SURVEY FOR TRANSITING EXTRASOLAR PLANETS IN STELLAR SYSTEMS: STELLAR AND PLANETARY CONTENT OF THE OPEN CLUSTER NGC 1245

DISSERTATION

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By

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ABSTRACT

An investigation into the stellar and planetary content of the open cluster NGC 1245 using BV\textit{I} photometry from the MDM 1.3m and 2.4m telescopes. Color magnitude diagram observations provide the basis for the exploration of the cluster stellar content. Based on detailed isochrone fitting, I find NGC 1245 has a slightly sub-solar metallicity, \([\text{Fe}/\text{H}] = -0.05 \pm 0.03 \) (statistical) \(\pm 0.08 \) (systematic) and an age of \(1.04 \pm 0.02 \pm 0.09 \) Gyr. In contrast to previous studies, I find no evidence for significant differential reddening. I determine an extinction of \(A_V = 0.68 \pm 0.02 \pm 0.09\) and a distance modulus of \((m - M)_0 = 12.27 \pm 0.02 \pm 0.12\), which corresponds to a distance of \(2.8 \pm 0.2\) kpc. I derive a logarithmic mass-function slope for the cluster of \(\alpha = -3.12 \pm 0.27\), where a Salpeter slope is \(\alpha = -1.35\). Fits to the radial surface-density profile yield a core radius of \(r_c = 3.10 \pm 0.52 \) arcmin (2.57 \pm 0.43 pc). NGC 1245 is highly relaxed and contains a strongly mass segregated population. The mass function for the inner cluster has a very shallow slope, \(b = -0.56 \pm 0.28\). Whereas the outer periphery of the cluster is enriched with low mass members and devoid of high mass members out to the tidal radius, \(r_t = 20 \) arcmin (16.5 pc). Based on the observed surface-density profile and an extrapolated mass function, I derive a total cluster mass, \(M = 1300 \pm 90 \pm 170M_\odot\).
Analyzing the stellar content of NGC 1245 supports the goal to assess the planetary content of the cluster. I undertook a 19-night photometric search for transiting extrasolar planets in the cluster. An automated transit search algorithm with quantitative selection criteria finds six transit candidates; none are bona fide planetary transits. I fully analyze this null result to derive upper limits on the fraction of cluster members with close-in Jupiter-radii, \( R_J \), companions. I characterize the survey detection probability via Monte Carlo injection and recovery of realistic limb-darkened transits. Simulating realistic transits requires the accurate stellar radii determined from the exploration of the stellar content of the cluster for every star in the sample. The transit survey sample contains \( \sim 740 \) cluster members.

I calculate 95% confidence upper limits on the fraction of stars with planets by assuming the planets have an even logarithmic distribution in semimajor axis over the Hot Jupiter (HJ - 3.0 < \( P < 9.0 \) day), Very Hot Jupiter (VHJ - 1.0 < \( P < 3.0 \) day), and an as of yet undetected Extremely Hot Jupiter (EHJ - \( P_{\text{Roche}} < P < 1.0 \) day) period ranges. For 1.5 \( R_J \) companions, I limit the fraction of cluster members with companions to <1.5%, <6.4%, and <52% for EHJ, VHJ, and HJ companions, respectively. For 1.0 \( R_J \) companions, I find <2.3% and <15% have EHJ and VHJ companions, respectively. From a careful analysis of the random and systematic errors of the calculation, the derived upper limits contain a \( \pm \frac{13}{7}\% \) relative error.
Dedicated to my parents for their love and support
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CHAPTER 1

INTRODUCTION

Open clusters are excellent laboratories for many different aspects of astrophysics. First, open clusters form a coeval set of stars with homogeneous properties. This homogeneity makes it possible to determine with relative ease the age and metallicity of the cluster (Yi et al. 2001). Second, owing to their relatively small relaxation times, open clusters provide an opportunity to study stellar systems in various stages of dynamical evolution (Binney & Tremaine 1987). Finally, theory predicts the star formation process within an open cluster is strongly affected by dynamical interactions, supernovae explosions, and UV radiation (Adams & Myers 2001). This work is part of the Survey for Transiting Extrasolar Planets in Stellar Systems (STEPSS). The project uses the transit technique to discover extrasolar planets and concentrates on stellar clusters since they provide a large sample of stars of homogeneous metallicity, age, and distance. Overall, the project’s goal is to assess the frequency of close-in extrasolar planets around main-sequence stars in several open clusters. By concentrating on main-sequence stars in open clusters of known (and varied) age, metallicity, and stellar density, we will gain insight into how these various properties affect planet formation, migration, and survival. These empirical
Because NGC 1245 is a rich open cluster and readily observable from the project’s primary observing site, it is the first target of the STEPSS project. The large angular size (tens of arcminutes to several degrees) of open clusters has traditionally made it difficult and time consuming to observe a substantial fraction of an open cluster’s members. However, recently available large-format CCD imagers now allow for complete studies of an open cluster with relatively small amounts of observing time. In this study I analyze two datasets. The first dataset is wide-field $BVI$ photometry of the open cluster NGC 1245 using the MDM 8K Mosaic imager on the MDM 2.4m Hiltner telescope and supplemental $BVI$ photometry obtained under photometric conditions on the MDM 1.3m McGraw-Hill telescope. This first dataset explores the stellar content of NGC 1245 through the use of the color magnitude diagram. The second dataset is a 19-night photometric search for transiting extrasolar planets in NGC 1245 using the MDM 8K Mosaic imager on the MDM 2.4m Hiltner telescope. An automated transit search algorithm with quantitative selection criteria finds six transit candidates; none are bona fide planetary transits. In addition, I derive upper limits on the fraction of cluster members with close-in
Jupiter-radii, $R_J$, companions. I characterize the survey detection probability via Monte Carlo injection and recovery of realistic limb-darkened transits.

1.1. Relation to Previous Work

NGC 1245 is a rich, old open cluster with an approximately solar metallicity population (Janes, Tilley, & Lyngå 1988; Wee & Lee 1996). The cluster is located toward the galactic anticenter ($\ell = 147^\circ, b = -9^\circ$) at a distance, $R \sim 2.5$ kpc (Janes, Tilley, & Lyngå 1988; Wee & Lee 1996). With its relatively large Galactocentric distance, NGC 1245 is particularly useful in constraining the Galactic metallicity gradient (Friel 1995; Wee & Lee 1996). With the first dataset, I improve on the observations for this cluster by covering a six-times greater area, obtaining the first CCD $I$-band photometric data, and acquiring the first CCD data under photometric conditions. With these significant improvements over earlier studies, I am able to determine more precisely the physical parameters of the cluster as a whole, not just the inner cluster core (Nilakshi et al. 2002).

A variety of techniques exist to detect extrasolar planets (Perryman 2000). Most of the extrasolar planets have been discovered using the radial velocity technique. Radial velocity detections indicate that $1.2\% \pm 0.3\%$ of FGK main-sequence stars in the solar neighborhood have a “Hot” Jupiter-mass planet (HJ) orbiting within 0.1 AU (Marcy et al. 2005). At this small separation from the central star, the high temperatures and low disk column densities prevent in situ formation for HJ
planets (Bodenheimer et al. 2000). Several mechanisms exist to exchange angular momentum between the protoplanet and natal disk, enabling the protoplanet to migrate from a more likely formation separation (several AU) to within 0.1 AU (Terquem et al. 2000). Due to tidal circularization, HJs have nearly circular orbits with an observed median ellipticity, $<e> = 0.07$. Whereas, the planets with larger separations have a median ellipticity, $<e> = 0.25$, much higher than the assumed prototypical solar system.

In addition to the detection statistics and planet properties, the extrasolar planet detections indicate several physical relationships between the stellar host properties and the frequency of extrasolar planets. The most apparent relationship so far is the probability for hosting an extrasolar planet increases rapidly with stellar metal abundance, $P \propto N_{Fe}^2$ (Fischer & Valenti 2005). The frequency of planets may also depend on the stellar mass. Butler et al. (2004) and Bonfils et al. (2005) point out the deficit of $M_J$ planets orbiting M dwarf stars. However, the increasing number of lower Neptune-mass planets discovered around M dwarf stars suggest the frequency of planets orbiting M dwarfs may be similar to FGK dwarfs, but the typical planet mass is less, thus escaping detection given the detection limitations of the current radial velocity surveys Bonfils et al. (2005). Additionally, none of the M dwarfs harboring planets are metal rich Bonfils et al. (2005).

A coherent theory of planet formation and survival requires not only reproducing the physical properties of the planets, but reproducing any trends in the physical
properties on the host environment. Despite the knowledge and constraints on extrasolar planets that radial velocity surveys provide, radial velocity surveys have their limitations. The high resolution spectroscopic requirements of the radial velocity technique limit its use to the solar neighborhood and orbital periods equivalent to the lifetime of the survey. A full consensus of the planetary formation process requires relying on additional techniques to detect extrasolar planets in a larger variety of conditions prevalent in the Universe.

For instance, microlensing surveys are sensitive to extrasolar planets orbiting stars in the Galactic disk and bulge with distances of many kpc away (Mao & Paczyński 1991; Gould & Loeb 1992). Two objects consistent with Jupiter-mass companions have been detected via the microlensing technique (Bond et al. 2004; Udalski et al. 2005). Additional information is obtained from studying the microlensing events that did not result in extrasolar planet detections. Microlensing surveys limit the fraction of M dwarfs in the Galactic bulge with Jupiter-mass companions orbiting between 1.5 to 4 AU to < 33% (Albrow et al. 2001; Gaudi et al. 2002).

Although limited to the solar neighborhood, attempts to directly image extrasolar planets are sensitive to planets with semimajor axis beyond 20 AU. The light from the parent star limits detecting planets interior to the seeing disk. Adaptive optics observations of young (∼ 1 Myr) stars provide the best opportunity to directly image extrasolar planets since the young planets are still relatively bright.
while undergoing a rapid, cooling contraction. Although the interpretation relies on theoretical modeling of these complex planetary objects, the broad-band colors and spectra are consistent with having detected three 1-42 Jupiter-mass objects in nearby star forming regions (Neuhäuser, et al. 2005; Chauvin et al. 2005a,b). The contrast ratios necessary for extrasolar planet detection are difficult to reach, and results for detecting higher mass brown dwarfs are more complete. An analysis of the Cornell High-Order Adaptive Optics Survey (CHAOS) derives a brown dwarf companion upper limit of 10% orbiting between 25 and 100 AU of the parent star (Carson et al. 2005). McCarthy & Zuckerman (2004) estimate 1% ± 1% of G,K, and M stars have brown dwarf companions orbiting between 75 and 300 AU, but this estimate may not account for the full range of orbital inclination and eccentricities possible (Carson et al. 2005). At greater separations, > 1000 AU, brown dwarfs are equally as frequent as stellar companions to F-M0 main-sequence stars (Gizis et al. 2001).

After the radial velocity technique, the transit technique has had the most success in detecting extrasolar planets (Konacki et al. 2005). The transit technique can detect $R_J$ transits in any stellar environment where $\lesssim 1\%$ photometry is possible. Thus, it provides the possibility of detecting extrasolar planets in the full range of stellar conditions present in the Galaxy; Solar neighborhood, thin and thick disk, open clusters, halo, bulge, and globular clusters are all potential targets for a transit survey. A major advantage for the transit technique is the current large-format
mosaic CCD imagers provide multiplexed photometric measurements with sufficient accuracy across the entire field of view.

The first extrasolar planet detections with their original discovery via the transit technique began with the candidate list provided by the OGLE collaboration (Udalski et al. 2002). However, confirmation of the transiting extrasolar planet candidates requires radial velocity observations. Due to the well known equation-of-state competition between electron degeneracy and ionic Coulomb pressure, the radius of an object becomes insensitive to mass across the entire range from below $M_J$ up to the hydrogen-burning limit (Chabrier & Baraffe 2000). Thus, objects revealing a $R_J$ companion via transits may actually have a brown-dwarf mass companion when followed up with radial velocities. This degeneracy is best illustrated by the planet-sized brown dwarf companion to OGLE-TR-122 (Pont et al. 2005). The first radial-velocity confirmations of planets discovered by transits (Konacki et al. 2003; Bouchy et al. 2004) provided a first glimpse at a population of massive, very close-in planets with $P < 2.5$ day and $M_p > M_J$ (“Very Hot Jupiters” - VHJ) that had not been seen by radial velocity surveys. Gaudi et al. (2005) demonstrated that, after accounting for the strong sensitivity of the transit surveys to the period of the planets, the transit detections were likely consistent with the results from the radial velocity surveys, implying that VHJs were intrinsically very rare. Subsequently, Bouchy et al. (2005b) discovered a VHJ with $P = 2.2$ day in a metallicity-biased
radial velocity survey around the bright star HD189733 that also has observable transits.

Despite the dependence of transit detections on radial velocity confirmation, radial velocity detections alone only result in a lower limit on the planetary mass, and thus do not give a complete picture of planet formation. The mass, radius information directly constrains the theoretical models, whereas either parameter alone does little to further constrain the important physical processes that shape the planet properties (Guillot 2005). For example the mass-radius relation for extrasolar planets constrains the size of the rocky core present. Also, the planet transiting across the face of its parent star provides the potential to probe the planetary atmospheric absorption lines against the stellar spectral features (Charbonneau et al. 2002; Deming et al. 2005a; Narita et al. 2005). Or, in the opposite case, the detection of emission from the planetary atmosphere when the planet orbits behind the parent star (Charbonneau et al. 2005; Deming et al. 2005b).

Despite these exciting results from detecting planets via the transit technique, the transit technique is significantly hindered by the restricted geometrical alignment necessary for a transit to occur. Thus, a transit survey contains several orders of magnitude more non-detections, and understanding these non-detections takes on increased importance. Several studies have taken steps toward sophisticated Monte Carlo calculations to quantify detection probabilities in a transit survey (Gilliland et al. 2000; Weldrake et al. 2005; Mochejska et al. 2005; Hidas et al. 2005; Hood
et al. 2005). Additionally, the null result in the globular cluster 47 Tucanae adds important empirical constraints to the trend of increasing probability of having a planetary companion with increasing metallicity (Gilliland et al. 2000; Santos et al. 2004). Unfortunately, these studies do not fully characterize the sources of error and systematics present in their analysis. Large errors are present since most studies do not accurately determine the number of dwarf main-sequence stars in their sample.

Additionally, most studies do not apply identical selection criteria when searching for transits amongst the observed light curves and when recovering injected transits as part of determining the survey sensitivity. Removal of false-positive transit candidates due to systematic errors in the light curve has typically involved subjective visual inspections, and these subjective criteria have not been applied to the recovery of injected transits when determining the survey sensitivity. This is statistically incorrect, and can in principle lead to overestimating the survey sensitivity. Even if identical selection criteria are applied to the original transit search and in determining the survey sensitivity, some surveys do not apply conservative enough selections to fully eliminate false-positive transit detections.

In this paper, I address these shortcomings of previous work during the analysis of a 19-night photometric search for transiting extrasolar planets in the open cluster NGC 1245. An automated transit search algorithm with quantitative selection criteria finds six transit candidates; none are bona fide planetary transits. In
addition, I describe a Monte Carlo calculation to derive upper limits on the fraction of cluster members with close-in Jupiter-radii, $R_J$, companions.

Leading up to the process of calculating the upper limit, I develop several new analysis techniques. First, I develop a differential photometry method that automatically selects comparison stars to reduce the systematic errors that can mimic a transit signal. In addition, I formulate quantitative transit selection criteria, which completely eliminate detection of systematic light-curve variability without human intervention. I characterize the survey detection probability via Monte Carlo injection and boxcar recovery of transits. Distributing the Monte Carlo calculation to multiple processors enables rapid calculation of the transit detection probability for a large number of stars.

1.2. **Scope of the Dissertation**

The outline of this dissertation is as follows: In Chapter 2, I discuss the observations and data analysis for the color magnitude diagram and extrasolar planet transit survey. In Chapter 3, I derive the cluster parameters, radial profile, and mass function. Chapter 4 details the procedure for transit detection and investigates the properties of the six transit candidates. Chapter 5 outlines the Monte Carlo calculation for determining the detection probability of the survey and presents results of the analysis along with a detailed error analysis. Chapter 6
compares the final results of this study to the expected transit detection rate before the survey began and discusses the future observations necessary to reach sensitivities similar to the radial velocity measured detection rate. Finally, Chapter 7 briefly highlights conclusions from this work.
Chapter 2

Observations and Data Analysis

2.1. Color Magnitude Diagram Data

2.1.1. Observations

I observed NGC 1245 on two occasions. The first set of observations was obtained in November 2001 using the MDM 8K mosaic imager on the MDM 2.4m Hiltner telescope. The MDM 8K imager consists of a 4x2 array of thinned 2048x4096 SITE ST002A CCDs (Crotts 2001). This instrumental setup yields a 26′x26′ field of view and a 0.36″ per pixel resolution employing the 2x2 pixel binning mode.

Table 2.1 shows for each night of observations the number of exposures in each filter, exposure time, median full width at half maximum (FWHM), and a brief comment on the observing conditions. None of the nights were photometric. Therefore, I reobserved NGC 1245 in February 2002 using the MDM 1.3m McGraw-Hill telescope with the 2048x2048 “Echelle” imaging CCD. This CCD provides a 17′x17′ field of view with 0.5″ per pixel resolution. Several nights were photometric, and I use Landolt (Landolt 1992) standard star observations to calibrate the photometry.
Table 2.2 details the images taken with the MDM 1.3m telescope. The relative placement between the 1.3m and 2.4m field of views is shown in Figure 2.1.

### 2.1.2. Data Reduction

I use the IRAF\(^1\) CCDPROC task for all CCD processing, and the 2.4m data reduction is described first. The stability of the zero-second image over the course of the 19 nights allows us to median combine 95 images to determine a master zero-second calibration image. For master flat fields, I median combine 11, 19, and 66 twilight sky flats taken throughout the observing run in the BVI passbands, respectively. I quantify the errors in the master flat field by examining the night to night variability between individual flat fields. The small-scale, pixel-to-pixel variations in the master flat fields are \(\sim 1\%\), and the large-scale, illumination-pattern variations reach the 3\% level. The large illumination-pattern error results from a strong sensitivity in the illumination pattern to telescope focus.

I use the DAOPHOT package (Stetson 1987) within IRAF for PSF fitting photometry, and I perform the photometry on each frame individually. To identify stars and to calculate stellar positions, I designate the highest signal-to-noise \(I\)-band image as input to the DAOFIND task. I transform the stellar positions determined

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\(^1\)IRAF is distributed by the National Optical Astronomy Observatories, which are operated by the Association of Universities for Research in Astronomy, Inc., under cooperative agreement with the National Science Foundation.
on the high signal-to-noise $I$-band image to all the other images as initial guesses for photometry. To calculate the model PSF, I select 25 bright, isolated stars evenly distributed on the reference image. I use the same PSF stars for all images. Slight offsets between images result in at least twenty stars in the PSF model calculation for all the images. Modeling the spatially constant PSF model involves three iterations with faint neighbors to the PSF stars being subtracted from the image between iterations. Using the resulting PSF model, the IRAF ALLSTAR task calculates the instrumental magnitudes by fitting the PSF model to all stars simultaneously. The reference image photometry defines the instrumental magnitude system, and I apply a zero-point offset to the individual image photometry to align them with the reference image photometry. A robust weighted mean using the errors output by ALLSTAR gives the final instrumental magnitude.

