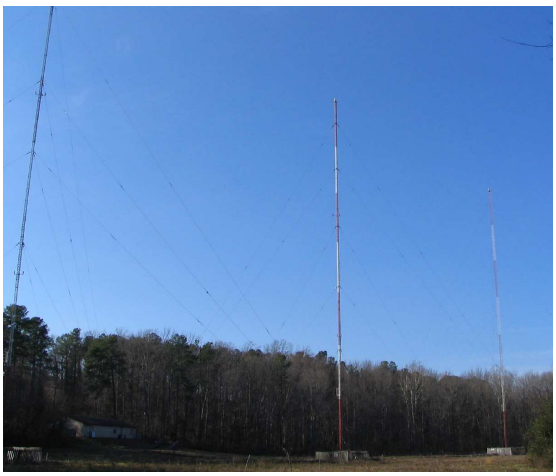
1 ASTRONOMY 822: Electromagnetic Radiation

1.1 Problem Set 4: due Wednesday, October 26

1) Commercial radio stations often broadcast their programs using 2 to 12 dipole antennas arranged in long rows:



a) Why do they use this arrangement, rather than a single, higher amplitude dipole? (Hint: this arrangement of dipoles is called a "directional antenna".)
b) I have been granted permission to broadcast radio programs to the town of Mozartville at a frequency $\nu = 1.0\,\mathrm{MHz}$. However, I am forbidden to send my signal to the town of Bachberg, only 10 kilometers away (there's a competing radio station there, broadcasting at the same frequency). After buying 4 identical dipole antennas, I will have enough money left to buy a plot of land 3 kilometers from the center of Mozartville. Where should I place my antennas, and how should I orient them? (My structural engineer, by the way, demands that the dipoles be placed in an exactly vertical direction.)

c) If my 4 dipoles are oscillating in phase, how far apart should I place them to minimize the power that I broadcast to Bachberg?

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2 a) An object with electric dipole moment $\vec{d}$ and moment of inertia I is rotating with angular frequency ω about a rotation axis tilted by an angle α with respect to the dipole vector $\vec{d}$. If the only source of energy is the object's kinetic energy of rotation, what is its characteristic time for spinning down, in terms of I, $|\vec{d}|$, ω, and α?
b) A water molecule has electric dipole moment $d = 1.86\,\mathrm{debye}$ and moment of inertia $I \approx 2 \times 10^{-40}\,\mathrm{g\,cm^2}$. If the water molecule could be treated classically, what would be the spindown time for a water molecule with $\omega = 1.0 \times 10^{10}\,\mathrm{s^{-1}}$?

3) An object with a magnetic dipole moment $\vec{M}$ radiates away energy at the rate

$$P = \frac{2|\ddot{\vec{M}}|^2}{3c^3} \ , \tag{1}$$

analogous to the rate at which an electric dipole radiates energy.
a) The Earth has magnetic dipole moment $M = 8 \times 10^{22}\,\mathrm{esu\,cm}$, and its rotation axis is tilted at an angle $\alpha = 11\,\mathrm{degrees}$ relative to its magnetic dipole vector. What is the Earth's spindown time due to its magnetic dipole radiation? (If you want, you can assume that the Earth is a perfect sphere of uniform density.)
b) A millisecond pulsar ($\omega = 10^3\,\mathrm{s^{-1}}$) has a magnetic dipole moment $M = 5 \times 10^{29}\,\mathrm{esu\,cm}$ that is tilted at an angle $\alpha = 45\,\mathrm{degrees}$ relative to the pulsar's rotation axis. If the pulsar mass is $M = 1.4\,\mathrm{M_\odot}$ and its radius is $R = 12\,\mathrm{km}$, what is its spindown time due to its magnetic dipole radiation? (You can assume that the pulsar is a perfect sphere of uniform density.)