DARK AND LUMINOUS MATTER IN GALAXIES AND LARGE SCALE STRUCTURE

DISSERTATION

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ABSTRACT

In standard cosmology, both the dark matter and baryons are important constituents of the universe. Although in the perspective of observation, dark matter and baryons are distinct, they are tightly correlated physically. This gives us a motivation to interpret the observations by considering them together and investigating their interactions. In this dissertation, I investigate the transverse proximity effect in Lyα transmitted flux, the baryon fraction and stellar mass-to-light ratio in early-type galaxies, and the relation between dark matter halos and optical observables of galaxy clusters.

The proximity effect is the observed reduction in absorption by HI in the Lyα forest in the proximity of QSOs. This effect was explained as the excess ionization from QSO and used to investigate the background QSO emission intensity. However, in some of the observations, there is only very weak or no proximity effect observed at all. This might arise from the QSOs residing in higher density regions. In this chapter, I investigate the effect of enhanced density close to QSOs with synthetic spectra from smoothed particle hydrodynamics (SPH) simulations at redshifts $z = 2$, $z = 3$, and $z = 4$. By modeling the halo mass-QSO luminosity
relationship, we compare the expected effect from the enhanced density and that from excess QSO photoionization. At all three redshifts, the transmitted flux close to low mass halos is smaller than the transmitted flux from randomly located pixels, while it is larger if the pixels are close to high mass halos. This means that we expect a proximity effect (higher than average flux) close to luminous QSOs in high mass halos, while for low luminosity QSOs in low mass halos the excess density is stronger and the average flux is below that at random locations. The influence of the QSO flux increases with redshifts, with more absorptions around low mass halos and much higher transmitted flux close to high mass halos.

To investigate the baryon fraction and the evolution of the stellar mass-to-light ratio in early-type galaxies, the joint gravitational lensing and stellar dynamical analysis of fifteen massive field early-type galaxies are used. The lenses are selected from the *Sloan Lens ACS* (SLACS) Survey – using *Hubble Space Telescope* ACS images and luminosity weighted stellar velocity dispersions obtained from the Sloan Digital Sky Survey database. The sample of lens galaxies is well-defined (see Treu et al. (2006)), with a redshift range of $z=0.06-0.33$ and an average stellar velocity dispersion of $\langle \sigma_{ap} \rangle = 263 \text{ km s}^{-1}$ (rms of 44 km s$^{-1}$) inside a 3-arcsec fiber diameter. The following numerical results are found: (i) A joint-likelihood gives an average logarithmic density slope for the total mass density of $\langle \gamma' \rangle = 2.01^{+0.02}_{-0.03}$ (68% C.L.; $\rho_{tot} \propto r^{-\gamma'}$) inside $\langle R_{\text{Einst}} \rangle = 4.2 \pm 0.4 \text{ kpc}$ (rms of 1.6 kpc). The inferred intrinsic rms spread in logarithmic density slopes is $\sigma_{\gamma'} = 0.12$, which might still
include some minor systematic uncertainties. A range for the stellar anisotropy parameter \( \beta = [-0.25, +0.25] \) results in \( \Delta \langle \gamma' \rangle = [+0.05, -0.09] \). Changing from a Hernquist to a Jaffe luminosity density profile increases \( \langle \gamma' \rangle \) by 0.05. (ii) The average position-angle difference between the light distribution and the total mass distribution is found to be \( \langle \Delta \theta \rangle = 0 \pm 3 \) degrees (rms of 10 degrees), setting an upper limit of \( \langle \gamma_{\text{ext}} \rangle \lesssim 0.035 \) on the average external shear. The total mass has an average ellipticity \( \langle q_{\text{SIE}} \rangle = 0.78 \pm 0.03 \) (rms of 0.12), which correlates extremely well with the stellar ellipticity, \( q_* \), resulting in \( \langle q_{\text{SIE}}/q_* \rangle = 0.99 \pm 0.03 \) (rms of 0.11) for \( \sigma \gtrsim 225 \) km s\(^{-1}\). At lower velocity dispersions, inclined S0 galaxies dominate, leading to a higher ratio (up to 1.6). This suggests that the dark-matter halo surrounding these galaxies is less flattened than their stellar component. Assuming an oblate mass distribution and random orientations, the distribution of ellipticities implies \( \langle q_3 \rangle \equiv \langle (c/a)_{\rho} \rangle = 0.66 \) with an error of \( \sim 0.2 \). (iii) The average projected dark-matter mass fraction is inferred to be \( \langle f_{\text{DM}} \rangle = 0.25 \pm 0.06 \) (rms of 0.22) inside \( \langle R_E \rangle \), using the stellar mass-to-light ratios derived from the Fundamental Plane as priors. (iv) Combined with results from the Lenses Structure & Dynamics (LSD) Survey at \( z \gtrsim 0.3 \), we find no significant evolution of the total density slope inside one effective radius for galaxies with \( \sigma_{ap} \geq 200 \) km s\(^{-1}\): a linear fit gives \( \alpha_{\gamma'} \equiv \frac{d\langle \gamma' \rangle}{dz} = 0.23 \pm 0.16 \) (1\( \sigma \)) for the range \( z=0.08–1.01 \). We conclude that massive early-type galaxies at \( z=0.06–0.33 \) on average have an isothermal logarithmic density slope inside half an effective radius, with an intrinsic spread of
at most 6% (1σ). The small scatter and absence of significant evolution in the inner density slopes suggest a collisional scenario where gas and dark matter strongly couple during galaxy formation, leading to a total mass distribution that rapidly converge to dynamical isothermality.