In order to place the MDM 2.4m instrumental magnitudes onto a standard system, I use the IRAF FITPARAMS task to determine the transformation to the photometric MDM 1.3m dataset (described below). Figure 2.1 shows a false-color image of NGC 1245 and shows the overlap between the MDM 2.4m and 1.3m datasets. The grid of eight numbered rectangles shows the field of view for the eight CCDs that make up the MDM 8K Mosaic imager. The large inscribed square is the field of view of the MDM 1.3m observations. The blue, green, and red color channels for the false-color image result from a median combined image of the $BVI$ passbands, respectively. The CCD 2 field has the greatest overlap with the photometric MDM
1.3m data. The rms scatter for an individual star around the best fit photometric transformation is 0.034, 0.032, and 0.028 mag in the $BVI$ passbands, respectively. This error represents how well an individual star approximates the photometric system defined by the 1.3m data described below. Approximately 500 stars were used for the fit. CCD 7 has the least overlap with the 1.3m dataset, and the rms scatter for an individual star around the photometric transformation is 0.026, 0.035, and 0.027 mag with $\sim 30$ stars in the fit. CCDs 1 and 3 suffer from a nonlinearity that reaches a level of 0.14 mag near full well. With the aid of the 1.3m dataset, this nonlinearity is correctable such that the errors in the photometric calibration for these and the remaining CCDs are similar to the above examples. The bottom panels of Figure 2.2 show the resulting color magnitude diagrams (CMDs) for the MDM 2.4m dataset. The stars shown have sample standard deviations in the $BVI$ photometry of less than 0.08 mag and at least 3, 3, and 5 measurements from the photometry package are returned without error, respectively. The error bars in Figure 2.2 show the median of the sample standard deviations in the photometric color determination for stars in bins of 0.5 mag at V. These error bars represent the expected deviation from the mean of a color measurement from a single set of $BVI$ exposures with similar signal to noise properties as this dataset. The ability to determine the mean color for a star requires dividing these errors by the $\sqrt{N}$ of the number of measurements.
An alternative method to analyze the MDM 2.4m dataset is to combine the images and perform the photometry on this master combined image. This allows photometry for significantly fainter stars at the expense of having to trust the error output from the photometry package. I median combine 15, 30, & 31 $BVI$ images, respectively, and perform the identical photometry procedure outlined above. Initial guesses for the stellar positions come from the I-band image. The instrumental magnitudes are fit to the calibrated MDM 1.3m dataset with a simple zeropoint and color term. The resulting CMDs are shown in Figure 2.3. As one can see these CMDs go significantly fainter, but the field star contamination becomes increasingly a problem, especially for the $V, B – V$ CMD. The low contrast of the cluster’s main sequence against the field star contamination limits the utility of these deeper CMDs in determining the physical parameters of NGC 1245. In this study I only use these deep CMDs for extending the mass function toward fainter magnitudes (see Section 3.4).

The analysis procedure for the MDM 1.3m dataset is similar to the MDM 2.4m dataset. The master zero-second image consists of a median combination of 49 images, and a master sky flat consists of a median combination of 12, 15, and 4 images for $BVI$ passbands, respectively. The PSF has strong spatial variations, so I choose the same set of $\sim$ 80 stars evenly distributed across a reference image to calculate a quadratically varying PSF model for each image individually. I employ the stand-alone ALLFRAME program (Stetson et al. 1998) to calculate the
instrumental magnitudes for all images simultaneously. The aperture corrections also are spatially variable. I fit a third order 2-D polynomial to the aperture corrections calculated on the evenly distributed PSF stars. The IRAF XYZTOIM task calculates the fit and provides the best fit solution as an image. The sample standard deviation of the aperture correction image is $\sim 0.02$ mag.

Aperture photometry measurements of the Landolt field SA101 (Landolt 1992) taken at three different airmasses, $X = 1.2, 1.5, \text{ and } 1.9$, on two separate photometric nights (see Table 2.2) provide a determination of the airmass coefficient. The airmass coefficients for the two nights are statistically identical; thus, I average both airmass coefficient determinations for the final calibration. Fixing the airmass coefficients, I calculate color terms for each night using observations of the Landolt fields G44-27, G163-50/51, and PG1407 (Landolt 1992). The color coefficients from the individual nights are also statistically identical, and I use the mean of both color coefficient determinations for the final calibration. The standard deviations in the calibrations are 0.015, 0.015, & 0.02 mag for the $BVI$ passbands, respectively. This represents how well an individual star approximates the photometric system outlined by Landolt (1992).

There are 2, 2, and 3 images in the $BVI$ passbands, respectively, of NGC 1245 that are photometric. After applying the aperture corrections to the ALLFRAME instrumental magnitudes for the photometric data, I take a weighted mean of the measurements and then apply the photometric calibration to derive an initial
standard-star catalog. The final standard-star catalog is obtained by applying the color term and aperture corrections to all nonphotometric MDM 1.3m data, fitting for the zero-point offset to the initial standard-star catalog, and calculating the robust weighted mean of all measurements.

In the process of determining the zero-point fit for the nonphotometric MDM 1.3m data to the initial standard-star catalog, I uncovered a 1.5% amplitude residual that correlates with spatial position on the CCD. The residual is negligible over most of the CCD, but the residual abruptly appears for stars with a position $x \lesssim 500$ in pixel coordinates. The residual linearly grows toward decreasing $x$ position on the detector reaching the 1.5% level at the edge. Due to the abrupt onset of this residual, even a cubicly varying PSF model did not reduce the amplitude of the residual. To reduce the effect of this systematic spatial variability, I limit the zero-point fit for the nonphotometric MDM 1.3m data to stars in the middle of the detector defined by $500 < x < 1500$ and $500 < y < 1500$ in pixel coordinates.

The top panels in Figure 2.2 show the resulting CMDs for the MDM 1.3m dataset. Only stars with at least 3 measurements returned from the photometry package without error and a sample standard deviation $< 0.08$ mag in all three passbands are shown.
2.1.3. **Comparison with Published Photometry**

There is some confusion in the literature regarding the photometric calibration of NGC 1245. Carraro & Patat (1994) perform the first CCD detector study of NGC 1245. They calibrate their photometry by comparison with the photoelectric observations of Hoag et al. (1961). Unfortunately, the data of Carraro & Patat (1994) available on the widely used WEBDA open cluster database is incorrect since it lacks the photometric zeropoint as used in publication (G. Carraro private communication). Thus, using the incorrect data from WEBDA, Wee & Lee (1996) concluded the calibration of Carraro & Patat (1994) was incorrect. My independent photometric calibration agrees with the earlier photoelectric studies. For 5 and 3 stars in common with the photoelectric studies of Hoag et al. (1961) and Jennens & Helfer (1975), respectively, I find $V$-band differences of $0.03 \pm 0.06$ and $-0.012 \pm 0.007$ mag, respectively, and $(B - V)$ color differences of $-0.01 \pm 0.01$ and $-0.001 \pm 0.02$ mag, respectively. The $(B - V)$ colors of the stars used in the comparison span the range from 0.6-1.2 mag. Thus, the photometric calibration for NGC 1245 is well determined.

The recent study by Subramaniam (2003) does not find any differences with the Carraro & Patat (1994) photometry in the $V$ band but does find a linear trend in the $(B - V)$ color difference as a function of $V$ magnitude with an amplitude of 0.35

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http://obswww.unige.ch/webda/
mag. It is unclear if the comparison with the Carraro & Patat (1994) photometry in Subramaniam (2003) is with the incorrect data from WEBDA. However, I do not find any (B-V) color difference trends with the Carraro & Patat (1994) data. Additionally, inspection of the Subramaniam (2003) CMD finds the giant clump is 0.35 mag redder in \((B − V)\) than in CMD of this study. The above comparisons call into question the Subramaniam (2003) color calibration.

### 2.2. Extrasolar Planet Transit Data

#### 2.2.1. Observations

I observed NGC 1245 for 19 nights between 24 Oct. and 11 Nov. of 2001 using the MDM 8K mosaic imager on the MDM 2.4m Hiltner telescope. The MDM 8K imager consists of a 4x2 array of thinned, 2048x4096, SITe ST002A CCDs (Crotts 2001). This instrumental setup yields a 26’x26’ field of view and 0.36” per pixel resolution in 2x2 pixel binning mode. Table 2.3 has an entry for each night of observations that shows the number of exposures obtained in the cousins \(I\)-band filter, median full-width-at-half-maximum (FWHM) seeing in arcseconds, and a brief comment on the observing conditions. In total, 936 images produced usable photometry with a typical exposure time of 300 s.
2.2.2. Data Reduction

I use the IRAF CCDPROC task for all CCD processing. The read noise measured in zero-second images taken consecutively is consistent with read noise measured in zero-second images spread through the entire observing run. Thus, the stability of the zero-second image over the course of the 19 nights allows us to median combine 95 images to determine a master, zero-second calibration image. For master flat fields, I median combine 66 twilight sky flats taken throughout the observing run. I quantify the errors in the master flat field by examining the night to night variability between individual flat fields. The small-scale, pixel-to-pixel variations in the master flat fields are $\sim 1\%$, and the large-scale, illumination-pattern variations reach the $3\%$ level. The large illumination-pattern error results from a sensitivity in the illumination pattern to telescope focus. However, such large-scale variations do not affect differential photometry with proper reference-star selection (as described in Section 2.2.3).

To obtain raw instrumental photometric measurements, I employ an automated reduction pipeline that uses the DoPHOT PSF fitting package (Schecter et al. 1993). Comparable quality light curves resulted from photometry via the DAOPHOT/ALLFRAME, PSF-fitting photometry packages (Stetson 1987; Stetson et al. 1998) in the background limited regime. DoPhot performs slightly better in terms of rms scatter in the differential light curve in the Poisson limited regime.
The photometric pipeline originated from a need to produce real-time photometry of microlensing events in order to search for anomalies indicating the presence of an extrasolar planet around the lens (Albrow et al. 1998). This study uses a variant of the original pipeline developed at The Ohio State University and currently in use by the Microlensing Follow Up Network (Yoo et al. 2004).

In brief, the pipeline takes as input a high signal-to-noise “template” image. A first pass through DoPHOT identifies the brightest, non-saturated stars on all the images. Using these bright-star lists, an automated routine (J. Menzies, private communication) determines the geometric transformation between the template image and all the other other images. A second, deeper pass with DoPhot on the template image identifies all the stars on the template image for photometric measurement. The photometric procedure consists of transforming the deep-pass star list from the template image to each frame. These transformed positions do not vary during the photometric solution. Next, an automated routine (J. Menzies, private communication) determines an approximate value for the FWHM and sky as required by DoPHOT. Finally, DoPHOT iteratively determines a best-fit, 7-parameter analytic PSF and uses this best-fit PSF to determine whether an object is consistent with a single star, double star, galaxy, or artifact in addition to the photometric measurement of the object.
2.2.3. Differential Photometry

In its simplest form, differential photometry involves the use of a single comparison star in order to remove the time variable atmospheric extinction signal from the raw photometric measurements (Kjeldsen & Frandsen 1992). The process of selecting comparison stars typically consists of identifying an ensemble of bright, isolated stars that demonstrate long term stability over the course of the observations (Gilliland & Brown 1988). This procedure is sufficient for studying many variable astrophysical sources; several percent accuracy is typically adequate. However, after applying this procedure on a subset of the data, systematic residuals remained in the data that were similar enough in shape, time scale, and depth to result in highly significant false-positive transit detections.

Removing $\lesssim 0.01$ mag systematic errors resembling a transit signal requires a time consuming and iterative procedure for selecting the comparison ensemble. Additionally, a comparison ensemble that successfully eliminates systematic errors in a light curve for a particular star fails to eliminate the systematic errors in the light curve of a different star. Testing indicates each star has a small number of stars or even a single star to employ as the comparison in order to reduce the level of systematics in the light curve. On the other hand, Poisson errors in comparison ensemble improve as the size of the comparison ensemble increases. Additionally, the volume of photometric data necessitates an automated procedure for deciding on
the “best” possible comparison ensemble. Given its sensitivity to both systematic and Gaussian noise and its efficient computation, I choose to minimize the standard deviation around the mean light curve level as the figure of merit in determining the “best” comparison ensemble.

I balance improving systematic and Poisson errors in the light curve using the standard deviation as the figure of merit by the following procedure. The first step in determining the light curve for a star is to generate a large set of trial light curves using single comparison stars. I do not limit the potential comparison stars to the brightest or nearby stars, but calculate a light curve using all stars on the image as a potential comparison star. All comparison stars have measured photometry on at least 80% of the total number of images. A sorted list of the standard deviation around the mean light-curve level identifies the stars with the best potential for inclusion in the comparison ensemble. Calculation of the standard deviation of a light curve involves 3 iterations eliminating 3-standard-deviation outliers between iterations. However, the eliminated measurements not included in calculation of the standard deviation remain in the final light curve.

Beginning with the comparison star that resulted in the smallest standard deviation, I continue to add in comparison stars with increasingly larger standard deviations. At each epoch, I median combine the results from all the comparison stars making up the ensemble after removing the average magnitude difference between target and comparison, $m_{\text{rel},i}$. I progressively increase the number of stars
in the comparison ensemble to a maximum of 30, calculating the standard deviation of the light curve between each increase in the size of the comparison ensemble.

The final light curve is chosen as the comparison ensemble size that minimizes the standard deviation. Less than 1% of the stars result in the maximum 30 comparison stars. The median number of comparison stars is $4 \pm 4$, with a modal value of 1.

Minimization of the standard deviation with a single comparison star emphasizes the importance of identifying the comparison stars that minimize systematic errors and achieve high accuracy.

**Comparison to a Similar Algorithm**

Independent of this study, Kovács et al. (2005) develop a generalized algorithm that shares several basic properties with the light-curve calculation just presented. They agree with the conclusion that optimal selection of comparison stars can eliminate systematics in the light curve. They also use the standard deviation of the light curve as their figure of merit (see their Equation 2). However, their more general implementation allows for the comparison star to have a real-valued, linear correlation coefficient ($c_i$ in their Equation 1) in the differential photometry, whereas the implementation outlined here forces binary values, 0 or 1, for the linear correlation coefficient. They solve for the real-valued, linear correlation coefficients by minimization of the standard deviation via matrix algebra, whereas the method given here relies on brute force minimization of the standard deviation.
A thorough comparison of the performance between these methods has not been done. However, I emphasize the modal number of stars in the comparison ensemble is a single star. Their algorithm restricts the comparison ensemble to a subset of the available stars. The restricted comparison ensemble may not capture the systematics present in a light curve. However, their real-valued, linear correlation coefficients may provide the degree of freedom lacking in the algorithm of this study necessary to cancel the systematics. Both algorithms possess an important caveat. The figure of merit cannot distinguish between improvements in the Poisson error or systematic error and therefore does not guarantee optimal elimination of the systematic deviations.

2.2.4. Additional Light-curve Corrections

Despite the improvements in removing systematic deviations in the light curves, some may still remain. For example, in good seeing, brighter stars display saturation effects. Whereas, in the worst seeing, some stars display light-curve deviations that correlate with the seeing. To correct for the light-curve seeing correlation, I fit a two-piece, third-order polynomial to the correlation. The median seeing separates the two pieces of the fit. After fitting the good-seeing piece unconstrained, the poor-seeing piece matches the good-seeing fit at the median seeing without constraint on the first or higher order derivatives. I excise measurements from the light curve that require a seeing-correlation correction larger than the robust standard deviation.
of the light curve. Additionally, measurements nearby bad columns on the detector also display systematic errors that are not removed by the differential photometry algorithm. Thus, measurements when the stellar center is within 6 pixels of a bad column on the detector are eliminated from the light curve. The final correction of the data consists of discarding measurements that deviate $>0.5$ mag from the average light curve level. This prevents detection of companions with radii $>3.5R_J$ around the lowest mass stars of the sample.

2.2.5. Light-curve Noise Properties

Figure 2.4 shows the logarithm standard deviation of the light curves as a function of the apparent $I$-band magnitude. Calculation of the standard deviation includes one iteration with 3-standard-deviation clipping. To maintain consistent signal to noise (S/N) at fixed apparent magnitude, the transformation between instrumental magnitude to apparent $I$-band magnitude only includes a zero-point value, since including a color term in the transformation results in stars of varying spectral shape and thus varying S/N in the instrumental $I$ band having the same apparent $I$-band magnitude. Each individual CCD in the 8K mosaic has its own zero point, and the transformation is accurate to 0.05 mag.

One CCD has significantly better noise properties than the others as evidenced by the sequence of points with improved standard deviation at fixed magnitude.
The instrument contains a previously unidentified problem with images taken in the binning mode. The data was taken with 2x2 native pixels of the CCD array binned to one pixel on readout. During readout, the control system apparently did not record all counts in each of the four native pixels. However, the single CCD with improved noise properties does not suffer from this problem; all the other CCDs do. Subsequent observations with large positional shifts allow photometric measurements of the same set of stars on the affected detectors and unaffected detector. Performing these observations in the unbinned and binning modes confirms that on the affected detectors, 50% of the signal went unrecorded by the data system. This effectively reduces the quantum efficiency by half during the binned mode of operation for seven of the eight detectors.

The two solid lines outlining the locus of points in Figure 2.4 provide further evidence for the reduction in quantum efficiency. These lines represent the expected noise due to Poisson error, sky error, and a 0.0015 mag noise floor. I vary the seeing disk and flat noise level until the noise model visually matches the locus of points for the detector with the lower noise properties. Then, the higher noise model results from assuming half the quantum efficiency of the lower noise model while keeping the noise floor the same. The excellent agreement between the higher noise model and the noise properties of the remaining detectors strongly supports the conclusion that half of the native pixels are not recorded during readout. This readout error would introduce significant errors in the limit of excellent seeing. However, only 4%
of the photometric measurements have FWHM < 2.5 binned pixels. Thus, even in the binning mode, I maintain sufficient sampling of the PSF to avoid issues resulting from the readout error. The different noise properties between detectors does not complicate the analysis. The transit detection method involves $\chi^2$ merit criteria (see Section 4.2) that naturally handle data with varying noise properties. Other than reducing the overall effectiveness of the survey, the different noise properties between the detectors does not adversely affect the results in any way.

In addition to the empirically determined noise properties, DoPhot returns error estimates that on average result in reduced $\chi^2 = 0.93$ for a flat light-curve model. The average reduced $\chi^2$ for light curves on all detectors agrees within 10%. Scaling errors to enforce reduced $\chi^2 = 1.0$ for each detector independently has a negligible impact on the results, thus I choose not to do so.

The upper and lower dash lines in Figure 2.4 show the transit depth assuming the star is a cluster member for 1.5 and 1.0 $R_J$ companions, respectively. In Figure 2.4, 3671 stars have light curves with a standard deviation less than the signal of a transiting $R_J$ companion.
Fig. 2.1.— A false color image of the open cluster NGC 1245 obtained with the MDM 8K Mosaic imager (the 4x2 array of CCDs are numbered) on the MDM 2.4m telescope. The large inscribed square is the field of view for the MDM 1.3m telescope data obtained with a single CCD imager. The blue, green, and red color channels of the false color image consist of combined images in the $BVI$ passbands, respectively. The circle shows the cluster center.
Fig. 2.2.— Color magnitude diagrams (CMDs) of the open cluster NGC 1245. Error bars represent the median sample standard deviation in the stellar color as a function of magnitude. 

*Top:* Data from the MDM 1.3m. *Bottom:* Data from the MDM 2.4m. *Left:* $V$ versus $(B-V)$. *Right:* $V$ versus $(V-I)$. 

Fig. 2.2.— Color magnitude diagrams (CMDs) of the open cluster NGC 1245. Error bars represent the median sample standard deviation in the stellar color as a function of magnitude. *Top:* Data from the MDM 1.3m. *Bottom:* Data from the MDM 2.4m. *Left:* $V$ versus $(B-V)$. *Right:* $V$ versus $(V-I)$. 

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Fig. 2.3.— Faint CMDs of the open cluster NGC 1245 calculated by median combining the MDM 2.4m data. Left: $V$ versus $(B - V)$. Right: $V$ versus $(V - I)$. 

Fig. 2.4.— Logarithm of the light-curve standard deviation as a function of the apparent $I$-band magnitude. The depth of a transit due to a 1.0 and 1.5 $R_J$ companion assuming the star is a cluster member (dash lines). Photometric noise models that match the empirically determined noise properties (solid lines).
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Table 2.1. MDM 2.4m CMD Observations.
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Table 2.2. MDM 1.3m CMD Observations.
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Table 2.3. MDM 2.4m Transit Observations
Chapter 3

Cluster Properties, Structure, and Dynamics

3.1. Isochrone Fit

NGC 1245 is located in the Galactic plane ($b = -9\,\text{deg}$), and therefore, the observed field contains a large number of foreground and background disk stars complicating the process of fitting theoretical isochrones to the CMDs. I attempted to statistically subtract the field-star population with a procedure similar to Mighell, Sarajedini, & French (1998). Unfortunately, because the contrast of the main sequence above the background field stars was relatively low (in comparison to the work by Mighell, Sarajedini, & French (1998)), I was unable to achieve a satisfactory background field subtraction.

