ASTRONOMY 822
Electromagnetic Radiation

Problem Set 1
due Wednesday, October 5
at class time

1) If you could construct a box opaque to neutrinos, then at a temperature T, the thermal neutrinos inside the box would have the specific intensity

$$I_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT) + 1} \ .$$

(Note the difference from the Planck distribution for photons; this results from the fact that neutrinos are fermions and photons are bosons).

a) Find the frequency $\nu_{\rm max}$ at which the peak of I_ν occurs, as a function of T. If $T = 10^{12}\,{\rm K}$ (a temperature which can be reached in a collapsing Type II supernova core), what is the numerical value of $\nu_{\rm max}$ for neutrinos?

b) Show that the energy density of neutrinos in the opaque box has the form

$$u(T) = a' T^4 \ .$$

What is the value of a' in terms of k, h, and c? If $T = 10^{12}\,{\rm K}$, how large a box of neutrinos would you need to have a total neutrino energy of $E = 10^{53}\,{\rm erg}$? (This is the typical energy radiated in neutrinos from a Type II supernova.)

2) a) Approximate the Sun as a blackbody with a temperature of $T = 5800\,{\rm K}$. What fraction of its flux F lies in the visible range of the electromagnetic spectrum, with $4.3 \times 10^{14}\,{\rm Hz} < \nu < 7.5 \times 10^{14}\,{\rm Hz}$?

b) Approximate the filament of an incandescent light bulb as a blackbody with a temperature of $T = 2900\,{\rm K}$. What fraction of the filament's flux lies in the visible range of the spectrum?

c) Approximate yourself as a blackbody with a temperature of $T = 310\,{\rm K}$. What fraction of your flux lies in the visible range of the spectrum?

[continued on back]

3) The star Vega has an angular diameter as seen from Earth of $\theta = 3.3 \times 10^{-3}$ arcsec. (This is determined using interferometric techniques.)

a) At a frequency $\nu = 1.4 \times 10^{14}$ Hz (corresponding to $\lambda = 2.2\,\mu$m), the measured flux from Vega is $F_\nu = 6.36 \times 10^{-19}\,\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}}$. What is the brightness temperature T_b of Vega at this frequency? (Note: the flux F_ν, like all the fluxes mentioned in this problem, is the flux passing through a window held perpendicular to the line of sight to Vega.)

b) At a frequency $\nu = 8.1 \times 10^{14}$ Hz (corresponding to $\lambda = 370$ nm), the measured flux from Vega is $F_\nu = 1.78 \times 10^{-20}\,\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}}$. What is the brightness temperature T_b of Vega at this frequency?

c) The measured energy flux from Vega, integrated over all frequencies, is $F = 2.53 \times 10^{-5}\,\mathrm{erg\,s^{-1}\,cm^{-2}}$. What is the flux F at the surface of Vega? What is the effective temperature T_{eff} of Vega?