Based on the galaxies hosted by halos more massive than $10^{13.5} M_\odot$ from the Millennium Run Simulation (MRS), the relations between the halo mass and cluster optical observables are investigated, at redshifts $z = 0$, $z = 0.3$, and $z = 0.5$ are investigated. Two simulated galaxy catalogs are used, with one from the Durham university group, and the other from Max Planck Institution for Astrophysics (MPA) group. The relations between halo mass and cluster luminosity, or halo mass and galaxy richness (galaxy number) can be well fit by power-law mean relations with lognormal scatter. The scatter is around 0.12 dex (Durham) and 0.15 dex (MPA) at cluster luminosity $L_{\text{tot}} 10^{1.4} L_\star$. The scatter in the Durham simulation decreases with increasing cluster luminosity, while no obvious trend appears in the MPA simulation. The central galaxy luminosity is also correlated with halo mass, but with larger scatter. At fixed halo mass, there is little or no correlation of average galaxy luminosity or central galaxy luminosity with richness. The mean relations and scatter show little evolution between $z = 0.5$ and $z = 0$. 


Dedicated to my family
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Table of Contents

Abstract .......................................................... ii
Dedication ............................................................ vi
Acknowledgments ..................................................... vii
Vita .................................................................. x
List of Tables ......................................................... xiv
List of Figures ........................................................ xv

Chapter 1 Introduction .............................................. 1
  1.1 Lyα Forest ....................................................... 4
  1.2 Gravitational Lensing .......................................... 6
  1.3 Large Scale Clustering ......................................... 8

Chapter 2 The Transverse Proximity Effect: Impact of the High-
Density QSO Environment ....................................... 12
  2.1 Introduction ..................................................... 12
  2.2 Method ............................................................ 15
    2.2.1 Simulation ................................................... 15
    2.2.2 Information Contained in Lyα Spectra .................. 16
    2.2.3 An Example of the Spectra Extraction .................. 18
    2.2.4 Probability Distribution Function of Flux Near QSOs ... 20
  2.3 Result ............................................................ 21
2.3.1 The General Effect and Velocity Bin Width .................................. 21
2.3.2 The Gravitational Effect ................................................................. 23
2.3.3 The Photoionization Effect ................................................................. 24
2.3.4 General Result ................................................................................. 26

2.4 Discussion ........................................................................................... 28
  2.4.1 How to Use the Calculation ............................................................... 28
  2.4.2 Compare with Data ........................................................................... 28