Rather than statistically subtracting field contamination, I select stars for the main sequence isochrone fit by the following procedure. Since binaries and differential reddening affects the $(B - V)$ color of a star less than its $(V - I)$ color, the $V, (B - V)$ CMD has a tighter main sequence than the $V, (V - I)$ CMD. The tighter main sequence in the $V, (B - V)$ CMD has a higher contrast above the
contaminating field stars. Thus, to select main sequence cluster members for the isochrone fit, I trace by eye the blue main sequence edge in the $V, (B - V)$ CMD. Shifting the blue main sequence edge redward by 0.12 mag defines the red edge selection boundary. Finally, I apply a 0.02 mag shift blueward to the blue edge selection boundary. These shifts were selected by eye to contain a high fraction of the single-star main sequence. The vertical light solid lines in Figure 3.1 show the resulting main sequence selection boundaries in the $V, (B - V)$ CMD.

Since the difference in color between the red-giant clump and the main sequence turnoff in the CMD places strong constraints on the cluster age, I designate a box in the $V, (B - V)$ CMD to select red giant members for the isochrone fit. The region with boundaries $13.7 < V < 14.4$ and $1.09 < (B - V) < 1.21$ defines the red-giant clump selection. The red-giant clump is saturated in the MDM 2.4m dataset, and therefore, I fit isochrones to the MDM 1.3m photometry only. There are 1004 stars that meet the main sequence and red-giant clump selection criteria.

For the isochrone fitting, I use the Yale-Yonsei (Y$^2$) isochrones (Yi et al. 2001), which employ the Lejeune, Cuisinier, & Buser (1998) color calibration. The Y$^2$ isochrones provide an interpolation scheme to calculate isochrones for an arbitrary age and metallicity within their grid of calculations. For a given set of isochrone
parameters (metallicity, age, distance modulus, $A_V$, and $R_V$), I define the goodness of fit as

$$\chi^2_{tot} = \sum_i \chi^2_i,$$

(3.1)

where the sum is over all stars selected for isochrone fitting, and $\chi^2_i$ is the contribution from star $i$,

$$\chi^2_i \equiv (B_{pred}(m_i) - B_{obs,i})^2 + (V_{pred}(m_i) - V_{obs,i})^2 + (I_{pred}(m_i) - I_{obs,i})^2.$$  \hspace{1cm} (3.2)

In the preceding equation, $BVII_{pred}$ are the isochrone predicted magnitudes using the stellar mass, $m_i$, as the independent variable, and $BVII_{obs,i}$ are the observed stellar magnitudes. I use the Brent minimization routine \cite{Press2001} to determine the stellar mass that minimizes $\chi^2_i$ for each star.

Theoretical isochrones and color calibrations contain systematic uncertainties that make it impossible to find a consistent fit to CMDs of the best studied open clusters over a wide range of colors and magnitudes \cite{deBruijne2001, Grochowski2003}. In order to avoid overemphasizing the main sequence turnoff, where the photometric errors are the smallest, I adopt an equal weighting to all stars.

To find the best fit isochrone parameters, I perform a grid search over age and metallicity. At fixed age and metallicity, I use the Powell multidimensional
minimization routine (Press et al. 2001) to determine the best fit distance modulus and reddening, $A_V$, by minimizing $\chi^2_{tot}$. To determine the reddening in the other passbands, $A_B$ and $A_I$, I must assume a value for $R_V$, the ratio of total to selective extinction (Cardelli, Clayton, & Mathis 1989). For the best fit solution, I fix $R_V = 3.2$. I discuss the reasoning for this choice and its impact on the results in the Section 3.2. I calculate $\chi^2_{tot}$ over a 40x40 evenly-log-spaced grid of metallicity and age that ranges $-0.26 \leq [\text{Fe/H}] \leq 0.13$, and $0.7943 \leq \text{Age(Gyr)} \leq 1.4125$.

I determine confidence limits on the model parameters by scaling the resulting chi-square statistic,

$$\Delta \chi^2 = \frac{\chi^2_{tot} - \chi^2_{min}}{\chi_{min}/\nu}, \quad (3.3)$$

where $\chi^2_{min}$ is the minimum $\chi^2_{tot}$ and $\nu$ is the number of degrees of freedom. Figure 3.2 is a contour plot showing the confidence region for the joint variation in metallicity and age. The three solid contours represent the 1-, 2-, and 3-$\sigma$ confidence limits corresponding to $\Delta \chi^2 = 2.3, 6.14, \text{and } 14.0$ for the two degrees of freedom, respectively. To further refine the best fit isochrone parameters, I ran another 40x40 metallicity and age grid at finer resolution: $-0.17 \leq [\text{Fe/H}] \leq 0.03$ and $0.9550 \leq \text{Age(Gyr)} \leq 1.2303$. I quantify the 1-$\sigma$ error bounds by fitting a paraboloid to the $\Delta \chi^2$ surface as a function of the four fit parameters $[\text{Fe/H}]$, Age, $(m - M)_0$, and $A_V$. The 1-$\sigma$ errors are then the projection of the $\Delta \chi^2 = 1.0$ extent of this paraboloid on the parameter axes. I note that this procedure assumes
that the $\Delta \chi^2$ surface near the minimum is parabolic, which I find to be a good approximation. Also, because I minimize on $A_V$ and $(m - M)_0$ at fixed age and metallicity, I am ignoring the additional variance from these parameters. Therefore, the errors are slightly underestimated. The resulting best fit parameters are $[Fe/H] = -0.05 \pm 0.03$, Age = 1.035 ± 0.022 Gyr, $(m - M)_0 = 12.27 \pm 0.02$ mag, and $A_V = 0.68 \pm 0.02$ mag. The best fit isochrone is overplotted on the $V, (B - V)$ and $V, (V - I)$ CMDs in Figure 3.1 and the best fit metallicity and age are shown as the filled square in Figure 3.2.

The isochrone fits well in the $V, (B - V)$ CMD, deviating slightly too red toward the lower main sequence. I investigate whether an adjustment to the photometric calibration color term can reconcile the observed main sequence and the isochrone by first calculating the color residual between the stars and the isochrone at fixed $V$ mag. A line fit to the color residuals as a function of $(B - V)$ color has a slope, $b = -0.065 \pm 0.007$. This calculation assumes that the V-band color term is correct, and only the B-band color term needs adjusting. The error in determining the color term, $1 - \sigma \sim 0.004$ in all passbands, suggests the large, $b = -0.065$, color term adjustment required to reconcile the observed main sequence and theoretical isochrone does not result from an incorrect determination of the color term alone.

The fit is poorer in the $V, (V - I)$ CMD. Visually, the best fit isochrone exhibits a tendency to lie redward of the main sequence in the $V, (V - I)$ CMD. This tendency toward the red partly results from the broader and more asymmetric main sequence.
in the $V, (V-I)$ CMD. Additionally, the overly red isochrone results from systematic errors in the shape of the isochrone. The isochrone fits best near $(V-I) \sim 0.9$ and deviates redward of the main sequence toward fainter and brighter magnitudes. A line fit to the $(V-I)$ color residual between the stars and isochrone at fixed $V$ mag as a function of $(V-I)$ color does not reveal any significant trend. Again, as with the $V, (B-V)$ CMD, the theoretical isochrones are unable to fit the detailed shape of the cluster main sequence from the turnoff down to the lowest observed magnitudes. The isochrone deviates from the observed main sequence by as much as 0.06 mag in $(V-I)$ color at fixed $V$ mag.

3.2. Systematic Errors

There are four main contributions to the systematic uncertainties in the derived cluster parameters: uncertainty in $R_V$, photometric calibration errors, theoretical isochrone errors, and binary star contamination. I address the effect of each of these uncertainties on the derived physical parameters in turn.

I can, in principle, determine $R_V$ by additionally minimizing $\chi^2_{tot}$ with respect to $R_V$. However, in doing so I find a best fit value of $R_V = 2.3$ and a metallicity of $[Fe/H] = -0.26$. Values of $R_V$ this low are rare in the galaxy (However, see Gould, Stutz, & Frogel (2001)).
I also can use infrared photometry from the 2MASS All-Sky Data Release Point-Source Catalog (Cutri et al. 2003) to place more robust constraints on $R_V$. As a side note, 2MASS photometry does not reach nearly as faint as the $BVI$ photometry of this study. Thus, I do not include it directly into the best fit solution for the physical parameters of NGC 1245. To aid in determining $R_V$, I constrain the sample of stars employed in the isochrone fit to stars with $V < 16$, which corresponds to $K_s \sim 14.5$ mag for turnoff stars. At this limiting magnitude, the typical photometric error in the 2MASS photometry is $\sigma_{K_s} \lesssim 0.07$. I transform the 2MASS $K_s$ passband magnitudes to the $K$-band system defined by Bessell & Brett (1988) (the system employed in the Lejeune, Cuisinier, & Buser (1998) color calibration) by adding 0.04 mag to the 2MASS photometry (Cutri et al. 2003). The photometric transformation of the 2MASS photometry does not require a color-term (Cutri et al. 2003). Using the $BVIK$ magnitudes of the $V < 16$ turnoff stars, I derive a best fit $R_V \sim 3.2$. However, the resulting best fit age is $\sim 1.2$ Gyr, somewhat older than the age of $\sim 1.0$ Gyr derived from the deeper $BVI$ data alone. Conversely, fixing the age at $\sim 1.0$ Gyr in the isochrone fit to the turnoff stars, I find $R_V \sim 3.4$ is the best fit solution. Figure 3.1 shows the best fit isochrone systematically deviates from the observed main sequence at the turnoff. Since the 2MASS photometry limits the $BVIK$ isochrone fit to within 1.5 mag of the turnoff, I suspect the difficulty in determining $R_V$ with an age consistent with the best fit

\footnote{http://www.ipac.caltech.edu/2mass/releases/allsky/doc/explsup.html}
solution arises from systematic errors in the isochrones. I therefore adopt a fiducial value of $R_V = 3.2$ for the best isochrone fit but explore the impact a systematic uncertainty in $R_V$ of 0.2 has on the cluster parameters in the following paragraph.

I quantify the effect of the systematic uncertainty in $R_V$ on the cluster parameters by recalculating the isochrone fit assuming $R_V = 3.0$, the 1-$\sigma$ lower limit in the derived value of $R_V$. The resulting best fit is $[\text{Fe/H}] = -0.1$, Age=1.036 Gyr, $(m - M)_0 = 12.29$ mag, and $A_V = 0.68$ mag. The open triangle in Figure 3.2 shows the resulting best fit metallicity and age for $R_V = 3.0$. Since the $(B - V)$ color is more sensitive to metallicity variations than the $(V - I)$ color, the color difference between the main sequence in the $V$, $(B - V)$ CMD and $V$, $(V - I)$ CMD is a good indicator of metallicity. Unfortunately, for a fixed $A_V$, variations in $R_V$ also alter the relative $(B - V)$ and $(V - I)$ color of the main sequence. This degeneracy between $R_V$ and metallicity in determining the main sequence color limits the ability to determine the metallicity.

To assess the systematic error associated with the photometric calibration uncertainty, I refit the isochrones assuming the $I$-band photometry is fainter by 0.02 mag. The resulting best fit is $[\text{Fe/H}] = 0.0$, Age=1.021 Gyr, $(m - M)_0 = 12.31$, and $A_V = 0.62$. The $I$-band offset solution is shown as the starred point in Figure 3.2.

I attempt to investigate the systematic uncertainties in the isochrones by fitting the Padova group isochrones [Girardi et al., 2000] to the CMDs. I find
that these isochrones do not match the shape of the observed main sequence. Systematic differences between the isochrones and data can reach up to 0.08 mag in \((B − V)\) and \((V − I)\) colors. The mismatch in shape to the observed main sequence prevents precise isochrone fits with the Padova isochrones. However, I find that the solar-metallicity, 1 Gyr isochrone of the Padova group is broadly consistent with the observed data.

As an alternative method to quantify the systematic errors in the isochrones, I calculate the physical parameters for the well studied Hyades open cluster using an identical procedure to the NGC 1245 data. The Hyades CMD data are selected for membership based on proper motion and eliminated of binary star contaminants based on spectroscopic observations (see Pinsonneault et al. 2003 for a discussion of the data and membership selection). Using a metallicity and age grid with resolution \(\Delta[Fe/H] = 0.01\) dex and \(\Delta \log(\text{Age}) = 0.006\) dex, I find best fit isochrone parameters of \([Fe/H] = 0.10\), \(\text{Age} = 670\) Myr, \((m − M)_0 = 3.23\) mag, and \(A_V = 0.03\) mag. The recent study by Paulson, Sneden, & Cochran (2003) and many previous studies determine a value of \([Fe/H] = 0.13 \pm 0.01\) for the metallicity of the Hyades using high resolution spectroscopy. Perryman et al. (1998) find an age of 625 \pm 50\ Myr using an independent set of isochrones. The Hipparcos distance to the Hyades is \((m − M)_0 = 3.33 \pm 0.01\) mag (Perryman et al. 1998), and the extinction for the Hyades is commonly cited as negligible. The difference in the physical parameters of the Hyades from the isochrone fit of this study to the more accurate determinations
of these parameters provides the relative systematic error in the isochrone fitting technique due to systematic errors in the isochrones.

The final systematic error source I address is the error resulting from contamination due to unresolved binaries. Unresolved binaries tend to lie redward of the single-star main sequence, and this binary “reddening” has a larger effect on the $(V - I)$ color than the $(B - V)$ color. Thus, the main sequence in a $V, (V - I)$ CMD is not as tight and shows greater intrinsic scatter than the $V, (B - V)$ CMD because the unresolved binaries are separated in $(V - I)$ color from the single-star main sequence more than they are in $(B - V)$ color. An additional selection of the main sequence in the $V, (V - I)$ CMD provides a sample of probable cluster members with reduced contamination by unresolved binaries. For determining the additional main sequence selection in the $V, (V - I)$ CMD, I trace by eye the blue boundary of the main sequence. The red main sequence boundary is defined by offsetting the blue boundary by 0.08 mag in color. A 0.15 mag color offset is needed to select most of the stars that meet the original main sequence selection based on the $V, (B - V)$ CMD alone. This additional main sequence selection reduces the stellar sample to 584 from the original 1004. The best fit isochrone using stars that meet the main sequence selection criteria in both CMDs has parameters $[Fe/H] = -0.05$, $Age = 1.052$ Gyr, $(m - M)_0 = 12.33$ mag, and $A_V = 0.62$ mag. The above solution is shown as the empty square in Figure 3.2.
To define the 1-σ systematic error on the fitted parameters, I use the difference between the best fit parameters and the parameters determined during the above discussion of the three sources of systematic error: uncertainty in $R_V$, photometric calibration and binary star contamination. For the 1-σ systematic error in the theoretical isochrones, I use the relative difference between the isochrone fit parameters for the Hyades from this study and the quoted values from the literature. To derive an overall systematic error in the cluster parameters, these four sources of systematic error are added in quadrature. The resulting overall 1-σ systematic errors are $\sigma_{[\text{Fe}/\text{H}]} = 0.08$, $\sigma_{\text{Age}} = 0.09$ Gyr, $\sigma_{(m-M)_0} = 0.12$ mag, and $\sigma_{A_V} = 0.09$ mag. The photometric calibration and $R_V$ uncertainty dominate the systematic uncertainty in metallicity. The systematic uncertainty in the isochrone dominates the systematic uncertainties in the age and distance. Unresolved binary contamination and photometric calibration dominate the uncertainty in the reddening.

**3.2.1. COMPARISON WITH OTHER DETERMINATIONS**

The best fit metallicity I derive from isochrone fitting to NGC 1245 agrees with the independent metallicity determination using spectroscopy of individual red-giant members (Marshall et al. 2005). Medium-resolution spectroscopic indices calibrated from high-resolution spectroscopy indicate $[\text{Fe}/\text{H}] = -0.06 \pm 0.12$, where the error is the 1-σ systematic error in the spectral-index-metallicity calibration. Wee & Lee (1996) measure the metallicity of NGC 1245 using Washington photometry and
obtain $[Fe/H] = -0.04 \pm_{\text{stat}} 0.05 \pm_{\text{syst}} 0.16$. I note that NGC 1245 is commonly quoted in the literature as having a super-solar metallicity of $[Fe/H] = +0.14$ as given by the Lyngå (1987) open cluster database\(^2\). The origin of this high metallicity for NGC 1245 is unclear; the source for the high metallicity as given in Lyngå (1987) does not pertain to NGC 1245.

The best fit age for NGC 1245 is in agreement with the Age = 1.1 ± 0.1 Gyr found by Wee & Lee (1996). Carraro & Patat (1994) and Subramaniam (2003) find a younger age of Age = 800 Myr for NGC 1245. In the case of Carraro & Patat (1994), the younger age most likely results from their assumption of $[Fe/H] = +0.14$ for the cluster. In the case of Subramaniam (2003), the distance in color between the main sequence turnoff and the red-giant clump requires the younger age. I believe the red-giant clump of Subramaniam (2003) is too red by 0.35 mag in $(B - V)$ color (as discussed in Section 2.1.3). The smaller color difference between the main sequence turnoff and the red-giant clump results in the older age for NGC 1245.

The main differences between this study and previous investigations of NGC 1245 are that I find a lower overall reddening and I find no evidence for significant differential reddening. I find $E(B - V) = 0.21 \pm 0.03$, where the error includes the systematic uncertainty in $R_V$ and $A_V$. Previous studies find higher reddenings $E(B - V) = 0.28 \pm 0.03$ (Wee & Lee 1996), marginally consistent with the determination from this study. The previous claims for differential reddening across

\(^2\)http://vizier.u-strasbg.fr
NGC 1245 have not been highly significant and even conflicting. Carraro & Patat (1994) determine a north/south \((B - V)\) color gradient of 0.04 mag arcmin\(^{-1}\) with redder colors toward the south, and Wee & Lee (1996) find a north/south \((B - V)\) color gradient of 0.03±0.16 mag arcmin\(^{-1}\), except the color gradient is in the opposite sense, with redder colors to the north.

I follow the same procedure as in Carraro & Patat (1994) and Wee & Lee (1996) to determine the differential reddening across NGC 1245 using the 1.3m dataset. To quantify the differential reddening across NGC 1245, I start with the same sample of stars that were used for the isochrone fit. I further restrict the sample to stars with \(15.0 < V < 17.0\) mag. The differential reddening is modeled as a linear trend between the \((B - V)\) color of stars as a function of north/south position. I formally find a north/south \((B - V)\) color gradient of 0.0042±0.0006 mag arcmin\(^{-1}\) with redder colors to the north. The color gradient corresponds to a total change of \(\Delta(B - V) \sim 0.03\) over 9 arcmin. The color gradient in the east/west direction is half the north/south color gradient with a similar error. Therefore, I find a color gradient that is smaller by an order of magnitude than previous investigations. Furthermore, although formally significant, I am not convinced of the color gradient’s reality because the two-dimensional aperture correction map and PSF variations both contain errors of order 0.02 mag.
3.3. **Radial Profile**

Stellar encounters drive an open cluster toward equipartition of kinetic energy resulting in mass segregation of the cluster members (Binney & Tremaine 1987). The survey of Nilakshi et al. (2002) demonstrates that the prominent cores of open clusters generally contain the most massive stars, but 75% of a cluster’s mass lies in a surrounding corona containing only low mass members of the cluster. In this section, I search for the signatures of mass segregation in NGC 1245 by studying its radial surface-density profile as a function of the magnitude of the cluster stars.

The first step in deriving the radial profile is to locate the cluster center. Using the stellar positions for stars with high-quality photometry shown in the 1.3m CMD (Figure 2.2), I spatially smooth the number counts with a Gaussian of radius $\sim 75''$. I do not use the 2.4m dataset to calculate the cluster center because gaps in the CCD array bias the derived center. The open circle in Figure 2.1 denotes the cluster center. The cluster center is located 48$''$ east and 75$''$ south of the center determined by Carraro & Patat (1994) and 36$''$ east and 36$''$ south of the center determined by Subramaniam (2003). Since neither study gives exact details on how the cluster center was determined, I cannot determine the significance of these differences in the cluster center.

