1) The age of the universe (that is, the time since the Big Bang) is 13.7 billion years. The age of the Solar System is 4.56 billion years. Thus, the Solar System has existed for 33.3% of the age of the universe. For what percentage of the total age of the universe have the following things existed?
a) neutral (as opposed to ionized) atoms
b) the first galaxies to have formed
c) the Great Pyramid in Giza, Egypt (its date of completion is something you can look up)
d) you

This question is fairly easy plug-and-chug provided you keep track of what time period you need to calculate. In all 4 cases, you need to determine how long each object has existed. Specifically, this is the time elapsed since its creation until today. A word of caution: this is NOT the same as how much time elapsed before it existed! The exact answer you find will depend on what numbers you decide to use. For each part, we'll compute the answer with the following:

$$\text{percentage} = 100\% \times \left(\frac{\text{how long existed}}{\text{age of universe}}\right).$$

If necessary, we can compute (how long existed) = (age of universe) - (time created), such that:

$$\text{percentage} = 100\% \times \left(\frac{(\text{age of universe}) - (\text{time created})}{\text{age of universe}}\right) = 100\% \times \left(1 - \frac{\text{time created}}{\text{age of universe}}\right)$$

(a) Neutral atoms have existed since recombination (when universe became transparent and the CMB photons were "freed"), roughly 380,000 years after the Big Bang. Neutral atoms have thus existed for the present age of the universe *minus* 380,000 years (3.8×10^5 years). Plugging in numbers, we find:

$$\text{percentage} = 100\% \times \left(1 - \frac{3.77 \times 10^5 \cancel{\text{yr}}}{13.7 \times 10^9 \cancel{\text{yr}}}\right) = 100\% \times \left(1 - 2.752 \cdot 10^{-5}\right) = 99.997\%$$

(b) The first galaxies formed roughly 750 million years ($7.5 \cdot 10^8$ yr) after the Big Bang. Plugging in, the percentage of the current universe age galaxies have existed is:

$$\text{percentage} = 100\% \times \left(1 - \frac{7.5 \cdot 10^8 \cancel{\text{yr}}}{13.7 \times 10^9 \cancel{\text{yr}}}\right) = 100\% \times (1 - 0.05474) \approx 94.5\%$$

(c) The Great Pyramid in Giza, Egypt was completed around 2540 B.C.E. Since it's currently 2009, this pyramid has existed for 2540 + 2009 = 4549 years. Plugging this in, find:

$$\text{percentage} = 100\% \times \left(\frac{4549 \cancel{\text{yr}}}{13.7 \times 10^9 \cancel{\text{yr}}}\right) = 100\% \times (3.32 \cdot 10^{-7}) = 3.32 \cdot 10^{-5}\,\%$$

(d) I've been alive for roughly 26.5 years. As a percentage of the age of the universe, this is:

$$\text{percentage} = 100\% \times \left(\frac{26.5 \cancel{\text{yr}}}{13.7 \times 10^9 \cancel{\text{yr}}}\right) = 100\% \times (1.93 \cdot 10^{-9}) = 1.93 \cdot 10^{-7}\,\%$$

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2) The Whirlpool Galaxy is at a distance $d = 7.1$ Mpc from us. Using Hubble's law, what do you expect the radial velocity v of the Whirlpool Galaxy to be? What do you expect the redshift z of the Whirlpool Galaxy to be? When hydrogen is at rest, it produces an emission line with a wavelength $\lambda_0 = 656.281$ nm; what wavelength λ would you measure for the corresponding emission line from hydrogen in the Whirlpool Galaxy?

We first compute the radial velicty v directly from Hubble's Law, $v = H_0 \cdot d$. Using the values $H_0 = 71$ km/s/Mpc and $d = 7.1$ Mpc, I find:

$$v = H_0 \cdot d = 71 \times \left(\frac{\text{km/s}}{\cancel{\text{Mpc}}}\right) \times 7.1\ \cancel{\text{Mpc}} = 504.1\ \text{km/s} \approx \boxed{500\ \text{km/s} = v}$$

Next we compute the redshift, z. We know that $v = cz$, which can be rewritten $z = v\,/\,c$. Using this formula and $c = 3.0 \cdot 10^5$ km/s, I find:

$$z = \frac{v}{c} = \frac{504.1\ \cancel{\text{km/s}}}{3.0 \cdot 10^5\ \cancel{\text{km/s}}} = \boxed{0.00168 = z}$$

Lastly we compute the expected wavelength λ of a hydrogen emission line from this galaxy. From the notes, we know that:

$$\frac{\lambda - \lambda_0}{\lambda_0} = \frac{v}{c} = z$$

This can be rearranged to minimize the number of calculations and avoid mistakes (using either z or $v\,/\,c$) as follows:

$$\frac{\lambda - \lambda_0}{\lambda_0} = \frac{v}{c} \quad \rightarrow \quad \lambda - \lambda_0 = \lambda_0 \left(\frac{v}{c}\right) \quad \rightarrow \quad \lambda = \lambda_0\,(1 + \frac{v}{c}) = \lambda_0\,(1 + z)$$

