ASTRONOMY 822
Electromagnetic Radiation
Problem Set 2
due Wednesday, October 12
at class time

1) Consider a plane electromagnetic wave of the form

$$\vec{E} = \hat{e}_y E_0 \cos(kx - \omega t)$$

and

$$\vec{B} = \hat{e}_z E_0 \cos(kx - \omega t) \ .$$

The wavelength of the light is measured to be $\lambda = 700\,\mathrm{nm}$ and its time averaged energy density is $\langle u \rangle = 20\,\mathrm{erg\,cm^{-3}}$.

a) An electron is placed at the origin, initially at rest ($\vec{x} = 0$ and $\vec{v} = 0$ at $t = 0$). Let's start by assuming that the motions of the electron are highly nonrelativistic, and that the magnetic forces can be ignored. What is the force exerted on the electron, in this case? What is the maximum displacement of the electron from the origin? What is the maximum velocity of the electron? Was our assumption of nonrelativistic motions justified?

b) The most powerful "petawatt" lasers can produce energy densities as large as $\langle u \rangle \approx 3 \times 10^{17}\,\mathrm{erg\,cm^{-3}}$. Assuming this energy density is provided by a single plane wave of wavelength $\lambda = 1000\,\mathrm{nm}$, what would be the maximum velocity of an electron oscillating in the wave? What would be the maximum velocity of a proton?

2) In a hot, low density plasma, the primary source of opacity is Thomson scattering from free electrons, which provides an opacity of $\kappa = 0.40\,\mathrm{cm^2\,g^{-1}}$ in a pure hydrogen gas.

a) The hot intracluster gas within clusters of galaxies has a typical mass density of $\rho \approx 10^{-27}\,\mathrm{g\,cm^{-3}}$. If we approximate the intracluster medium as pure hydrogen whose opacity is due entirely to Thomson scattering, what is the scattering coefficient of the cluster?

b) If the cluster is a sphere 3 megaparsecs in diameter, what fraction of the photons passing through the cluster center will undergo a scattering inside the cluster?

[continued on back]

3) a) In a uniform magnetic flux density $\vec{B}$, show that particles of electric charge q and mass m follow circular orbits in the plane perpendicular to $\vec{B}$ with an angular frequency ω that depends only on B, c, and the charge-to-mass ratio q/m of the particles.

b) In a uniform magnetic field, will electrons and positrons orbit in the same direction or opposite directions? Explain your answer.

c) An early cyclotron built by Ernest Lawrence and his collaborators had a magnetic flux density $B = 10^4$ gauss and a radius of $r = 4.5\,\mathrm{cm}$. What maximum proton speed could this cyclotron produce?