I construct a radial surface-density profile by determining the surface density of stars in concentric annuli of width $\sim 25''$. The top left panel in Figure 3.3 shows the
surface-density profile derived from both the 1.3m and 2.4m datasets. I approximate the errors as $N_i^{1/2}$, where $N_i$ is the number of stars in each bin. Area incompleteness sets in due to the finite size of the detector for annuli $\gtrsim 7.3'$ in the 1.3m dataset and $\gtrsim 11.6'$ for the 2.4m dataset, thereby increasing the errors. Generally, the surface-density profiles derived from the two datasets agree well, except near the center of the cluster and a small overall scaling. The scaling difference results from the fainter limiting magnitude in the 2.4m dataset (see Figure 2.2). The discrepancy in the inner annuli arises from the fact that the cluster center falls very close to a gap in the CCD array for the 2.4m detector. Therefore, incompleteness affects the first few radial bins of the 2.4m surface-density profile. I disregard the first two radial bins for all surface-density profiles derived from the 2.4m dataset.

As expected, the radial profile of the cluster exhibits a gradual decline to a constant field-star surface density. I fit the surface-density profile to the model,

$$\Sigma(r) = \Sigma_f + \Sigma_0 \left[ 1 + \left( \frac{r}{r_c} \right)^\beta \right]^{-1},$$

(3.4)

where $\Sigma_f$ is the surface density of field stars, $\Sigma_0$ is the cluster surface density at $r = 0$, and $r_c$ is the core radius. In fitting Equation (3.4) I require a positive $\Sigma_f$. I fit this profile to both the 1.3m and 2.4m datasets. Although the 2.4m surface-density profile extends to larger radii, the incompleteness near the center generally results in less well-constrained parameters, in particular the core radius $r_c$. 

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Both fits are excellent: for the 1.3m dataset, $\chi^2 = 15.9$ for 28 degrees of freedom (dof), whereas for the 2.4m dataset, $\chi^2 = 39.3$ for 36 dof. Table 3.1 tabulates the best fit parameter values and 1-$\sigma$ uncertainties for all datasets. A parabola fit to the envelope of $\chi^2$ values as a function of the parameter of interest determines the parameter’s error. The solid line in the top left panel of Figure 3.3 shows the best fit model to the 1.3m profile. The logarithmic slope of the surface density is $\beta = 2.36 \pm 0.71$, and the core radius is $3.10 \pm 0.52$ arcmin. The field and central surface densities are $\Sigma_f = 4.0 \pm 0.8$ arcmin$^{-2}$ and $\Sigma_0 = 14.7 \pm 2.9$ arcmin$^{-2}$, respectively. At the derived distance to the cluster, $d = 2850$ pc, the conversion from angular to physical distances is 1.207 arcmin/pc. Thus, NGC 1245 has a core radius of $\sim 2.6$ pc. The core radius agrees with the previous determination of $2.7 \pm 0.13$ arcmin from Nilakshi et al. (2002).

To investigate mass segregation, I next divide the high-quality photometry sample of stars into two subsamples: bright stars with $V < 17.8$ and faint stars with $V > 17.8$. There are roughly an equal number of cluster stars in each subsample. The surface-density profiles for both the 1.3m and 2.4m data are shown in the bottom left and middle left panels of Figure 3.3. I find a significant difference between the best fit logarithmic slopes of the bright and faint subsamples. For the 1.3m dataset, I find $\beta = 3.9 \pm 1.3$ for the bright sample, and I find $\beta = 1.3 \pm 0.9$ for the faint sample. This provides evidence, at the $\sim 2 - \sigma$ level, that the faint cluster stars in NGC 1245 are considerably more spatially extended than the brighter stars.
The fainter stars being spatially extended is almost certainly due to the effects of mass segregation, which I expect to be important for a cluster of this mass and age. I provide additional evidence for mass segregation in Section 3.4 and discuss the implications of this mass segregation more thoroughly in Section 3.5.

The radial profile of NGC 1245 implies that the faint cluster members may actually extend significantly beyond the field of view. The radial profile for the 1.3m data predicts that, at 1-σ, 40 to 100% of the faint \((V > 17.8)\) stars outside 10′ of the cluster center and within the field of view are cluster members. The strong cluster contribution in the outer periphery of the field hampers obtaining an accurate statistical subtraction of the field star contamination necessary to determine the mass function for NGC 1245. I discuss this additional complication and the solution in Section 3.4.

### 3.4. Mass Function

Typically an assumed mass-luminosity relation allows the transformation of the observed luminosity function to derive the present day mass function (Chabrier 2003). In contrast, I assign a mass to each star individually using the best fit isochrone. Minimizing a star’s individual \(\chi^2_i\) (Equation 3.2) using the best fit isochrone parameters determines the star’s mass. To determine the mass for each star, I use the high quality 2.4m photometry shown in the bottom panel CMDs of...
Figure 2.2 as it covers a much larger field of view. I reduce field star contamination of the cluster mass function by restricting the sample to stars with $\chi_i^2 < 0.04$. This $\chi_i^2$ cutoff selects a region in the $V, (B-V)$ CMD roughly similar to the main sequence selection used in the isochrone fit but naturally selects stars based on their proximity to the isochrone in all three passbands.

Despite the large field of view of the 2.4m data, the radial profile of the cluster suggests that there exists a significant number of low mass cluster members out to the limits of the field of view. Lacking observations of a field offset from the cluster, the Besançon theoretical galaxy model (Robin & Crézé 1986) provides an alternative method to correct the observed mass function for field star contamination. I obtain from the Besançon galaxy model electronic database the stellar properties and photometry for a one-square-degree field centered on the cluster’s galactic coordinates without observational errors. A spline fit to the median photometric error in 0.5 mag bins in each passband allows us to simulate the observed errors in the theoretical galaxy field photometry. I add a zero-mean, unit-standard-deviation Gaussian random deviate scaled to the observed photometric error model to the theoretical galaxy field photometry. I apply the same error selection criteria ($\sigma < 0.08$) as was applied to the observed CMD and place a saturation cutoff at $V=14.0$ mag to the theoretical CMD.

To make a qualitative comparison between the theoretical field CMD and the observed CMD, I adjust the theoretical CMD for the smaller observed 2.4m field of
view. In the comparison, a star is included in the theoretical CMD only if a uniform random deviate is less than the ratio between the two field of views. Figure 3.4 compares the CMDs of the theoretical galaxy model in the top panels and the observations for stars beyond 12.7 arcmin of the cluster center for the 2.4m data in the bottom panels. The dashed line shows the best fit isochrone for reference. As shown in Figure 3.4, the theoretical and observed field CMDs are qualitatively very similar.

To calculate the field star contamination in the mass function, I first obtain the mass for the stars in the theoretical field CMD and apply the same $\chi^2_i < 0.04$ selection in the same manner as the observed field CMD. For both the theoretical field CMD and observed cluster CMD, I calculate mass functions using even log-spaced intervals of $0.071 \log(M_\odot)$. Number counts from the entire one-square-degree field of view provide the basis for the theoretical galaxy mass function. The larger field of view of the theoretical galaxy model than the observed field requires scaling of the number counts and corresponding Poisson error of the theoretical galaxy mass function by the ratio of field of views. Subtracting the scaled theoretical galaxy field number counts from the the observed number counts provides the cluster mass function. The cluster mass function is shown as the heavy solid line in the upper panel of Figure 3.5. The top axis shows the median apparent V magnitude for the corresponding mass bin. The vertical dashed line delineates mass bins that are complete to better than 90%. Comparing the cluster mass function to a mass
function calculated by subtracting the theoretical galaxy number counts without applying the photometric error selection criteria determines the completeness of a mass bin. The long-dashed line labeled ‘Salpeter’ in Figure 3.5 illustrates the typical observed mass function slope, \( \alpha = -1.35 \). A power law in linear-mass, linear-number-count space fit with weights to the mass function bins not affected by incompleteness yields a slope steeper than Salpeter, \( \alpha = -3.12 \pm 0.27 \).

To verify the validity of using the theoretical galaxy field in place of an observed off-cluster field, I calculate an alternative mass function using the observed field outside 12.7 arcmin of the cluster center for field contamination subtraction. The entire 2.4m field of view again provides the basis for the observed number counts. The smaller field of view of the outer periphery requires scaling the field contamination number counts and corresponding Poisson error by the relative area factor, \( f = 6.4 \). The light-solid line in the upper panel of Figure 3.5 shows the resulting cluster mass function using the outer periphery for determining the field contamination. The mass bin positions are offset slightly for clarity. The two mass functions disagree most at the low mass end. The theoretical galaxy subtracted mass function contains greater number counts at the low mass end, consistent with the conclusion that the outer regions of the field of view contain a significant number of low mass cluster stars. Both mass functions contain a drop in the number counts at \( \log(M) = 0.071 \). I question the significance of this drop in the number counts.
The drop occurs at a magnitude with some of the heaviest field star contamination, making the field star contamination subtraction less certain.

To further the evidence for mass segregation in NGC 1245, I calculate the mass function inside and outside a radius of 4.2 arcmin from the cluster center. According to the cluster radial profile, this radius roughly separates half of the cluster members. The inner and outer mass functions are shown as the solid and short-dashed lines, respectively, in the lower panel of Figure 3.5. The fit to the inner region mass function yields a shallower slope that is consistent with Salpeter, \( \alpha = -1.37 \pm 0.2 \). Ignoring the highest mass bin, the best fit slope for the inner region is even shallower, \( \alpha = -0.56 \pm 0.28 \). I find that the outer region is highly enriched in low mass stars. The outer region mass function has a very steep slope, \( \alpha = -7.1 \pm 1.2 \). The mass bins with the down pointing arrows have negative values, and the point represents the 2-\( \sigma \) upper confidence limit. Since the mass function is fit in linear space, the fit includes bins with negative values.

The radial profile and mass function demonstrate that a significant fraction of the low mass cluster members reside in the outer periphery of the cluster. Previous studies of NGC 1245 generally find Salpeter and shallower slopes for the mass function (Carraro & Patat 1994; Subramaniam 2003). As warned in Subramaniam (2003), the smaller field of view of these previous studies from which to determine the field contamination biases the mass function against low mass members that reside in the outer periphery of the cluster. The mass functions from the previous
studies agree well with the mass function for the interior of NGC 1245, but I conclude that a steeper slope than Salpeter, $\alpha = -3.12 \pm 0.27$, is more appropriate for NGC 1245 as a whole down to the completeness limit of $M = 0.8M_\odot$.

The deeper CMD obtained from combining the MDM 2.4m data (Figure 2.3) allows us to extend the mass function to even lower masses. Defining the mass function to lower masses improves the mass estimate of the cluster, calculated in the following section. To calculate the mass function, I follow the same procedure as outlined above. At these fainter magnitudes, the theoretical galaxy field overpredicts the number of stars. Thus I must resort to using the outer periphery of the MDM 2.4m field of view for estimating the field contamination. The heavy solid line in Figure 3.6 shows the resulting mass function based on the deeper CMD. The light solid line reproduces the equivalent mass function from Figure 3.5 based on the shallower CMD data. The vertical dashed line represents the completeness limit at $M = 0.56M_\odot$. The deep mass function has a slope, $\alpha = -0.5 \pm 0.4$, significantly steeper than the slope for the shallow mass function, $\alpha = -2.8 \pm 0.8$. The significantly differing slopes suggest there is a turnover in the mass function at $\log(M/M_\odot) = -0.05$. Fitting to the last three complete mass bins in the deep mass function yields a slope, $\alpha = 1.8 \pm 0.5$. The low mass end of the deep mass function likely does not fall as steeply as measured. I use the outer periphery of the MDM 2.4m field of view, which may contain many low mass cluster members, to quantify
the field contamination. Thus, the measured slope at the low mass end represents an upper limit.

3.5. TOTAL MASS AND DYNAMICS

Numerous dynamical processes take place in open clusters. Interactions between the cluster members dramatically affect binary and planetary systems, mass segregation within the cluster, and the timescale for cluster dissolution (Hurley & Shara 2002a,b; Giersz & Heggie 1997; Portegies Zwart et al. 2001). The cluster’s initial total mass sets the timescale for these dynamical processes and governs its dynamical evolution. Unfortunately, several difficulties prevent a completely empirical determination of the total mass of NGC 1245. First, as I show in Section 3.3 the surface-density profile suggests the cluster extends significantly beyond the field of view. Second, a substantial population of low mass cluster members exists below the completeness limit of the mass function. Third, the data do not resolve binary systems. Therefore, the total mass in observed cluster members (∼820M☉) only places a lower limit to the current cluster mass. Finally, even overcoming the heretofore difficulties, stellar evolution and evaporation results in additional complications for determining the initial birth mass of the cluster.

In order to provide a rough estimate of the current total cluster mass, I must adopt a number of assumptions in order to extrapolate the observed mass to the
total mass. I must account for stars outside the survey area and stars fainter than
the magnitude limit. The total cluster mass determined by integrating the radial
profile model I fit in Section 3.3 slowly converges and is unrealistically large for an
integration to infinity. In reality, the tidal field of the Galaxy truncates the outer
radius of the cluster. Therefore, I refit the observed dataset to the King profile (King
1962),

$$\Sigma = \Sigma_f + \Sigma_0 \left\{ \left[ 1 + \left( \frac{r}{r_c} \right)^2 \right]^{-1/2} - \left[ 1 + \left( \frac{r_t}{r_c} \right)^2 \right]^{-1/2} \right\}^2,$$

(3.5)

where \( r_t \) is the tidal radius of the cluster. Note that, as \( r_t \to \infty \), this profile reduces
to the radial profile given in Equation 3.4 for \( \beta = 2 \). For a given set of parameters;
\( \Sigma_0, r_c, \) and \( r_t \); the integral over Equation 3.5 determines the total number of stars,

$$N = \pi r_c^2 \Sigma_0 \left\{ \frac{\ln(1 + x_t) - \left[ 3(1 + x_t)^{1/2} - 1 \right] (1 + x_t)^{1/2} - 1}{1 + x_t} \right\},$$

(3.6)

where \( x_t \equiv (r_t/r_c)^2 \) (King 1962). The total mass is then \( M = \langle m \rangle_{obs} N \), where
\( \langle m \rangle_{obs} = 1.00 M_\odot \) is the average mass of observed cluster stars.

Unfortunately, King profiles fit to either dataset yield only lower limits for
the value of \( r_t \). The lower limit only for \( r_t \) is primarily due to the covariance
between \( r_t \) and \( \Sigma_f \). Therefore, I need an additional constraint on either the surface
density of field stars or the tidal radius in order to determine the total cluster mass.
Attempting to constrain the fit using values of \( \Sigma_f \) determined from the theoretical
galaxy field star counts (see Section 3.4) results in unrealistically large values of $r_t > 40$ arcmin. Thus, I adopt a different approach.

The tidal radius of the cluster is set by both the local tidal field of the Galaxy and the total mass of the cluster. I can estimate the tidal radius as the radius of the Lagrange point (Binney & Tremaine 1987),

$$r_t = \left( \frac{M}{3 M_{MW}} \right)^{1/3} D,$$

where $D$ is the distance of the cluster from the center of the Galaxy, $M$ is the total mass of the cluster, and $M_{MW}$ is the mass of the Galaxy interior to $D$. For the derived distance of $d = 2850$ pc, assuming $R_0 = 8$ kpc, and Galactic coordinates ($l = 147.6^\circ, b = -8.9^\circ$), the Galactocentric distance of NGC 1245 is $D = 10.50$ kpc. For a flat rotation curve with $v = 220$ km s$^{-1}$, I find $M_{MW} = 1.18 \times 10^{11} M_\odot$, and thus $r_t \simeq 20$ pc($M/2500 M_\odot)^{1/3}$.

I use equations (3.6) and (3.7) to solve for the total cluster mass $M$ and tidal radius $r_t$, while simultaneously constraining $r_c, \Sigma_f,$ and $\Sigma_0$ from fitting Equation 3.5 to the observed radial surface-density profile. However, I must first account for low mass stars that are below the detection limit. To do this, I use the best fit mass function ($dN/d \log M \propto M^\alpha$, with $\alpha = -3.12$) to the turnover in the mass function
at 0.85\(M_\odot\). For stars between the hydrogen-burning limit and 0.85\(M_\odot\), I assume \(\alpha = 1.0\). Thus, the assumed cluster mass function is

\[
\frac{dN}{d\log M} \propto \begin{cases} 
M^{-3.12}, & M > 0.85M_\odot, \\
M^{1.0}, & 0.08M_\odot < M < 0.85M_\odot
\end{cases}
\]  

(3.8)

The adopted mass function slope at the low mass end, \(\alpha = 1.0\) is a compromise between the measured lower limit to the slope, \(\alpha = 1.8\) (see Section 3.4), and other clusters that have an observed mass function slope as shallow as \(\alpha 0.4\) at the low mass end (Prisinzano et al. 2001; Bouvier et al. 2003). This yields a total cluster mass (in the field of view) of \(\sim 1312M_\odot\) and a correction factor of 1.61 to the observed mass in cluster stars.

I use an iterative procedure to determine \(M\) and \(r_t\). I assume a value of \(r_t\) and then fit the King profile (Equation 3.5) to the observed radial density profile. The best fit King profile yields trial values of \(r_c, \Sigma_f, \Sigma_0\), and the total number of cluster stars via Equation 3.6. I then apply the correction factor to account for stars below the completeness limit to determine the total cluster mass, \(M = 1.61\langle m\rangle_{\text{obs}}N\). I then use Equation 3.7 to predict a new value for \(r_t\). The procedure iterates until convergence with a tidal radius of \(r_t \simeq 20\) arcmin (16.5 pc) and a total mass of \(M = 1312 \pm 90M_\odot\), where the error is the 1-\(\sigma\) statistical error from the fit. I determined the total cluster mass using the shallow 2.4m dataset as this is the dataset used to determine the mass correction factor and surface-density profile.
The mass correction factor, sensitive to the mass function slope at the low mass end, dominates the uncertainty in the cluster mass. For example, varying the logarithmic slope of the mass function below $0.85M_\odot$ in the range $0.6 \leq \alpha \leq 1.4$ changes the derived total mass by $+200 - 140M_\odot$.

