 \text{ AU/pc}) - 111.3 \text{ AU} = 2.67 \times 10^{5} \text{ AU}$$

Next, use this value in the rate equation along with the second conversion factor from above to cancel units and get travel time as follows:

$$t_{travel} = \frac{d_{remain}}{speed} = \frac{2.67 \times 10^{5} \text{ AU} \times (1.5 \times 10^{8} \text{ km/AU})}{17.08 \text{ km/s}} = \frac{4.01 \times 10^{13} \text{ km}}{17.08 \text{ km/s}}$$

Simplyfing this expression, find that:

$$t_{travel} \approx 2.34 \times 10^{12} \text{ s} \approx 74,000\text{yr},$$

just like the other method.