Chapter 3 The Baryon Fractions and Mass-to-Light Ratios of Early-Type Galaxies ................................................................. 40
  3.1 Introduction ......................................................................................... 40
  3.2 Data and Method ................................................................................ 42
    3.2.1 Data .............................................................................................. 42
    3.2.2 Method of Analysis ....................................................................... 43
  3.3 Results ................................................................................................ 51
    3.3.1 Properties of Individual Galaxies .................................................... 52
    3.3.2 Homogeneity .................................................................................. 53
    3.3.3 The Stellar Mass Fraction and Mass-to-Light Ratio ....................... 56
  3.4 Discussion ........................................................................................... 58

Chapter 4 Optical Cluster Observables as indicators of Halo Mass ................................................................. 79
  4.1 Introduction ......................................................................................... 79
  4.2 Galaxy and halo catalogs ..................................................................... 83
  4.3 The Relations between Optical Observables and Halo Mass ............... 87
    4.3.1 Galaxy Richness, Cluster Luminosity and Halo Mass ..................... 87
    4.3.2 Scatter in Mass-L and Mass-N relations ....................................... 89
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.3.3 Distributions of the Observables</td>
<td>93</td>
</tr>
<tr>
<td>4.3.4 Galaxy Luminosity and Halo Mass-to-light Ratio</td>
<td>95</td>
</tr>
<tr>
<td>4.4 Redshift and Magnitude Cut-off Influence</td>
<td>99</td>
</tr>
<tr>
<td>4.5 Conclusion</td>
<td>101</td>
</tr>
</tbody>
</table>

**Bibliography** ........................................................................................................ 117
List of Tables

2.1 Halo Mass, QSO Luminosity and Excess Ionization .......................... 38
3.1 Lens property data I ................................................................. 75
3.2 Lens property data II ................................................................. 77
List of Figures

2.1 Example of One Line of Sight Pixel Temperature, Density, and Spectra (1 of 7) ................................................................. 31