The derived mass does not account for the presence of unresolved binaries. Without knowledge of the properties of the binary population, it is difficult to assess the magnitude of the correction. However, for a large binary fraction, the correction could be as large as 50%. I have also not attempted to correct the derived present-day mass for evolutionary effects, such as evaporation or stellar evolution, to derive an initial cluster mass. For the adopted mass function (Equation 3.8), the correction to the total cluster mass due to stellar evolution is small, $\sim 12\%$. About 10% of the cluster stars are lost to the Galaxy per relaxation time (Spitzer 1987; Portegies Zwart et al. 2001). Since the current age of NGC 1245 is approximately 7.5 times larger than the current relaxation time (see below), it is possible that the cluster has lost an appreciable amount of mass over its lifetime. The numerical N-body simulations of Portegies Zwart et al. (2001) predict the initial cluster mass based on the current number of red-giant stars. Using their Equation 1, an age of 1 Gyr for NGC 1245 and a total of 40 observed red-giant stars (no correction for background contamination) yields an initial total cluster mass, $M_o = 3120M_\odot$. 
The relevant quantity that determines the timescale for dynamical evolution of a stellar cluster is the half-mass relaxation timescale (Spitzer 1987),

\[ t_{rlx} = 0.138 \left( \frac{r_{hm}}{G\langle m \rangle} \right)^{1/2} \frac{N^{1/2}}{\ln \Lambda}. \]  

(3.9)

Here \( r_{hm} \) is the radius enclosing half of the total mass of the cluster, \( \langle m \rangle \simeq 0.63M_\odot \) is the average mass of cluster members, and \( \ln \Lambda \simeq \ln 0.4N \) is the Coulomb logarithm. This is roughly the average time over which the velocity of a typical cluster member changes by order unity due to random encounters with other cluster members. For NGC 1245, \( r_{hm} = 3.8 \text{ pc} \), and \( t_{rlx} \simeq 130 \text{ Myr} \). Since the age of NGC 1245 is \( \sim 1 \text{ Gyr} \), I expect this cluster to be dynamically quite evolved. Therefore, the evidence of mass segregation found in Sections 3.3 and 3.4 is not surprising.
Fig. 3.1.— Best fit isochrone solution overlying the color magnitude diagrams based on the MDM 1.3m data (heavy line). Main sequence selection boundaries (light line).
Fig. 3.2.— Contour lines represent the 1-, 2-, & 3-sigma confidence regions on the joint variation in metallicity and age. Best fit solution \textit{(filled square)}. Best fit solution after reducing binary star contamination \textit{(open square)}. Best fit solution assuming a ratio of selective-to-total extinction of $R_V = 3.0$ instead of $R_V = 3.2$ \textit{(open triangle)}. Best fit solution assuming an $I$-band calibration fainter by 0.02 mag \textit{(asterisk)}. 
Fig. 3.3.— *Left:* Radial surface-density profiles of stars in the NGC 1245 field for three different samples of stars. *Top:* Radial profile for all stars. *Middle:* Profile for the subset of these stars with $V < 17.8$. *Bottom:* Profile for stars with $V > 17.8$. The solid circles are derived from the 1.3m data, while the open squares are from the 2.4m data. In each panel, the solid line shows the best fit model to the 1.3m data, which is composed of a cluster profile (dotted line) plus a constant field-star surface density (dashed line). *Right:* The 68%, 95%, and 99% confidence regions on the fitted logarithmic slope of the density profile $\beta$ and core radius for the 1.3m data. Best fit model for the 1.3m data (filled point). Best fit model for the 2.4m dataset (open square). The shaded region is the 68% confidence region for the fit to the 2.4m dataset. In the upper-right panel, we also show the 68% confidence regions for the bright and faint samples.
Fig. 3.4.— *Top:* Besançon theoretical galaxy model $V, (B-V)$ and $V, (V-I)$ CMDs for the Galaxy position of NGC 1245 (Robin & Crézé 1986). Photometric errors and selection criteria are included in the theoretical galaxy to match the observed photometric errors. *Bottom:* Observed $V, (B-V)$ and $V, (V-I)$ CMDs for stars beyond 12.7 arcmin of the cluster center using the 2.4m data. Best fit isochrone (*dash line*).
Fig. 3.5.— *Top*: Mass function using a theoretical galaxy model (*heavy solid line*) and outer periphery of the field of view (*light solid line*) for determining the field contamination. The vertical line delineates mass bins complete to better than 90%. *Bottom*: Mass function for stars inside (*heavy solid line*) and outside (*light solid line*) 4.3 arcmin. The down pointing arrows represent mass bins with negative values, and the point on the tail represents the 2-sigma upper limit. Salpeter mass function offset for clarity (*dash line*).
Fig. 3.6.— Mass function calculated from the deep 2.4m data (Figure 2.3) using the outer periphery of the field of view for determining the field contamination (heavy solid line). The vertical line delineates mass bins complete to better than 90%. The equivalent mass function based on the shallow 2.4m data (light solid line) as shown as the light solid line in the upper panel of Figure 3.5. Salpeter mass function offset for clarity (dash line).
<table>
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<th>Dataset</th>
<th>Sample</th>
<th>$\Sigma_f$</th>
<th>$\Sigma_0$</th>
<th>$r_c$</th>
<th>$\beta$</th>
<th>$\Sigma_f$ (model)</th>
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<td></td>
<td></td>
<td>#/arcmin$^2$</td>
<td>#/arcmin$^2$</td>
<td>arcmin</td>
<td>#/arcmin$^2$</td>
<td>#/arcmin$^2$</td>
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<td>All</td>
<td>4.0 ± 0.8</td>
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<td>3.1 ± 0.5</td>
<td>2.4 ± 0.7</td>
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<td>(5.9 ± 1.2)</td>
<td>(21 ± 4)</td>
<td>(2.6 ± 0.4)</td>
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<td>(7.40 ± 0.06)</td>
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<td>3.3 ± 0.5</td>
<td>3.9 ± 1.3</td>
<td>2.19 ± 0.02</td>
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<td></td>
<td></td>
<td>(3.3 ± 0.3)</td>
<td>(11 ± 2)</td>
<td>(2.3 ± 0.4)</td>
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<td>(3.19 ± 0.04)</td>
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<tr>
<td>1.3m</td>
<td>Faint</td>
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<td>8 ± 3</td>
<td>4.6 ± 2.4</td>
<td>1.3 ± 0.9</td>
<td>2.89 ± 0.03</td>
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<td></td>
<td>(0.7 ± 2.8)</td>
<td>(11 ± 5)</td>
<td>(3.8 ± 2.0)</td>
<td></td>
<td>(4.20 ± 0.04)</td>
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<td>2.2 ± 1.4</td>
<td>1.7 ± 0.8</td>
<td>5.44 ± 0.04</td>
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<td>(6.8 ± 1.6)</td>
<td>(28 ± 18)</td>
<td>(1.8 ± 1.2)</td>
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<td>(7.92 ± 0.06)</td>
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<tr>
<td>2.4m</td>
<td>Bright</td>
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<td>8 ± 3</td>
<td>2.8 ± 1.0</td>
<td>2.5 ± 1.1</td>
<td>2.05 ± 0.02</td>
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<td></td>
<td>(2.3 ± 0.4)</td>
<td>(11 ± 4)</td>
<td>(2.3 ± 0.8)</td>
<td></td>
<td>(2.99 ± 0.04)</td>
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<tr>
<td>2.4m</td>
<td>Faint</td>
<td>1.5 ± 3.1</td>
<td>29 ± 43</td>
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<td>0.7 ± 1.3</td>
<td>3.38 ± 0.03</td>
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<td>(2.2 ± 4.6)</td>
<td>(42 ± 63)</td>
<td>(0.3 ± 4.2)</td>
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<td>(4.92 ± 0.04)</td>
</tr>
</tbody>
</table>

Table 3.1. Fitted Parameters and 1 − $\sigma$ Errors to Surface-Density Profiles.
4.1. Transit Detection

In Section 2.2.3 I describe a procedure for generating light curves that reduces systematic errors that lead to false-positive transit detections. However, systematics remain that result in highly significant false-positive transit detections. This section describes the algorithm for detecting transits and methods for eliminating false positives based on the detected transit properties. There are two types of false-positives I wish to eliminate. The first is false-positive transit detections that result from systematic errors imprinted during the signal recording and measurement process. The second type of false-positive results from true astrophysical variability that does not mimic a transit signal. For example, sinusoidal variability can result in highly significant transit signals in transit search algorithms. I specifically design the selection criteria to trigger on transit photometric variability that affects a minority of the measurements and that are systematically faint. However, the selection criteria do not eliminate false-positive transit signals due to true astrophysical
variability that mimic the extrasolar planet transit signal I seek (grazing eclipse, diluted eclipsing binary, etc.).

For detecting transits I employ the box-fitting least squares (BLS) method (Kovács et al. 2002). Given a trial period, phase of transit, and transit length the BLS method provides an analytic solution for the transit depth. Instead of using the Signal Detection Efficiency (Equation 6 in Kovács et al. 2002) for quantifying the significance of the detection, I use the resulting $\chi^2$ of the solution as outlined by Burke et al. (2005).

This section begins with a discussion of the parameters affecting the BLS transit detection algorithm. I set the BLS algorithm parameters by balancing the needs of detecting transits accurately and of completing the search efficiently. The next step involves developing a set of selection criteria that automatically and robustly determines whether the best-fit transit parameters result from bona fide astrophysical variability that resembles a transit signal. A set of automated selection criteria that only pass bona fide variability is a critical component of analyzing the null-result transit survey and has been ignored in previous analyses.

Due to the systematic errors present in the light curve, statistical significance of a transit with a Gaussian noise basis is not applicable. In addition, the statistical significance is difficult to calculate given the large number of trial phases, periods, and inclinations searched for transits. Given these limitations, I empirically
determine the selection criteria on the actual light curves. I cannot formally assign a probability to the significance of the empirical selection criteria. However, given < 1 false detections occur over all ∼6700 light curves, the probability of a detection happening by chance $P_{det} < 0.0001$. Although it is impossible to assign a formal false alarm probability to the selection criteria, the exact values for the selection criteria are not important as long as the cuts eliminate the false positives while still maintaining the ability to detect $R_J$ objects and the same criteria are employed in the Monte Carlo detection probability calculation.

4.1.1. BLS Transit Detection Parameters

The BLS algorithm has two parameters that determine the resolution of the transit search. The first parameter determines the resolution of the trial orbital periods. The BLS algorithm (as implemented by Kovács et al. 2002) employs a period resolution with even frequency intervals, $\frac{1}{P_2} = \frac{1}{P_1} - \eta$, where $P_1$ is the previous trial orbital period, $P_2$ is the subsequent (longer) trial orbital period, and $\eta$ determines the frequency spacing between trial orbital periods. During implementation of the BLS algorithm, I adopt an even logarithmic period resolution by fractionally increasing the period, $P_2 = P_1 \times (1 + \eta)$. The original implementation by Kovács et al. (2002) for the orbital-period spacing is a more appropriate procedure, since even frequency intervals maintain constant orbital phase shifts of a measurement between subsequent trial orbital periods. The even logarithmic period resolution I employ
results in coarser orbital phase shifts between subsequent trial orbital periods for the shortest periods and increasingly finer orbital phase shifts toward longer trial orbital periods. Either period-sampling procedure remains valid with sufficient resolution. I adopt $\eta = 0.0025$, which, given the observational baseline of 19 days, provides $<10\%$ orbital phase shifts for orbital periods as short as 0.5 day.

The second parameter of the BLS algorithm determines the resolution of the orbital phase by binning the phase-folded light curve. Binning of the data in orbital phase drastically improves the numerical efficiency, but not without loss in determining the correct transit properties. Kovács et al. (2002) give a thorough examination of how the sensitivity in recovering transits varies with orbital-phase binning resolution. To search for transit candidates in the light curves I adopt $N_{\text{bins}} = 400$ orbital-phase bins. I verify with tests that the above parameters accurately recover boxcar signals in the light curves. After injection of boxcar signals in the light curves, I calculate the $\chi^2$ of the solution returned by the BLS method with the $\chi^2$ of the injected model. Tests show that the BLS method with the above parameters return a $\chi^2$ within 30%, typically much better, of the injected model’s $\chi^2$. Tests of the Monte Carlo calculation (see Section 5.1.1) reveal a reduction of the BLS parameters to $N_{\text{bins}} = 300$ and $\eta = 0.004$ still yield absolute accuracy in the detection probability calculation of 0.3% in comparison to tests with several times more accurate BLS parameters.
4.2. Selection Criteria

I apply the BLS method following the description in the previous section to search for transit candidates in 6787 stars with light curves. A visual inspection of a light curve folded at the best-fit transit period readily discriminates between bona fide astrophysical variability and a false detection of systematic errors. However, a proper statistical assessment of the sensitivity of a transit search requires that the exact same set of selection criteria that are applied to cull false positives are also applied when assessing the detection probability via, e.g., Monte Carlo injection and recovery of artificial signals. Due to the large number of artificial signals that must be injected to calculate the detection probability properly, using selection criteria based on visual inspection of light curves is practically very difficult or impossible. Therefore, quantitative, automated detection criteria that mimic the visual criteria must be used.

I employ four selection criteria that eliminate all false detections while still maintaining the ability to detect $R_J$ companions. The four selection criteria are $\chi^2$ improvement of a transit model over a constant flux model, $\Delta \chi^2$, ratio between the $\Delta \chi^2$ of the best-fit transit model and the $\Delta \chi^2$ of the best-fit anti-transit model, transit period, and a significant fraction of the transit model $\chi^2$ during the transit must come from more than a single night.
The first of the selection criteria, $\chi^2$ improvement between a constant flux model and a transit model, $\Delta \chi^2$, is similar to the Signal Residue, SR, of Kovács et al. (2002). Burke et al. (2005) derive $\Delta \chi^2$ and its relation to SR. I prefer $\Delta \chi^2$ over SR as the former allows a direct comparison of the transit detection significance between light curves with different noise properties. Given the correlated systematics in the data, I cannot rely on analytical formulations with a Gaussian statistics basis for the statistical significance of a particular $\Delta \chi^2$ value. I empirically determine $\Delta \chi^2$ in combination with the other selection criteria to fully eliminate false detections.

For a transit detection I require $\Delta \chi^2 > 95.0$. This selection criterion corresponds to a $S/N \sim 10$ transit detection. Figure 4.1 shows the $\Delta \chi^2$ of the best fit transit for all light curves along the x-axis. The vertical line designates the selection criteria on this parameter. There are a large number of objects that result in $S/N > 10$ false-positive transit detections.

False-positive transits can result from systematics in the data analysis that underestimate the apparent flux of the star. In contrast, when systematics result in overestimating the apparent flux of a star, an anti-transit signal (brightening) appears in the light curve. Also, sinusoidal variables will have highly significant transit and anti-transit signals. Thus, the best-fit anti-transit signal provides a rough estimate of the systematics present in the light curve or of the amplitude of variability. A highly significant transit signal has a negligible anti-transit signal (long periods of photometric stability). Thus, I require the best-fit transit to have...
a greater significance than the best-fit anti-transit. I accomplish this by requiring
transit detections to have $\Delta \chi^2 / \Delta \chi^2 > 2.75$, where $\Delta \chi^2$ is the $\chi^2$ improvement of
the best-fit anti-transit. For a given trial period, phase of transit, and length of
transit, the BLS algorithm returns the best-fit transit without restriction on the
sign of the transit depth. Since the BLS algorithm simultaneously searches for the
best-fit transit and anti-transit, determining $\Delta \chi^2$ has no impact on the numerical
efficiency.

The y-axis of Figure 4.1 shows $\Delta \chi^2$ of the best fit transit. The diagonal line
demonstrates the selection on the ratio $\Delta \chi^2 / \Delta \chi^2 = 2.75$. Objects toward the lower
right corner of this Figure pass the selection criteria. The objects with large $\Delta \chi^2$
typically have correspondingly large $\Delta \chi^2$. This occurs for sinusoidal-like variability
that has both times of bright and faint measurements with respect to the mean
light-curve level.

Requiring observations of the transit signal on separate nights also aids in
eliminating false-positive detections. I quantify the fraction of a transit that occurs
during each night based on the fraction of the transit’s $\chi^2$ significance that occurs
during each night. The parameters of the transit allow identification of the data
points that occur during the transit. I sum $\chi^2_i = (m_i / \sigma_i)^2$ values for data points
occurring during the transit to derive $\chi^2_{\text{tot}}$, where $m_i$ is the light curve measurement
and $\sigma_i$ is its error. Then I calculate the same sum for each night individually,
$\chi^2_{\text{4th night}}$. I identify the night that contributes the greatest to $\chi^2_{\text{tot}}$ by the following
fraction, $f = \chi^2_{k\text{th night}}/\chi^2_{\text{tot}}$. For the final selection criteria, I require $f < 0.65$. This roughly corresponds to seeing a full transit on one night and at least half a transit on another night assuming both nights have equivalent noise. Alternatively, this criterion is also met by observing $2/3$ of a transit on one night and $1/3$ of the transit on a separate night, or observing a full transit on one night and $1/6$ of transit on a separate night with three times improvement in the photometric error. Figure 4.2 shows the $f$ value for all light curves along the y-axis. The horizontal line designates the selection on this parameter.

The red points in Figure 4.2 show objects that pass the $\Delta \chi^2 > 95.0$ selection. I find they are clustered around the 1.0 day orbital period. A histogram of the best-fit transit periods amongst all light curves reveals a high frequency for 1.0 day and 0.5 day periods. Visual inspection of the phased light curves reveals a high propensity for systematic deviations to occur on the Earth’s rotational period and 0.5 day alias. I do not fully understand the origin of this effect, but I can easily conjecture on several effects that may arise over the course of an evening as the telescope tracks from horizon to horizon following the Earth’s diurnal motion. For the third selection criteria, I require transit detections to have periods that are not within $1.0 \pm 0.1$ and $0.5 \pm 0.025$ day. The horizontal lines designate these avoided period ranges.
4.3. Transit Candidates

Six out of 6787 stars pass all four selection criteria. All of these stars are likely real astrophysical variables whose variability resembles that of planetary transit light curves. However, I find that none are bona fide planetary transits in NGC 1245. After describing the properties of these objects I will describe the procedure for ruling out their planetary nature. Figure 4.3 shows the phased light curves for these six stars. Each light-curve panel in Figure 4.3 has a different magnitude scale with fainter flux levels being more negative. The upper left corner of each panel gives the detected transit period as given by the BLS method. The upper right corner of each panel gives an internal identification number. The panels from top to bottom have decreasing values in the ratio between the improvement of a transit and anti-transit model, $\Delta \chi^2 / \Delta \chi^2$.

Table 4.1 lists the positions and photometric data for the stars shown in Figure 4.3. The selection criteria values for the best-fit transit identified in the transit candidates is given in Table 4.2. The green diamonds in Figures 4.1 and 4.2 represent the selection criteria for the six transit candidates. The $\chi^2_{\text{mem}}$ entry in Table 4.1 measures the photometric distance of a star from the isochrone that best fits the cluster CMD. A lower value of this parameter means a star has a position in the CMD closer to the main sequence. Heavy points in Figure 4.4 denote stars with $\chi^2_{\text{mem}} < 0.04$, and I designate these stars as potential cluster members.
Based on $\chi^2_{\text{mem}}$, star 20513 and star 70178 have photometry consistent with cluster membership, thus I also list the physical parameters of those stars in Table 4.3. Section 3.4 details the procedure for determining the physical parameters of a star based solely on the broad-band photometry and the best-fit cluster isochrone. However, the validity of the stellar physical parameters only applies if the star is a bona fide cluster member.

Figure 4.5 shows a finding chart for each star with a light curve in Figure 4.3. The label in each panel gives the identification number, and the cross indicates the corresponding object. The offset from center of star 20274 results from its location near the detector edge. The field of view of each panel is 54". North is toward the right, and East is toward the bottom. The panels for stars 20065, 20398, and 20513 (located near the cluster center) provide a visual impression of the heaviest stellar crowding encountered in the data. Figure 4.4 shows the V and $B-V$ CMD, of the cluster field. The large open stars denote the locations of the objects that exceed the transit selection criteria.

### 4.4. Consistency of Transit Parameters with Cluster Membership

Only stars 20513 and 70718 have $\chi^2_{\text{mem}}$ values consistent with cluster membership. Additionally, the transit depth in both stars indicates potential for
having a $R_J$ companion. However, qualitatively, the length of the transit duration in comparison to the orbital period and knowledge of the parent star’s physical properties rule out the planetary nature of the companion. I quantify ruling out the planetary nature by first estimating the stellar radius from the CMD. An additional independent estimate of a lower limit on the stellar radius comes from the properties of the light curve. The CMD stellar radii for both stars are well below the light-curve-based stellar-radius lower limit.