Plugging $\lambda_0 = 656.281$ nm and $1 + z = 1.00168$, I find:

$$\lambda = 656.281\ \text{nm} \times 1.00168 = 657.384\ \text{nm} \approx \boxed{657.4\ \text{nm} = \lambda}$$

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3) We can detect a star with our naked eyes as long as its flux is above some minimum threshold, F_{min}. The flux of the Sun would be equal to F_{min} if it were at a distance of 17 parsecs from us. In other words, the maximum distance at which you would be able to see the Sun with your naked eyes is $r_{sun} = 17$ pc. The luminosity of a supernova (that is, an exploding star) is $L_{super} = 3.6 \times 10^9\ L_{sun}$. What is the maximum distance r_{super} at which you would be able to see a supernova with your naked eye? If a supernova went off in the Andromeda Galaxy, would we be able to see it here on Earth without the aid of a telescope?

This is very similar to a question from Problem Set 4. I'll approach this one in the same fashion. We're looking for r_{super}, which is the distance at which a supernova has flux F_{min}. Any fainter and the supernova would not be naked-eye visible. Mathematically, we need r_{super} where

$$F_{min} = \frac{L_{super}}{4\pi\, r_{super}^2}$$

We're given this flux in terms of the Sun's luminosity and a maximum distance at which the Sun would be visible. Mathematically, this means:

$$F_{min} = \frac{L_{sun}}{4\pi\, r_{sun}^2}$$

The value of F_{min} in the two above equations is the same; it's the minimum flux detectable by the human eye. We use this fact to combine the above equations as follows:

$$\frac{L_{super}}{4\pi\, r_{super}^2} = \frac{L_{sun}}{4\pi\, r_{sun}^2}$$

To solve this, we want to separate the r terms from the L terms. After a little algebra, we find:

$$\frac{L_{super}}{L_{sun}} = \frac{\cancel{4\pi}\, r_{super}^2}{\cancel{4\pi}\, r_{sun}^2} = \frac{r_{super}^2}{r_{sun}^2} \quad \longrightarrow \quad r_{super}^2 = r_{sun}^2 \cdot \left(\frac{L_{super}}{L_{sun}}\right)$$

From the question we know that $L_{super} = 3.6 \times 10^9\ L_{sun}$. Plugging in $\frac{L_{super}}{L_{sun}} = 3.6 \times 10^9$ leaves:

$$r_{super}^2 = r_{sun}^2 \cdot (3.6 \times 10^9)$$

Next, take the square root of both sides. Note that $3.6 \times 10^9 = 36 \times 10^8$, a perfect square. Find:

$$r_{super} = r_{sun} \cdot \sqrt{3.6 \times 10^9} = r_{sun} \cdot (6 \times 10^4)$$

Lastly, we know from the question that $r_{sun} = 17$ pc. Plugging this in we find:

$$r_{super} = r_{sun} \cdot (6 \times 10^4) = 17\ \text{pc} \cdot (6 \times 10^4) = \boxed{1.02 \times 10^6\ \text{pc} = 1.02\ \text{Mpc} = r_{super}}$$

You could thus see a supernova up to 1.02 Mpc away with your naked eye! The Andromeda Galaxy is roughly 780 kpc (0.78 Mpc) away. Since 0.78 Mpc < 1.02 Mpc, you would have no trouble seeing a supernova in the Andromeda Galaxy with your naked eye.

4) Our old friend "Flat-Earth Fred" is up to some new tricks. He now believes that the Big Bang Model is bogus; he thinks that he lives in a static universe that is both infinitely large and eternally old. Describe what evidence you could provide that would convince Fred that the universe *cannot* be static, infinitely large, and eternally old. (Remember, skeptical Fred prefers evidence that he can see directly with his own eyes.)

The most readily available evidence that we do not live in a static, infinitely large, and infinitely old universe is that the sky is dark at night. If the universe were infinitely large and uniformly filled with objects (galaxies), then *every* possible line of sight in the sky would "hit" a star in some galaxy. If the universe were also infinitely old, then light would have had time to reach us along all of these sightlines! (If the universe had finite age, the light from sufficiently distant stars would not have had time to reach us and some lines of sight would appear dark.) Since the night sky *is*, in fact, dark (except for the small fraction covered by stars), the universe *cannot* be both infinitely large and infinitely old.

If you could convince Fred to accept evidence other than what he can see with his own eyes, more options become available. By observing Doppler shifts in galaxies, you could (as Edwin Hubble did) show that all galaxies appear to be receding with a velocity proportional to their distance from us. This could indicate that we are extremely lucky observers positioned in the center of a static universe with all galaxies fleeing from us. The more reasonable explanation (as we have seen), however, is that the universe is expanding.

Lastly, with access to a microwave antenna, you could apprise Fred of the existence of the Cosmic Microwave Background (CMB). As was discussed in a previous problem set, the CMB poses a number of insurmountable problems for the static universe model. See the solution to Problem Set 5 for more details.