AST 822: Electromagnetic Radiation

Problem Set 5: due Wednesday, November 9

1) An observer in the inertial frame of reference K sees a particle moving in the xy plane, with velocity $u_x = u\cos\theta$ and $u_y = u\sin\theta$. The inertial frame of reference K' is moving relative to the frame K at a velocity $v = \beta c$ along the x axis. In the K' frame, the particle's velocity is $u'_x = u'\cos\theta'$ and $u'_y = u'\sin\theta'$.
a) If the particle being viewed is a photon, what are $\sin\theta$ and $\cos\theta$ in terms of θ' and β?
b) What is the *aberration angle* $\Delta\theta \equiv \theta' - \theta$ in the limit $\beta \ll 1$?
c) If the Earth is moving on a circular orbit with a velocity $v = 30\,\mathrm{km\,s^{-1}}$, what is the maximum aberration you expect in the position of stars over the course of a year? (This effect is the "aberration of starlight", first measured by James Bradley in AD 1728.)

2 a) In an inertial frame K', a neutron is at rest, and thus has a lifetime $t \approx 10^3\,\mathrm{s}$. In an inertial frame K, the neutron is highly relativistic, and has an energy $E = 10^{21}\,\mathrm{eV}$. (This is approximately the energy of the most energetic cosmic ray particles.) What is the lifetime of the neutron in the K frame of reference?
b) Could the neutron have come from a source within the disk of our galaxy, without having decayed along the way? Could it have come from the Coma cluster? (If you don't know the distance to Coma, you can look it up.)

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3) A rich cluster of galaxies has an intracluster medium with density $\rho \approx 10^{-27}\,\mathrm{g\,cm^{-3}}$, radius $R \approx 1\,\mathrm{Mpc}$, and temperature $T \approx 10^8\,\mathrm{K}$.

a) If the intracluster gas consists entirely of ionized hydrogen, what is the total thermal energy of the gas? What is the rate at which the gas radiates its energy by thermal bremsstrahlung? What is the cooling time for the intracluster gas, if thermal bremsstrahlung is the only mechanism? (Go ahead – assume the Gaunt factor is equal to one.)

b) The thermal energy per electron in the intracluster gas is much greater than the average energy of a Cosmic Microwave Background photon ($h\nu \approx 6 \times 10^{-4}\,\mathrm{eV}$). In this case, encounters between electrons and photons result in a loss of energy by the electron and a gain in energy by the photon (the process known as "inverse Compton scattering"). The average energy lost by a non-relativistic electron in a single inverse Compton scattering is

$$\Delta e = h\nu \frac{4kT}{m_e c^2} \ . \tag{1}$$

What is the rate at which the gas loses energy by inverse Compton scattering of CMB photons, given that the number density of CMB photons is $n_\gamma = 411\,\mathrm{cm^{-3}}$? What is the cooling time for the intracluster gas, if inverse Compton cooling is the only mechanism?

c) Suppose that the density and temperature of the intracluster gas have remained constant with redshift, while the number density and average energy of the CMB photons have had their usual cosmological dependence: $h\nu \propto 1 + z$, and $n_\gamma \propto (1 + z)^3$. At what redshift z was the inverse Compton cooling time equal to the thermal bremsstrahlung cooling time?