To derive a lower limit on the stellar radius from the light curve, I build on the work by Seager & Mallén-Ornelas (2003). They provide a relationship (their Equation 8) between the orbital semimajor axis, $a$, and stellar radius, $R_*$, based on a purely geometric relation by assuming a circular orbit with a given orbital period, $P$, depth of transit, $\Delta F$, and total duration of the transit (first to fourth contact), $\tau$. By assuming the impact parameter, $b = 0$, I arrive at a lower limit.

$$R_* > \frac{(M_* + m_p)^{1/3} P^{2/3} \sin(\pi \tau / P)}{(1 + \sqrt{\Delta F})},$$  \hspace{1cm} (4.1)

where Kepler’s Third Law replaces the semi-major axis with the mass of the star, $M_*$, and mass of the companion, $m_p$, given in $M_\odot$, and $P$ in years. Assuming to first order $\sin(\pi \tau / P) \sim \pi \tau / P$,

$$R_* > \frac{\pi (M_* + m_p)^{1/3} \tau}{P^{1/3}(1 + \sqrt{\Delta F})}. \hspace{1cm} (4.2)$$
Parameters on the right hand side of the above equation contain substantial uncertainties. Replacing the parameters by their maximum plausible deviation from their measured values in such a manner as to decrease $R_\star$ increases the robustness of the lower limit. The orbital period determination has the largest uncertainty. Tests of recovering transits in the light curves reveal a 10% chance for the BLS method to return an orbital period, $P'$, at the $1/2P$ and $2P$ aliases of the injected orbital period, and a $< 1\%$ chance of detecting the $1/3P$ and $3P$ aliases. Misidentification of the correct orbital period results from gaps in the observing window function. Replacing $P$ in the above equation with $3P'$, where $P'$ is the orbital period returned by the BLS algorithm, provides the maximum plausible deviation of this quantity and increases the robustness of the lower limit. In addition, the stellar mass determination based on the CMD potentially has contamination from a binary companion. Thus, I replace $M_\star$ with $0.5M'_\star$, where $M'_\star$ is the stellar mass estimate from the CMD. I take $m_p = 0$, and do not modify $\tau$ or $\Delta F$. The BLS algorithm fits a boxcar transit model to the light curve via a $\chi^2$ minimization. Since, in the limit of zero noise, any non-zero boxcar height fit to a transit can only result in an increasing $\chi^2$ when the length of the boxcar exceeds the length of the transit, $\tau$ underestimates the true transit length. Making the above replacements the lower limit on the stellar radius,

$$R_\star > 7.3 \frac{(M'_\star/M_\odot)^{1/3}(\tau/1 \text{ day})}{(P'/1 \text{ day})^{1/3}(1 + \sqrt{\Delta F})} R_\odot.$$  \hspace{1cm} (4.3)
For star 20513, the above equation requires $R_\star > 1.04R_\odot$ if the star is a cluster member. Fits to the CMD yield a stellar radius $R_\star = 0.80R_\odot$. The lower limit for star 70718 $R_\star > 0.82R_\odot$, whereas the CMD yields $R_\star = 0.56R_\odot$. Clearly both stars lack consistency between the stellar radius based on the CMD location and the stellar radius based on the transit properties.

The planetary nature of the transits in 20513 and 70718 is also unlikely if the stars are field dwarfs. Tingley & Sackett (2005) provide a diagnostic to verify the planetary nature of a transit when only the light curve is available. The diagnostic of Tingley & Sackett (2005) compares the length of the observed transit to an estimate of the transit length by assuming a mass-radius relation for the central star. By assuming a radius of the companion $R_p = 1.0R_J$, I find $\eta_p = 4.0$ and $3.8$ for 20513 and 70718, respectively. Values of $\eta_p \lesssim 1$ correspond to planetary transits.

This radius confirmation is a conceptually different kind of selection criterion. The original selection criteria detect bona fide astrophysical variability that resembles the transit signal. Detecting the transit signal is independent of whether the detected signal results from a planetary companion. Inclusion or replacement of the radius confirmation to the set of selection criteria does not allow a significant reduction for the remaining criteria. Also, by definition the realistic transit injected into the light curve passes the radius confirmation. A negligible fraction of the transits recovered by the original selection criteria do not also pass the radius confirmation. Despite its limited utility in detecting transits, observing a cluster does provide an advantage.
over observing field stars. The additional constraint on the stellar radius from the cluster CMD provides a more reliable confirmation of the planetary nature than the light curve alone (Tingley & Sackett 2005).

4.5. INDIVIDUAL CASES

This section briefly discusses each object that met the selection criteria as a transit candidate but does not belong to the cluster. The V-shaped transit detected in star 30207 rules out a $R_J$ companion. Transiting $R_J$ companions result in a flat bottomed eclipse as the stellar disk fully encompasses the planetary disk. A closer inspection of the light curve also reveals ellipsoidal variations outside of the transit. This light curve matches the properties of a grazing eclipse, which is a typical contaminant in transit searches (Bouchy et al. 2005).

The remaining stars have depths too large for a $R_J$ companion and show evidence for secondary eclipses. I eliminate values beyond $\Delta m \pm 0.5$ mag in the light curve. This eliminates the eclipse bottom for star 20065. Keeping all the data for star 20065 clearly reveals the characteristics of a detached eclipsing binary. Additionally, the period BLS derives for star 20065 aligns the primary and secondary eclipses. Thus, the BLS period is not the true orbital period.

The eclipses in stars 20398 and 20274 do not perfectly phase up. The period search resolution prevents perfect alignment of the eclipses for such short periods.
This effect is inconsequential for detecting transits as they have orbital periods longer than 0.3 day. Other variables exist in the dataset. However, they do not meet the $\Delta \chi^2_{\text{min}} / \Delta \chi^2_{\text{min}}$ selection criterion. A future paper will present variables that exist in this dataset using selection criteria more appropriate for identifying periodic variability (Pepper & Burke 2005).
Fig. 4.1.— $\Delta \chi^2$ as a function of $\Delta \chi^2_-$ for the resulting best-fit transit parameters in all light curves, where $\Delta \chi^2$ and $\Delta \chi^2_-$ are the $\chi^2$ improvement between the flat light-curve model and the best-fit transit and anti-transit model, respectively. $\Delta \chi^2 = 95.0$ selection boundary (vertical line). $\Delta \chi^2/\Delta \chi^2_- = 2.75$ selection boundary (diagonal line). Objects in the lower right corner pass the selection criteria. The selection criteria for the six transit candidates (green diamonds). Resulting selection criteria values after recovery of the example injected transits with light curves shown in Figure 5.1 (blue stars). The label next to the blue stars corresponds to the label in the upper right corner of each panel in Figure 5.1. Blue star labeled 0 shows the selection criteria values before the example transits were injected.
Fig. 4.2.— $f$ as a function of best-fit transit orbital period, where $f$ is the fraction of the total $\chi^2$ improvement with the best-fit transit model that comes from a single night. Objects that pass the $\Delta \chi^2 > 95.0$ selection criteria (red points). The $f = 0.65$ selection boundary (horizontal line). The orbital period regions avoided due to false-positive transit detections (vertical lines). Same as in Figure 4.1 (blue stars and green diamonds).
Fig. 4.3.— Change in magnitude as a function of orbital phase for all stars that meet the transit candidate selection criteria. Negative values for $\Delta$ mag are toward fainter flux levels. The phased period is given in the upper left corner of each panel, and the number in the upper right corner of each panel gives the internal identification number.
Fig. 4.4.— CMD of the cluster NGC 1245. Potential cluster members having $\chi^2_{\text{mem}} < 0.04$ (heavy points). Objects that exceed the selection criteria for transit detection (open diamond).
Fig. 4.5.— Finding charts for transit candidates with light curves shown in Figure 4.3. Each panel shows 54” on a side. North is toward the right, and East is toward the bottom.
### Table 4.1. Transit candidate photometric data.

<table>
<thead>
<tr>
<th>ID</th>
<th>RA(2000.0)</th>
<th>Dec(2000.0)</th>
<th>V (mag)</th>
<th>B−V (mag)</th>
<th>V − I (mag)</th>
<th>χ²</th>
<th>χ²/∆χ²</th>
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<tr>
<td>30207</td>
<td>3:15:40.0</td>
<td>+47:21:18</td>
<td>18.1</td>
<td>1.17</td>
<td>1.31</td>
<td>0.137</td>
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<tr>
<td>20513</td>
<td>3:15:04.6</td>
<td>+47:15:09</td>
<td>18.6</td>
<td>1.10</td>
<td>1.27</td>
<td>0.028</td>
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<tr>
<td>20065</td>
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<td>+47:14:33</td>
<td>16.1</td>
<td>1.02</td>
<td>1.25</td>
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<td>20398</td>
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<td>+47:16:03</td>
<td>18.4</td>
<td>1.29</td>
<td>2.00</td>
<td>0.863</td>
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</tr>
<tr>
<td>20274</td>
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<td>+47:19:29</td>
<td>19.3</td>
<td>1.67</td>
<td>3.37</td>
<td>4.390</td>
<td></td>
</tr>
<tr>
<td>70718</td>
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<td>+47:06:59</td>
<td>21.1</td>
<td>1.28</td>
<td>1.79</td>
<td>0.017</td>
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### Table 4.2. Transit candidate best-fit transit data.

<table>
<thead>
<tr>
<th>ID</th>
<th>P(d)</th>
<th>Δf (mag)</th>
<th>τ(h)</th>
<th>φ</th>
<th>Δχ²/Δχ²</th>
<th>Δχ²</th>
<th>f</th>
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<td>4.614</td>
<td>0.030</td>
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<td>0.72</td>
<td>5.65</td>
<td>584</td>
<td>0.57</td>
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<tr>
<td>20513</td>
<td>1.637</td>
<td>0.018</td>
<td>4.71</td>
<td>0.95</td>
<td>3.85</td>
<td>840</td>
<td>0.49</td>
</tr>
<tr>
<td>20065</td>
<td>3.026</td>
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<td>0.59</td>
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<tr>
<td>20398</td>
<td>0.349</td>
<td>0.145</td>
<td>0.90</td>
<td>0.72</td>
<td>3.76</td>
<td>66056</td>
<td>0.29</td>
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<tr>
<td>20274</td>
<td>0.302</td>
<td>0.050</td>
<td>0.98</td>
<td>0.48</td>
<td>3.23</td>
<td>14063</td>
<td>0.21</td>
</tr>
<tr>
<td>70718</td>
<td>0.640</td>
<td>0.032</td>
<td>3.07</td>
<td>0.16</td>
<td>2.86</td>
<td>312</td>
<td>0.22</td>
</tr>
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</table>

### Table 4.3. Transit candidate stellar parameters.

<table>
<thead>
<tr>
<th>ID</th>
<th>M(M☉)</th>
<th>Log(R/R☉)</th>
<th>Teff(K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20513</td>
<td>0.91</td>
<td>-0.095</td>
<td>5400</td>
</tr>
<tr>
<td>70718</td>
<td>0.63</td>
<td>-0.253</td>
<td>4300</td>
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</table>
Chapter 5

Transit Detection Probability and Transit Survey Results

5.1. Transit Detection Probability Calculation

I did not detect any transit signals consistent with a $R_J$ companion. To analyze this null result, I develop a Monte Carlo detection probability calculation for quantifying the sensitivity of the survey for detecting extrasolar planet transits. The calculation provides the probability of detecting a transit in the survey as a function of the companion semimajor axis and radius. In addition to the photometric noise and observing window, the observed properties of the transit signal depends sensitively on the host mass, radius, limb-darkening parameters, and orbital inclination with respect to the line of sight. Without accurate knowledge of the stellar parameters, a detailed detection probability is not possible. The minimal expenditure of observational resources necessary for determining the stellar parameters for a cluster transit survey provides a significant advantage over transit surveys of the field.
Each star in the survey has a unique set of physical properties and photometric noise, thus I calculate the detection probability for all stars in the survey. This is the first study of its kind to do so. Given the detection probability for each star, the distribution of extrasolar planet semimajor axis, and frequency of extrasolar planet occurrence, the survey should have detected,

$$N_{\text{det}} = \sum_{i=1}^{N} P_{\text{det},i}.$$  \hspace{1cm} (5.1)

extrasolar planets, where the sum is over all stars in the survey,

$$P_{\text{det},i} = \int \int f_\star \frac{d^2p}{dR_p da} P_{\epsilon,i}(a, R_p) P_{T,i}(a, R_p) P_{\text{mem},i} dR_p da,$$  \hspace{1cm} (5.2)

$R_p$ is the extrasolar planet radius, $a$ is the semimajor axis, $f_\star$ is the fraction of stars with planets distributed according to the joint probability distribution of $R_p$ and $a$, $\frac{d^2p}{dR_p da}$. The Monte Carlo detection probability calculation provides $P_{\epsilon,i}(a, R_p)$, the probability of detecting a transit in a given light curve. The term $P_{T,i}(a, R_p)$ gives the probability for the planet to cross the limb of the host along the line of sight, and $P_{\text{mem},i}$ gives the probability the star is a cluster member. This framework for calculating the expected detections of the survey follows from the work of Gaudi et al. (2002). In the following sections I describe the procedure for calculating each of these probability terms.
5.1.1. Calculating Detection Probability

$P_{e,i}(a, R_p)$ is the probability of detecting a transit around the $i$th star of the survey averaged over the orbital phase and orbital inclination for a given companion radius and semimajor axis. I begin this section with a description of the procedure for injecting limb-darkened transits into light curves for recovery. After injecting the transit, I attempt to recover the transit employing the same BLS algorithm and selection criteria as employed during the transit search on the original data. It is critical to employ the same selection criteria during the recovery as the original transit search since only then can I trust the robustness and statistical significance of the detection. The fraction of transits recovered for fixed semimajor axis and $R_p$ determines $P_e$. Next, I characterize the sources of error present in $P_e$ and how I ensure a specified level of accuracy. Finally, I discuss the parallelization of the calculation to obtain $P_e$ for all stars in the survey in a reasonable amount of time.

Burke et al. (2005) discusses the importance of injecting realistic transits for recovery. Mandel & Agol (2002) provide analytic formulas for calculating realistic limb-darkened transits. I employ the functional form of a transit for a quadratic limb-darkening law as given in Section 4 of Mandel & Agol (2002). The quadratic limb-darkening coefficients come from Claret (2000). Specifically, I use the $I$-band limb-darkening coefficients using the ATLAS calculation for $\log g = 4.5$, $\log[M/\text{H}]=0.0$, and $v_{\text{turb}} = 2 \text{ km s}^{-1}$. I assume circular orbits for the companions. All
known extrasolar planets to date that orbit within 0.1 AU, have eccentricities <0.3, and the average eccentricity for these planets is 0.051.

After injecting the transit, I employ the BLS algorithm to recover the injected transit signal using the selection criteria described in Section 4.2. For numerical efficiency, I relax the resolution of the BLS search parameters. I adopt a fractional period step, $\eta = 0.004$, and phase space binning, $N_{\text{bins}} = 300$. Despite the reduced resolution, higher resolution, converged solutions reveal only a 0.003 lower probability resulting from the adopted parameters. I correct all probabilities for this systematic even though it is at an insignificant level compared to the other uncertainties.

Figure 5.1 visualizes the injected transits with increasing degrees of significance from top to bottom. This Figure shows light curves with an injected transit phased at the period as returned from the BLS algorithm. The solid line illustrates the injected limb-darkened transit signal. The top two panels and the bottom two panels illustrate 1.0 and 1.5 $R_J$ companions, respectively. The transit recovery in the top panel only slightly meets the selection criteria, thus giving a visual impression at the sensitivity of the survey. The resulting selection criteria values after recovery of the injected transits are shown in Figures 4.1 and 4.2 by the blue stars, and the labels next to the stars correspond to the panel label given in the upper right hand corner. The blue star, labeled 0 in Figures 4.1 and 4.2, represents the selection criteria values found before injecting the example transits shown in Figure 5.1. The

1http://www.obspm.fr/encycl/catalog.html
modeled transits shown in this Figure are injected into the light curve of the same V=16.6 potential cluster member, and the rms scatter in the light curve before transit injection is $\sigma = 0.003$.

As opposed to previous work, I carefully examine and control the errors present in the calculation. During injection of a transit a fixed semimajor axis, the transit can occur during any phase of the orbit. I ensure convergence of $P_\epsilon$ by injecting enough trial transits at random orbital phase by the following procedure. Based on binomial statistics, the error in the resulting probability at fixed orbital period depends on the actual probability and the number of trial transit phases, $\sigma_\epsilon = \sqrt{N_{\text{trial}} \epsilon_{\text{act}} (1 - \epsilon_{\text{act}})}$, where $N_{\text{trial}}$ is the number of trial transit phases and $\epsilon_{\text{act}}$ is the actual probability (unknown a priori). Maintaining the same error in the detection probability for differing $\epsilon_{\text{act}}$ requires a variable number of trial phases. I obtain an initial estimate for the probability, $\epsilon_{\text{est}}$, after $N_{\text{trial}} = 100$. To stop the calculation at fixed orbital period, $N_{\text{trial}}$ increases until the probability converges when $\sigma_\epsilon = \sqrt{N_{\text{trial}} \epsilon_{\text{est}} (1 - \epsilon_{\text{est}})} \leq 0.02$. The above procedure systematically overestimates $\epsilon_{\text{act}}$ when $\epsilon_{\text{act}} \gtrsim 0.95$ and systematically underestimates $\epsilon_{\text{act}}$ when $\epsilon_{\text{act}} \lesssim 0.05$. Increasing $N_{\text{trial}}$ before calculating $\epsilon_{\text{est}}$ reduces this systematic. However, this systematic is of order the adopted $\sigma_\epsilon = 0.02$ accuracy.

In addition to a random orbital phase, assuming a random orientation of the orbit requires taking into account an even distribution in $\cos i$, where $i$ is the inclination of the orbit. Only a narrow range of inclinations, $\cos i \leq (R_* + R_p)/a$,
results in a transit. Thus, I inject the transit with an even distribution in \( \cos i \) between

\[ 0 \leq \cos i \leq \left( \frac{R_* + R_p}{a} \right). \]

The previous discussion pertains to ensuring a prescribed accuracy at fixed semimajor axis. However, the expected detection rate also requires an integral over semimajor axis, which must be sampled at high enough resolution to ensure convergence of the integral. I calculate the probability at even logarithmic intervals, \( \delta \log a = 0.011 \) AU. In comparison to high resolution, converged calculations, this semimajor axis resolution results in an absolute error in the integrated detection probability \( \sigma_e = 0.003 \). I inject transits with semimajor axis from the larger of 0.0035 AU and 1.5\( R_* \) to 0.83 AU. The best-fit isochrone to the cluster CMD determines the parent star radius.

Generating the light curve from the raw photometric measurements is numerically time consuming. Thus, I inject the transit after generating the light curve. This procedure has the potential to systematically reduce or even eliminate the transit signal, because generating the light curve and applying a seeing decorrelation tend to “flatten” a light curve. To quantify the significance of this effect, I inject transits in the raw photometric measurements before the light curve generation procedure on several stars in the sample. Comparing the detection probability obtained by injecting transits before light curve generation to the detection probability obtained by injecting the transit after light curve generation reveals that injecting the transit after generating the light curve overestimates the
detection probability by \( \sim 0.03 \). I decrease the calculated probability at fixed period by 0.03 to account for this systematic effect.

The 0.03 systematic overestimate in the detection probability becomes increasingly important for correctly characterizing the detection probability at long orbital periods. For instance, the detection probability for a star of median brightness will be overestimated by \( >15\% \) for orbital periods \( >4.0 \) day and 1.5 \( R_J \) companions if this systematic effect is not taken into account. The detection probability is overestimated by \( >50\% \) for orbital periods \( >8.0 \) day without correction. The results for 1.0 \( R_J \) companions are even more severe. The detection probability would be overestimated by 50\% for periods beyond 1.8 day for a star of median brightness without correction.

Based on the CMD of NGC 1245, this study contains light curves for \( \sim 2700 \) stars consistent with cluster membership. Initially, I calculate the detection probability for 2 possible companion radii: 1.0 and 1.5 \( R_J \). For each star, on average I inject 50000 transits for a single companion radius at 150 different semimajor axes. In total, I inject and attempt to recover \( \sim 2.7 \times 10^8 \) transits. Current processors allow injection and attempted recovery on order of 1 s per transit. A single processor requires \( \sim 3000 \) days for the entire calculation. Fortunately, the complete independence of a transit injection and recovery trial allows parallelization of the calculation. I accomplish a parallel calculation via a server and client architecture. A server injects a transit in the current light curve and sends it to a client for recovery.
Based on the computing resources available, I employ two different methods for communication between the server and clients. Using a TCP/IP UNIX socket implementation for communication between the server and clients allows access to \( \sim 40 \) single-processor personal workstations connected via a local area network within the department of astronomy at The Ohio State University. Additionally, the department of astronomy at The Ohio State University has exclusive access to a 48 processor Beowulf cluster via the Cluster Ohio program run by the Ohio Supercomputer Center. The Message Passing Interface (MPI) libraries provide communication between the server and clients on the Beowulf cluster. A Beowulf cluster belonging to the Korean Astronomy Observatory also provided computing resources for this calculation. C programming source code for either client-server communication implementation is available upon request from the author.

The light solid line in Figure 5.2 shows the detection probability, \( P_e(a, R_p) \), for three representative stars in order of increasing apparent magnitude from top to bottom and for the 2 companion radii, 1.0 and 1.5 \( R_J \), from left to right, respectively. In general, the probability nears 100% completion for orbital periods \( \lesssim 1.0 \) day and then has a power law fall off toward longer orbital periods. The large drop in the detection probability around 0.5 and 1.0 day orbital periods results from the selection criteria I impose. The narrow, non-zero spikes in the detection probability near the 0.5 and 1.0 day orbital periods result from injecting a transit at this period, but the BLS method returns a best-fit period typically at the \( \sim 0.66 \) day alias.
The falloff in the detection probability toward longer orbital periods results from the requirement of observing more than one transit. Figure 5.2 shows the detection probability with 3.3 times higher resolution in orbital period and a lower, 1%, error in the detection probability at fixed orbital period than the actual calculation. Thus, the figure resolves variability in the detection probability as a function of orbital period for probabilities \(\gtrsim 1\%\). However, such fine details have negligible impact on the results.

5.1.2. Calculating Transit Probability

The probability for a transit to occur is

\[
P_T = \frac{(R_* + R_p)}{a}.
\]

This transit probability assumes the transit is equally detectable for the entire possible range of orbital inclinations that geometrically result in a transit. As \(\cos i\) for the orbit approaches \((R_* + R_p)/a\) the transit length and depth decreases, degrading the transit S/N. I address this when computing \(P_\epsilon\) by injecting the transit with an even distribution in \(\cos i\) between the geometric limits for a transit to occur. Thus, \(P_T\) represents the overall probability for a transit with high enough inclination to begin imparting a transit signal, while the detailed variation of the light curve signal for varying inclination takes place when calculating \(P_\epsilon\). \(P_T\) is shown as the dashed light line in Figure 5.2. The heavy solid line in Figure 5.2 is a product of \(P_\epsilon\) and \(P_T\).
5.1.3. Calculating Membership Probability

The Monte Carlo calculation requires knowledge of the stellar properties, and the given properties are only valid if the star is in fact a bona fide cluster member. An estimate of the field-star contamination from the CMD provides only a statistical estimate of the cluster membership probability. Based on the study of the mass function and field contamination in Chapter 3, I estimate the cluster membership probability, \( P_{\text{mem}} \), as a function of stellar mass. In brief, I start with a subsample of stars based on their proximity to the best-fit cluster isochrone (selection on \( \chi^2_{\text{mem}} < 0.04 \), see Section 4.3). This sample contains \( N_\star \sim 2700 \) potential cluster members, and the heavy points in Figure 4.4 mark this cluster sample in the CMD. The best-fit isochrone allows an estimate of the stellar mass for each member of the cluster sample, and I separate the sample into mass bins. Repeating this procedure on the outskirts of the observed field of view, scaled for the relative areas, provides an estimate of the field-star contamination in a given mass bin. I fit \( P_{\text{mem}} \), given in discrete mass bins, with a smooth spline fit for interpolation.

The solid line in Figure 5.3 shows \( P_{\text{mem}} \) as a function of stellar mass. The corresponding probability is given on the right side ordinate. The clear histogram shows the distribution of the potential cluster members as a function of mass. The lower shaded histogram shows the product of the potential cluster members
histogram and $P_{\text{mem}}$. This results in effectively $N_{\text{eff}} \sim 870$ cluster members in total. For reference, the corresponding apparent $I$-band magnitude is given along the top.

5.2. Transit Survey Results

5.2.1. Results Assuming a Power-law Orbital-period Distribution

The previous section describes the procedure for calculating the sensitivity of the survey to detect $R_J$ companions as a function of semimajor axis. The results from this calculation enable us to place an upper limit on the fraction of cluster members harboring close-in companions given the null result. However, calculating the upper limit over a range of orbital periods necessitates assuming a distribution of orbital periods for the extrasolar planet companions. Radial velocity surveys characterize the distribution of extrasolar planets in period as $dn \propto P^{-\gamma}dP$, with $0.7 \leq \gamma \leq 1.0$, corresponding to $dn \propto a^{-\beta}da$, with $0.5 \leq \beta \leq 1.0$ (Stepinski & Black 2001; Tabachnik & Tremaine 2002). These studies fit the entire range of orbital periods ranging from several days to several years. More recently, after an increase in the number of extrasolar planet discoveries, Udry et al. (2003) confirm a shortage of planets with $10 \leq P \leq 100$ day orbits. Thus, the period distribution may take on different values of $\gamma$ in the $P \lesssim 10$ day and $P \gtrsim 100$ day regimes.
At even shorter orbital periods, the radial-velocity extrasolar-planet period distribution has an apparent cutoff for $P \lesssim 3.0$ day. In contrast to the radial velocity results, the initial extrasolar planet discoveries via the transit technique have periods less than 3.0 day (Konacki et al. 2004). Due to the small number of transiting extrasolar planet discoveries to date and the strong drop in sensitivity for longer period companions of the transit technique (Figure 5.2), Gaudi et al. (2005) shows consistency between the apparent lack of VHJ companions in the radial velocity surveys and their discovery in transit surveys. Due to the incomplete knowledge of the actual period distribution of extrasolar planets and its possible dependence on the properties of the parent star, I provide upper limits assuming an even logarithmic distribution of semimajor axis. Thus, I assume a form of the joint probability distribution of the semimajor axis and $R_p$ given by

$$\frac{d^2p}{dR_p da} = k\delta(R_p - R_p')a^{-1}, \quad (5.3)$$

where $k$ is the normalization constant, $\delta$ is the Dirac delta function, and $R_p'$ is the planet radius. I initially give results for $R_p' = 1.0$ and 1.5 $R_J$.

Figure 5.4 shows the probability for detecting a $1.0 \leq P \leq 3.0$ day companion with an even logarithmic distribution in semimajor axis as a function of apparent $I$-band magnitude. The left and right panels show results for a 1.5 and 1.0 $R_J$ companion, respectively. The top panels of Figure 5.4 show the probability for detecting an extrasolar planet, $P_{\det}$, assuming $P_{\text{mem}} = 1.0$. The bottom panels show
$P_{\text{det}}$ after taking into account $P_{\text{mem}}$. The results for 1.0 $R_J$ companions broadly scatter across the full range of detection probability. However, the 1.5 $R_J$ companion results delineate a tight sequence in detection probability as a function of apparent magnitude.

The 1.5 $R_J$ companion signal lies many times above the rms scatter in the light curve (see Figure 2.4). Thus, a single measurement contributes a large fraction of the S/N required for detection. In this limit, the observing window function mainly determines the detection probability, and as I show in Section 6.2 the result is similar to results obtained by the theoretical detection probability framework of Gaudi (2000). However, the 1.0 $R_J$ companion transit signal comes closer to the detection threshold. Pepper & Gaudi (2005) describe the sensitivity of a transit survey as a function of planet radius. The sensitivity of a transit survey depends weakly on $R_p$ until a critical radius is reached when the S/N of the transit falls rapidly. The sensitivity of the survey for 1.0 $R_J$ is near this threshold, hence the large scatter in the detection probability.

With the detection probabilities for all stars in the survey for the assumed semimajor axis distribution, I can calculate the expected number of detections scaled by the fraction of cluster members with planets. Thus, from the Poisson distribution, the result $N_{\text{det}} = 0$ is inconsistent at the $\sim$95\% level when $N_{\text{det}} \sim 3$. This allows us
to solve for the 95% confidence upper limit on the fraction of cluster members with planets,

\[ f_{<.95} = 3.0/\sum_{i=1}^{N_*} P_{\text{det},i}. \] (5.4)

Figure 5.5 shows the 95% confidence upper limit on the fraction of stars with planets in NGC 1245 for several ranges of orbital period. I again assume an even logarithmic distribution of semimajor axis. The solid and dash lines give results for 1.5 and 1.0 \( R_J \) companions, respectively. I follow Gaudi et al. (2005) and show results for HJ (3.0<\( P <9.0 \) day) and VHJ (1.0<\( P <3.0 \) day) ranges. In addition, I show results for a more extreme population of companions with \( P_{\text{Roche}} < P < 1.0 \) day, where \( P_{\text{Roche}} \) is the orbital period at the Roche separation limit, which I designate as Extremely Hot Jupiter (EHJ). Assuming a negligible companion mass, the Roche period depends solely on the density of the companion. Jupiter, Uranus, and Neptune have nearly the same \( P_{\text{Roche}} \sim 0.16 \) day. For 1.5 \( R_J \) companions I limit the fraction of cluster members with companions to <1.5%, <6.4%, and <52% for EHJ, VHJ, and HJ companions, respectively. For 1.0 \( R_J \) companions, I find <2.3% and <15% have EHJ and VHJ companions, respectively.

The detection probability decreases rapidly with orbital period beyond 1.0 day. As a result, the survey does not reach the sensitivity in order to place an upper limit on 1.0 \( R_J \) companions beyond \( P > 3.0 \) day. I further divide the VHJ period range and show upper limits for 1.0<\( P <2.0 \) day, \( P_{12} \), and 2.0<\( P <3.0 \) day, \( P_{23} \) period
ranges. For 1.5 \( R_J \) companions I limit \( f_* \) to <5.2% and <11% for \( P_{12} \) and \( P_{23} \), respectively. For 1.0 \( R_J \) companions I limit \( f_* \) to <19% and <47% for \( P_{12} \) and \( P_{23} \), respectively. I also divide the HJ period range and limit \( f_* \) for 1.5 \( R_J \) companions in the 3.0 < \( P \) < 6.0 day to <36%.

### 5.2.2. Results for Other Companion Radii

Due to computing limitations I calculate detection probabilities for the entire cluster sample only for 1.5 and 1.0 \( R_J \) companions. In Section 5.2.3 I show that an upper limit determination using a subsample of the stars with size \( N_{\star, SS} = 100 \) approximates results based on the entire stellar sample. Thus, I calculate upper limits for a variety of companion radii using \( N_{\star, SS} = 100 \) randomly chosen stars in the sample. Instead of showing upper limit results over a range of orbital periods, I derive upper limits at fixed period by replacing the semimajor axis distribution with a \( \delta(a) \) function in Equation 5.3. Figure 5.6 shows the upper limit on the fraction of stars with planets in the survey as a function of orbital period. The lines show results for various values of the companion radius in terms of \( R_J \) as indicated by the label next to each line along the top of the figure. The shaded regions denote orbital periods removed by the selection criteria in order to eliminate false-positive transit detections that occur around the diurnal period and 0.5 day alias. At smaller companion radii, the transit S/N \( \propto R_c^2 \) drops quickly. Toward larger companion radii the S/N of the transit saturates and the observational
window function increasingly dominates the survey effectiveness. The survey cannot
detect companions with $R_c > 3.5R_J$ as the transit/eclipse becomes too deep given
the removal of measurements that deviate by more than 0.5 mag from the mean
light-curve level.

5.2.3. ERROR IN THE UPPER LIMIT

In this section I discuss several sources of error present when determining an
upper limit on the fraction of stars with planets. Computing power limitations
discourage calculating detection probabilities over the entire cluster sample. Thus,
I first characterize the error associated with determining an upper limit using only
a subset of the entire cluster sample. Starting with Equation 5.4 I derive an error
estimate when using a subsample by the following means. Replacing the summation
over $P_{i,\text{det}}$ with the arithmetic mean, $\bar{P}_{\text{det}}$, Equation 5.4 becomes

$$f_{<.95} = 3.0/(N_\star \bar{P}_{\text{det}}).$$

(5.5)

By propagation of errors, the error in the upper limit is given by

$$\sigma_f = \frac{3.0}{N_\star \sigma_{\bar{P}_{\text{det}}}},$$

(5.6)

where $\sigma_{\bar{P}}$ is the error in the mean detection probability. The error in the mean
detection probability scales as $\sigma_{\bar{P}} = \sigma_P/\sqrt{N_{\star,\text{SS}}}$, where $\sigma_P$ is the intrinsic standard
deviation of the distribution of $P_{i,\text{det}}$ values and $N_{\star,\text{SS}}$ is the size of the subsample.
I empirically test this error estimate by calculating the upper limit with subsamples of increasing size. The small points in Figure 5.7 show the upper limit on the fraction of stars with planets as a function of the subsample size. The upper limit calculation assumes an even logarithmic distribution of semimajor axis for companions with \( 1.0 \leq P \leq 3.0 \) day for 1.5 and 1.0 \( R_J \) radius companions, top and bottom panels, respectively. Neighboring columns of upper limits differ by a factor of 2 in the subsample size. I randomly draw stars from the full sample without replacement, making each upper limit at fixed sample size independent of the others. The dash line represents the upper limit based on the full cluster sample.

The distribution of upper limits around the actual value possesses a significant tail toward higher values. This tail results from the significant number of stars with \( P_{\text{tot}} = 0.0 \). At fixed sample size, the large square point represents the mean upper limit. Estimates of an upper limit with \( N_{\star, \text{SS}} \lesssim 20 \) systematically overestimates the upper limit. The open star symbol represents the \( 1 - \sigma \) standard deviation of the distribution at fixed sample size. The solid line shows the error estimate from Equation 5.6. Despite the non-Gaussian nature of the underlying distribution, the error estimate in the upper limit roughly corresponds with its empirical determination especially toward increasing \( N_{\star, \text{SS}} \) where the systematic affects become negligible. From Figure 5.7, \( N_{\star, \text{SS}} \gtrsim 100 \) provides adequate control of the random and systematic errors in calculating an upper limit without becoming numerically
prohibitive. This verifies the procedure for estimating the upper limit for a variety of companion radii in Section 5.2.2.

### 5.2.4. Error in Determining Sample Size

Up to this point, I have mainly addressed sources of error directly associated with determining $P_e$. However, the upper limit error budget contains an additional source of error from uncertainties in determining $P_{\text{mem}}$. This additional source of error directly relates to the accuracy in determining the number of single main-sequence stars in the survey.

I characterize this error as follows. At fixed orbital period, $P_{\text{det}} = P_{\text{mem}} \times P_e \times P_T$. Given the independence of $P_{\text{mem}}$ from the other terms, the previous average is separable, such that $P_{\text{det}} = \overline{P_{\text{mem}}} \times \overline{P_{\text{det}}'}$. This separation changes the derived upper limit by a negligible 0.3% relative error. The separation allows us to rewrite Equation (5.5) as

$$f_{<,95} = 3.0/(N_{*,\text{eff}} \overline{P_{\text{det}}'})$$

(5.7)

where $N_{*,\text{eff}} = N_\star \times \overline{P_{\text{mem}}}$ is the effective number of cluster members in the sample after taking into account background contamination. Thus, $N_{*,\text{eff}}$ carries equal weight with $\overline{P_{\text{det}}'}$ in the upper-limit error budget.
The ability to determine \( N_{\ast,\text{eff}} \) accurately provides an advantage for transit surveys toward a rich stellar cluster rather than toward a random galactic field. Even though methods based on the cluster CMD statistically determine cluster membership, they concentrate on a narrow main-sequence region to search for planets where the cluster counts significantly outweigh the background contamination counts. By concentrating on the main sequence of a cluster, this survey has only \( \sim 68\% \) contamination by background stars. In contrast, random galaxy fields contain \( \gtrsim 90\% \) contamination by sub-giant and giant stars for \( V<11 \) surveys (Gould & Morgan 2003). Overall, \( N_{\ast,\text{eff}} \) has an 8% error, which propagates to a relative error of 8% in the upper limit.

### 5.2.5. Error Due to Blends and Binaries

The final source of error I address results from stellar blends due to physical binaries or chance, line-of-sight associations. The additional light from an unresolved blend dilutes a transit signal from one component of the blend. Thus, I overestimate the ability to detect a transit around blends. However, a compensatory effect arises since the extra light from a blend results in an overestimate in the stellar mass and radius, which in turn results in modeling a shallower transit.

Due to the steep dependence of luminosity on stellar mass, the dilution of the transit signal takes precedence. For instance, a transiting companion orbiting a
single component of an equal mass binary, \( q = 1 \), results in a transit signal \( 1/4 \) the signal of the single star case. However, in practice the transit signal is not overestimated by a factor of 4, since I overestimate the stellar radius by a factor \( \sim 3/2 \) assuming \( L \propto R^{7/4} \) when treating the light from a \( q = 1 \) blend as a single star.

Instead of modeling a completely undiluted transit, the method naturally results in a transit diluted by a factor of \( 4/9 \). Thus, overall, the modeled transit signal is only a factor of \( \sim 2 \) higher than if I properly model the \( q = 1 \) binary. The \( q = 1 \) binary represents the extreme, and the effect quickly becomes negligible for smaller mass ratios.

Detailed modeling of this effect is beyond the scope of this paper, but I can estimate the number of stars affected. Finding charts in Figure 4.5 demonstrate the stellar crowding conditions of the survey. Due to low stellar crowding, I estimate chance blends have a negligible effect in comparison to physically associated binaries (Kiss & Bedding 2005).

The latest Coravel radial velocity survey dedicated to F7-K field dwarfs (Halbwachs et al. 2004) and the visual binary and common proper motion pairs survey of Eggenberger et al. (2004) provide the basis for the binary star estimates. Overall they find a binary frequency of 56% for systems with \( \log(P) \leq 6.31 \). However, due to the strong dependence of luminosity on the stellar mass only systems with mass ratio, \( q > 0.6 \), significantly contribute light to dilute the transit signal. When taking binaries across the entire range of orbital periods the mass-ratio
distribution peaks near $q \sim 0.2$ and slowly drops toward higher $q$ (Duquennoy & Mayor 1991). From Figure 10 in Duquennoy & Mayor (1991), only $\sim 20\%$ of their binary systems have $q > 0.6$. Thus, if the binary statistics for the cluster matches the field dwarfs, transit dilution occurs for $\sim 11\%$ of the stellar sample. The radial velocity survey for binaries in the Pleiades and Praesepe clusters reveals consistency with the frequency of binaries in the field survey (Halbwachs et al. 2004).

In principle, the data from this survey can also answer whether the binary statistics of the cluster matches the field dwarfs. However, the statistical methods and selection criteria described in this study do not optimally detect interacting and eclipsing binaries. Additionally, in order to reach planetary companion sensitivities, I remove light-curve deviations beyond 0.5 mag as discrepant, which removes the deep eclipses. A detailed calculation is beyond the scope of this study, but a preliminary analysis of the light curves retaining measurements beyond 0.5 mag deviation and selection criteria based on the Stetson (1996) $J$ statistic and the ANOVA period search algorithm by Schwarzenberg-Czerny (1996) allows some rough comparisons (Pepper & Burke 2005). For stars consistent with cluster membership, $\chi^2_{\text{mem}} < 0.04$, 6 stars have variability consistent with interacting or eclipsing binaries. Another four stars have significant variability that cannot be unambiguously attributed to binarity. Given the 68% field contamination in the sample $\sim 2.5$ stars have variability attributable to binarity and belong to the cluster.
The longest period out of the 6 potential cluster-member eclipsing binaries is 1.8 day. I use the middle left panel of Figure 5.2 to roughly estimate the detection probability for \( P < 2.0 \) day, resulting in \( P_e \sim 0.8 \). A companion with a stellar radius enhances the inclination alignment probability, \( P_T \), over the planetary companion case. Estimating a 1.5 times increase over the planetary companion case, I estimate \( P_T \sim 0.25 \). Overall, I detect \( P_e \times P_T \sim 0.2 \) of the stars in the sample that have a binary companion with \( P < 2.0 \) day. Since I detect \( \sim 2.5 \), the survey sample contains \( \sim 12 \) binaries with \( P < 2.0 \) day.

I can compare this result to the field study of Halbwachs et al. (2004). Their Figure 4 and the normalization that 13.5% of field dwarfs have a companion with \( P < 10 \) year indicate that 0.6% of stars have a companion with \( P < 2 \) day. With \( \sim 870 \) cluster members in the survey, consistency with the Halbwachs et al. (2004) result for field dwarfs predicts \( \sim 5 \) cluster members have companions with \( P < 2.0 \) day. This indicates that the cluster potentially contains more binaries than the local field dwarfs, but this result is only at the 1-\( \sigma \) level.

5.2.6. Overall Error

The errors involved with determining the number of cluster members dominates the error budget in determining the upper limit. However, as discussed in Section 5.1.1, this is only true if one quantifies and corrects for the systematic
overestimate in detection probability due to a reduction in the transit signal from the procedures of generating and correcting the light curve. For instance, at the median stellar brightness, the detection probability is overestimated by >15% for orbital periods >4.0 day and >1.0 day for 1.5 and 1.0 $R_J$ companions, respectively, without correction. Since I characterize this systematic effect, the error in determining the number of cluster members dominates the error budget.

Additionally, the potential for a large contamination of binaries diluting the transit signal necessitates an asymmetrical error bar. I roughly quantify the error estimate resulting from binary contamination from the field dwarf binary statistics. I adopt 11% of the sample containing a $q > 0.6$ binary as a $1 - \sigma$ systematic fractional error due to binary star contamination. Overall, combining this systematic error with the 7% fractional error in determining the cluster membership, upper limits derived from the full stellar sample contain a $\sigma_{-7}^{+13}$ fractional error.
Fig. 5.1.— Phased light curves showing the recovery of transits injected in the light curve by the Monte Carlo calculation. The injected limb-darkened transit signal (solid line). The top two panels and bottom two panels show results for 1.0 and 1.5 $R_J$ companions, respectively. The transit recovery in the top panel slightly meets the selection criteria and gives an impression for the sensitivity of the survey. The labels in the upper right corner of the panels designate the resulting selection criteria values shown in Figures 4.1 and 4.2.
Fig. 5.2.— Detection probability as a function of the orbital period (heavy solid line). This is a product of the probability for a transit to occur (dash line) and the probability an injected transit meets the selection criteria (light solid line). The panels from top to bottom show representative stars in order of increasing apparent magnitude. Left: 1.5 $R_J$ companion. Right: 1.0 $R_J$ companion.
Fig. 5.3.— Distribution of the potential cluster members as a function of stellar mass (open histogram). Membership probability (right hand ordinate) as a function of stellar mass (solid line). Product of the potential cluster member histogram and the cluster membership probability (shaded histogram). The corresponding apparent $I$-band magnitude is given along the top.
Fig. 5.4.— *Top:* Probability for transit detection as a function of the apparent $I$-band magnitude assuming an even logarithmic distribution in semimajor axis from $1.0 < P < 3.0$ day and assuming $P_{\text{mem}} = 1.0$. *Left:* $1.5 R_J$ companion. *Right:* $1.0 R_J$ companion. *Bottom:* Same as top panels, but taking into account $P_{\text{mem}}$. 
Fig. 5.5.— Upper limit (95% Confidence) on the fraction of stars in the cluster with companions for several ranges in orbital period assuming an even logarithmic distribution in semimajor axis. 1.5 $R_J$ companion (solid line). 1.0 $R_J$ companion (dash line).
Fig. 5.6.— Upper limit (95% Confidence) on the fraction of stars in the cluster with companions for several companion radii as label along the top. The result for 1.0 $R_J$ companion is based on the entire sample, whereas the results for the other companion radii are based on $N_* = 100$ sample. The shaded regions denote orbital periods removed by the selection criteria in order to eliminate false-positive transit detections that occur around the diurnal period and 0.5 day alias.
Fig. 5.7.— Top: Estimate for the upper limit (95% Confidence) on the fraction of stars in the cluster with 1.5 $R_J$ companions assuming an even logarithmic distribution in semimajor axis between $1.0 < P < 3.0$ day orbital period as a function of the sample size employed in making the estimate (small points). Upper limit based on the entire sample (dash line). Average upper limit at fixed sample size (square points). Standard deviation in the distribution of upper limits at fixed sample size (open stars). Error model estimate for the standard deviation in the upper limit (solid line). Bottom: Same as top panel, but for 1.0 $R_J$ companions.
Chapter 6

Discussion of Transit Results and Planning
Future Surveys

Transit surveys have still not achieved the planet discovery-rate predictions estimated just a few years ago (Horne 2003). Given the plethora of null results for transit surveys understanding the null results becomes a necessity. Along with this work, several other transit surveys have quantified their detection probability from actual observations in an attempt to constrain the fraction of stars with planets or quantify the consistency with the solar neighborhood radial velocity planet discoveries (Gilliland et al. 2000; Weldrake et al. 2005; Mochejska et al. 2005; Hidas et al. 2005; Hood et al. 2005). Unfortunately, a direct comparison of upper limits from this work with these other transit surveys cannot be made. Until this study, none of the previous studies have quantified the random or systematic errors present in their techniques in sufficient detail to warrant a comparison. Additionally, previous studies do not have quantifiable selection criteria that completely eliminate
false-positive transit detections due to systematic errors in the light curve, a necessary component of an automated Monte Carlo calculation.

6.1. Initial Expectations vs. Actual Results

In the meantime, I can discuss why the initial estimate of finding 2 $R_J$ companions assuming 1% of stars have $R_J$ companions evenly distributed logarithmically between 0.03 to 0.3 AU (Burke et al. 2003) compares to the actual estimate based on the results from this study of 0.1 $R_J$ companions. The initial estimates for the detection rate are based on the theoretical framework of Gaudi (2000). Given a photometric noise model, observational window, and S/N of the transit selection criteria, the theoretical framework yields an estimate of the survey detection probability. This theoretical detection probability coupled with a luminosity function for the cluster determines the expected number of detections. As I show next, the initial estimates did not account for the light curve noise floor or detector saturation, contain optimistic estimates for the sky background and luminosity function, and the theoretical detection probability begins to break down near the critical threshold of detection.

The top panels of Figure 6.1 compare the detection probability of the Monte Carlo calculation of this study to the initial theoretical estimate. The small points replicate the Monte Carlo results from the top panels of Figure 5.4 while the dash line shows the detection probability based on the initial theoretical expectations.
The initial theoretical expectations clearly overestimate the detection probability. The bright end continues to rise due to ignoring the effects of detector saturation and the photometric noise floor. The faint end does not cutoff due to an underestimated sky brightness. The initial estimate of the sky brightness, $19.5 \text{ mag arcsec}^{-2}$, compares optimistically to the range of sky brightnesses encountered during the actual observations. The sky varied between 17.5 and 19.0 mag arcsec$^{-2}$ over the course of the observations. The full lunar phase took place near the middle of the observation, and the Moon came within 40 degrees of the cluster when nearly full.

The initial estimate for the cluster luminosity function simply selected cluster members via tracing by eye lines that bracket the main sequence in the CMD. This crude technique led to an estimated 3200 cluster members down to $I \sim 20$. A careful accounting of the field star contamination results in only $\sim 870$ cluster members in the survey. The luminosity function overestimate and the expected sensitivity to transits around the bright and faint cluster members leads to a factor of 4-5 overestimate in the number of cluster members in the survey. Additionally, the factor of 4-5 overestimate of the initial detection probability when compared to binned average detection probability for the Monte Carlo results (open stars in Figure 6.1), easily accounts for the factor of 20 difference in the overall expected detections.
6.2. Improving Theoretical Expectations

Clearly, accurate and realistic transit detection statistics requires more detailed analysis than these early estimates and more careful theoretical work has already been done (Pepper & Gaudi 2005). In the case of an open cluster, delineating cluster membership by tracing the main sequence in the CMD overestimates the number of cluster members. A careful subtraction of the field contamination is necessary in order to extract an accurate cluster-member count. Transit surveys that observe galactic fields must be content with empirical determinations of the dwarf star population. Transit surveys that observe bright, \( V < 11 \), field stars may have up to 90% contamination of their sample by sub-giant and giant stars (Gould & Morgan 2003).

A photometric noise model that accurately reflects the quality of observations is the next step in correctly calculating a theoretical detection probability. From Figure 2.4, I estimate the actual photometric noise present in the data. This includes the proper sky measurement and systematic floor in the photometric precision. With a noise model similar to the lower solid line in Figure 2.4, we recalculate the theoretical detection probability. The dot dash line in Figure 6.1 shows the resulting detection probability still overestimates the Monte Carlo results. However, it does agree with the faint-end cutoff of the Monte Carlo calculation. I impose the
bright-end cutoff due to saturation effects at the same magnitude as the observed increase in light curve rms as shown in Figure 2.4.

For these results I include an additional effect not taken into account by Gaudi (2000). I multiply the transit S/N selection criteria, Equation 5 of Gaudi (2000), by \( \sqrt{\max(N_{\text{obs}}, 1.7)} \), where \( N_{\text{obs}} \) is the typical number of transits detected throughout the observing run. The \( N_{\text{obs}} = 1.7 \) floor in this factor corresponds to the \( f = 0.65 \) selection criteria employed in the Monte Carlo calculation. For simplicity, I take \( N_{\text{obs}} = N_{\text{tot}} / P \times 0.2 \), where \( N_{\text{tot}} = 16 \), the length of the observing run in days, and the factor of 0.2 accounts for the actual observational coverage encountered during the run.

Given the theoretical calculation still overestimates the Monte Carlo results, to increase the realism of the theoretical detection probability, I include a linear limb-darkening law, which effectively weakens the transit depth. I solve for the factor \( G \), Equation 6 of Gaudi (2000), assuming a linear limb-darkening parameter, \( \mu = 0.6 \), for all stars. The inclusion of limb darkening significantly impacts the theoretical detection probability as the solid line in Figure 6.1 demonstrates. Although the theoretical detection probability still overestimates the upper envelope of results from the Monte Carlo calculation, the level of agreement, after including an accurate photometric noise model and limb darkening, shows significant improvement over the initial estimates.
Despite the improved agreement, the Monte Carlo detection probability calculation shows significant scatter at fixed magnitude. The theoretical probability treats all stars at fixed magnitude as having the same noise properties. With the theoretical detection probability I can address whether the scatter in detection probability at fixed magnitudes results from the observed scatter in noise properties at fixed magnitude as shown in Figure 2.4. Thus, I calculate a theoretical detection probability for each star individually using the measured rms in the light curve for each star to determine the theoretical transit S/N selection criteria, Equation 5 of Gaudi (2000). The small points in the bottom panels of Figure 6.1 show the resulting theoretical detection probability.

Some of the scatter in detection probability results from the scatter in noise properties as a function of magnitude. The heavy star points represent the average Monte Carlo detection probability in 0.25 magnitude bins. In the case of the 1.5 $R_J$ companions, the signal is large in comparison to the photometric noise. The left panels of Figure 6.1 demonstrate the theoretical detection probability overestimates the Monte Carlo detection probability by only 20%. However, the closer the transit signal approaches the systematic and rms noise, the theoretical detection probability strongly overestimates the actual detection probability. In the case of 1.0 $R_J$ companions (right panels of Figure 6.1), the theoretical calculation overestimates the Monte Carlo results by 80%. Thus, I urge caution when relying on a theoretical
detection probability when the survey is near the critical threshold for transit
detection. Such is the case for $1.0 \ R_J$ companions in the survey.

6.3. Planning Future Surveys

Even though the theoretical calculation overestimates the absolute detection
probability by a factor of $<2$, tests on a small sample of stars with the Monte
Carlo calculation reveal it provides a much higher relative accuracy. Thus, the
computationally efficient theoretical calculation allows us to examine the relative
change in the detection probability for a given change in survey parameters. For
planning future surveys it is essential to decide between increasing the number
of stars by observing another cluster or improving the detection probability by
increasing the length of observations on a single cluster. As shown in Section 5.2.3,
the upper limit scales linearly with the sample size, thus keeping everything else
constant, increasing the sample size by a factor of 2 improves the upper limit by a
factor of 2.

Using the theoretical detection probability framework, I can quantify the
improvement in sensitivity for a survey twice as long. I assume a survey twice as
long consists of an observing window identical to the current survey for the first
half and repeats the observing window of the current for the latter half. The upper
limit improves only by a factor of 1.3 for a logarithmic distribution of VHJ planets.
However, the upper limits for HJs with 3.0 and 9.0 day orbital periods decrease by a factor of 2.6. Thus, not only is it more efficient to observe this cluster twice as long, but the analysis of Gaudi et al. (2005) reveals a 5-10 times larger HJ population than the VHJ population. This strongly suggests transit surveys with a single observing site require month long runs for maximum efficiency in detecting HJ companions.

Figure 6.1 reveals little improvement in the detection probability occurs for increasing the photometric precision, at least for 1.5 $R_J$ companions. To first order, the photometric precision determines the faint-end cutoff in the detection probability. Thus, a lower sky background or improved photometric precision predominately effects the number of stars in the survey rather than the detection probability. However, improving the photometric precision does lead to increasing the sensitivity for smaller radius companions. In the case of 1.0 $R_J$ companions, the rms in the light curve typically is $\lesssim 1.8$ times lower than the transit signal. As shown in the previous section, the theoretical detection probability breaks down for such low precision. In the case of 1.5 $R_J$ companions, the rms in the light curve typically is $\lesssim 4$ times lower than the transit signal. Thus, for the 1.0 $R_J$ results to reach the same sensitivity as the 1.5 $R_J$ results, improvement in the light curve rms is necessary until the transit S/N is above a critical threshold when the detection probability is weakly dependent on $R_p$ (Pepper & Gaudi 2005).
According to a recent review of radial velocity detected planets, $1.2 \pm 0.3\%$ of solar neighborhood stars have HJ companions (Marcy et al. 2005). This survey of NGC 1245 reached an upper limit of 52\% of the stars having $1.5 \ R_J$ HJ companions. As mentioned previously, a survey lasting twice as long can reduce this upper limit to 21\%. Reaching similar sensitivity as the radial velocity results requires observing additional clusters in order to increase the number of stars in the sample. This survey has $\sim 870$ cluster members and $\sim 740$ of them have nonzero detection probability for $1.5 \ R_J$ VHJ companions. Hence a total sample size of $\sim 7400$ dwarf stars observed for a month will be needed to help constrain the fraction of stars with planets to a 2\% level (comparable to radial velocity results). Assuming the validity of 1\% of stars having $1.5 \ R_J$ HJ extrasolar planets remains valid for a variety of stellar environments, I expect to detect one planet every 5000 dwarf stars observed for a month. Results for $1.0 \ R_J$ companions without substantial improvement in the photometric precision likely will require a small factor larger sample size.
Fig. 6.1.— *Top:* Probability for transit detection as a function of the apparent $I$-band magnitude assuming an even logarithmic distribution in semimajor axis from $1.0 < P < 3.0$ day and $P_{\text{mem}} = 1.0$ using the Monte Carlo calculation of this study (*small points*). Binned average of the Monte Carlo results (*open stars*). Expected probability for transit detection based on a theoretical calculation prior to this survey (*dash line*). Theoretical probability for transit detection assuming a photometric noise model appropriate for the survey (*dot dash line*). Theoretical probability for transit detection with an accurate photometric noise model for the survey and including the effects of limb darkening (*solid line*). *Left:* $1.5 R_J$ companion. *Right:* $1.0 R_J$ companion. *Bottom:* Theoretical probability for transit detection allowing each star of the survey to have its empirically determined photometric noise and including the effects of limb darkening (*small points*). Open stars reproduced from the top panels.
This study thoroughly examines the stellar and planetary content of the open cluster NGC 1245. The photometric data resolve some of the confusion in the literature regarding the correct photometric zeropoint for this cluster (Wee & Lee 1996; Subramaniam 2003). Based on isochrone fits employing the $Y^2$ calculations (Yi et al. 2001), I confirm the findings of Wee & Lee (1996) that this cluster has a slightly subsolar metallicity, $[Fe/H] = -0.05 \pm 0.03$ (statistical) $\pm 0.08$ (systematic). The best fit age is $1.04 \pm 0.02 \pm 0.09$ Gyr. In contrast to previous studies, I do not find evidence for significant differential reddening. I find a $V$-band extinction of $A_V = 0.68 \pm 0.02 \pm 0.09$, and with the aid of 2MASS $K_s$ photometry, I constrain the ratio of absolute to selective extinction to be $R_V = 3.2 \pm 0.2$. The resulting absolute distance modulus is $(m - M)_0 = 12.27 \pm 0.02 \pm 0.12$.

With the large field of view provided by the MDM 8K Mosaic imager, I confirm the finding of Nilakshi et al. (2002) that this cluster is highly relaxed. A majority of the low mass cluster members reside in an extended halo. The mass function slope, $\alpha = -3.12 \pm 0.27$, down to $M = 0.85M_\odot$ is steeper than the
Salpeter value of $\alpha = -1.35$, found by earlier studies. The previous studies did not have a sufficient field of view to detect the low mass cluster members that preferentially reside in the outer periphery of the cluster. Based on the observed stellar surface-density profile and an extrapolated mass function, I derive a total cluster mass, $M = 1300 \pm 90 \pm 170M_\odot$.

With the knowledge of the stellar properties for NGC 1245, I examine a 19-night search for transiting extrasolar planets orbiting members of the cluster. An automated transit search algorithm with quantitative selection criteria finds six transit candidates; none are bona fide planetary transits. Thus, this work also details the procedure for analyzing the null-result transit search in order to determine an upper limit on the fraction of stars in the cluster harboring close-in $R_J$ companions. In addition, I outline a new differential photometry technique that reduces the level of systematic errors in the light curve.

A reliable upper limit requires quantifiable transit selection criteria that do not rely on visual, qualitative judgments of the significance of a transit. Thus, I develop completely quantitative selection criteria that enable us to calculate the detection probability of the survey via Monte Carlo techniques. I inject realistic limb-darkened transits in the light curves and attempt their recovery. For each star I inject 100,000 transits at a variety of semimajor axes, orbital inclination angles, and transit phases, to fully map the detection probability for 2700 light curves consistent with cluster membership based on their position in the CMD. After characterizing
the field contamination, I conclude the sample contains ∼870 cluster members, and ∼740 cluster members have nonzero transit detection probability.

When calculating a 95% confidence upper limit on the fraction of stars with planets, I assume companions have an even logarithmic distribution in semimajor axis over several ranges of orbital period. I adopt the period ranges as outlined by Gaudi et al. (2005), for HJ and VHJ companions, and an as of yet undetected population with P < 1.0 day, which I denote as Extremely Hot Jupiters (EHJ). For NGC 1245, I limit the fraction of cluster members with 1.0 $R_J$ companions to <3.2% and <24% for EHJ and VHJ companions, respectively. I do not reach the sensitivity to place any meaningful constraints on 1.0 $R_J$ HJ companions. For 1.5 $R_J$ companions I limit the fraction of cluster members with companions to <1.5%, <6.4%, and <52% for EHJ, VHJ, and HJ companions, respectively.

I also fully characterize the errors associated with calculating the upper limit. I find the overall error budget separates into two equal contributions from error in the total number of single dwarf cluster members in the sample and the error in the detection probability. After correcting the detection probability for systematic overestimates that become increasingly important for detecting transits toward longer orbital periods (see Section 5.1.1), I conclude random and systematic errors in determining the number single dwarf stars in the sample dominate the error budget. Section 5.2.6 details the error analysis, and overall, I assign a $\sigma^{+13\%}_{-7\%}$ fractional error in the upper limits.
In Section 5.2.5 I provide a preliminary analysis of the light curves searching for large amplitude periodic variables. This reveals evidence for a higher population of close-in, P < 2.0 day, binaries in NGC 1245 than in the field star binary statistics of Halbwachs et al. (2004) and Eggenberger et al. (2004), but with only a 1-σ significance.

In planning future transit surveys, I determine observing NGC 1245 for twice as long will reduce the upper limits for the important HJ period range more efficiently than observing an additional cluster of similar richness as NGC 1245 for the same length of time as this data set. To reach a ~ 2% upper limit on the fraction of stars with 1.5 $R_J$ HJ companions (radial velocity surveys currently measure 1.3% Marcy et al. (2005)), I conclude a total sample size of ~ 7400 dwarf stars observed for a month will be needed. If 1% of stars have 1.5 $R_J$ HJ extrasolar planets, I expect to detect one planet every 5000 dwarf stars observed for a month. Results for 1.0 $R_J$ companions without substantial improvement in the photometric precision likely will require a small factor larger sample size.

The ability to detect extrasolar planets and place constraints on their existence is hampered in open clusters due to their modest number of members. The techniques developed in this study enable combining results from transit surveys in other clusters in a statistically meaningful way. Only by combining results from a variety of stellar environments can the transit technique reach its full potential.
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