2.2 Transmitted flux computed from different velocity bin (2 of 7) .... 32

2.3 Transmitted flux in randomly located pixels (3 of 7) ............... 33

2.4 Transmitted flux in in close-to-halo pixels at z=2 (4 of 7) .......... 34

2.5 Transmitted flux in close-to-halo pixels at z=3 (5 of 7) ............. 35

2.6 Transmitted flux in close-to-halo pixels at z=4 (6 of 7) ............. 36

2.7 Estimated transmitted flux ratio at different redshifts (7 of 7) .... 37

3.1 Mass Fraction and Concentration Contour (1 of 10) .............. 65

3.2 Mass Fraction and Concentration Contour (2 of 10) .............. 66

3.3 Stellar M/L and Concentration Contour (3 of 10) ............... 67

3.4 Stellar M/L and Concentration Contour (4 of 10) ............... 68

3.5 Errors in Luminosity and Velocity Dispersion (5 of 10) ......... 69

3.6 Likelihood of Homogeneity (6 of 10) ................................. 70

3.7 3D Density Slope (7 of 10) ............................................. 71

3.8 Probability Distribution of Stellar Mass Fraction (8 of 10) ...... 72

3.9 Probability Distribution of the Density Slope (9 of 10) ........... 73

3.10 Stellar M/L Ratio and Evolution (10 of 10) ......................... 74

4.1 Halo mass function (1 of 14) ........................................... 103

4.2 Color-Magnitude Graph (2 of 14) ...................................... 104
4.3 Observables and Halo Mass Fit (3 of 14) ......................... 105
4.4 Halo Mass Scatter (4 of 14) ...................................... 106
4.5 Observable Scatter at Given Halo Mass (5 of 14) ............ 107
4.6 Cluster Luminosity Distribution (6 of 14) ...................... 108
4.7 Galaxy Richness Distribution (7 of 14) ......................... 109
4.8 Halo Mass Bias by Matching Luminosity and Halo Mass(8 of 14) . 110
4.9 Halo Mass Bias by Matching Galaxy Richness and Halo Mass (9 of 14) 111
4.10 Average Galaxy Luminosity (10 of 14) ....................... 112
4.11 Central and Satellite Galaxies (11 of 14) ..................... 113
4.12 Central Galaxy Luminosity (12 of 14) ......................... 114
4.13 OM Relation Evolution Durham (13 of 14) .................. 115
4.14 OM Relation Evolution MPA (14 of 14) ..................... 116
Understanding large scale observations is one of the major tasks in cosmology. Standard cosmology (SC) is well-developed and powerful for this purpose, which is based on two basic assumptions: (a) The large-structure of the universe is essentially determined by gravitational interactions and hence can be described by Einstein’s theory of gravity. (b) The distribution of the matter in the universe is homogeneous and isotropic at sufficiently large scale. As pointed out in Narlikar & Padmanabhan (2001), the first assumption requires that the geometry or the matter distribution in the phase space (including both the common coordinate space and the time) of the universe could be determined by Einstein’s field equation and the metric of the universe could be indicated in the form of separation in time and space. Then, combined with the equation-of-state, the evolution of the universe could be described. If the basic assumptions hold, to fully understand the question, certain boundary conditions, i.e., the cosmology parameters are needed. In total, there are seven cosmology parameters: Hubble constant $H_0$, baryon parameter $\Omega_b$, dark matter parameter $\Omega_{dm}$, radiation parameter $\Omega_R$, cosmological constant $\Omega_{\Lambda}$, initial fluctuation amplitude $A$, and power $n$. The first five parameters describe the
content of the universe and the last two represent the initial conditions of the mass fluctuation. Combined with the equation-of-state, the state of the universe at any given time or redshift could be determined.

Different branches of physics and astronomy enter at different revolutionary stages of the universe. The equation-of-state of the particles in the early universe should be determined by particle physicists (especially in GeV Energy). When the universe cools down to (few) MeV, the big bang nucleosynthesis (BBN) takes place and leaves some footprints on the abundances of the helium, deuterium, and lithium in the early universe. When the temperature of the universe approaches $10^3$K, the photons decouple from matter and become the cosmic microwave background (CMB) in the current universe. After that, the stars, galaxies, galaxy clusters and even larger scale structures are formed, which give the astronomers more facets to study the universe. Then, different observational tools are created by different objects and phenomena to offer good constraints on the cosmological parameters.

The high redshift type Ia supernova observations imply the expansion of the universe (e.g. Riess et al. (1998); Perlmutter et al. (1999); Riess et al. (2004)). The light emitted by Quasars formed at high redshifts will be intercepted by the ubiquitous hydrogen, with the main features as the Ly$\alpha$ absorptions (e.g. Gunn & Peterson (1967)). Therefore, the Ly$\alpha$ forest is a naturally powerful test on the Baryonic content of the universe, since hydrogen is about 75% of all the baryons. The light from higher redshift sources will be deflected by the foreground masses, specifically,
the foreground galaxies and clusters. Then the distorted images of the background objects inform us about the foreground mass which serves as a lens (e.g. Blandford & Narayan (1992)). According to the strength of the distortion, the effects could be illustrated as strong gravitational lensing (e.g. Narayan et al. (1988); Cen & Ostriker (1994)) and weak lensing (e.g. Kaiser & Squires (1993); Bartelmann et al. (1996)). On the other hand, the observations of the large scale structures such as the abundance of the galaxy clusters could give us the information about the value of $\Omega_M$ (which includes both $\Omega_B$ and $\Omega_{DM}$) and $\sigma_8$ (the rms relative mass fluctuation within $8h^{-1}\text{Mpc}$ top-hat radius in early universe (e.g. Fan et al. (1997); Bahcall et al. (2003))). Besides all the methods above, the CDM or $\Lambda$CDM simulation based on the SC bridges the theory and observations in cosmology.

As mentioned above, the content of the universe determines the formation and evolution of the cosmological structures. In particular, I am most interested in the baryon and dark matter content in different structures, such as in the intergalactic medium, in the early type galaxies and in large scale clusters. In this dissertation, I focus on several specific questions: transverse proximity effect in Ly$\alpha$ absorption, baryon fraction and stellar mass-to-light ratio evolution in early-type galaxies from
lensing data, and the relations between dark matter halo mass and the cluster optical observables from MRS simulation.

1.1. Ly\(\alpha\) Forest

The light emitted in high redshift sources travels a long distance into our detector and the light will be redshifted as it propagates. Since hydrogen is ubiquitous in the universe, the intervening of hydrogen atoms on the light is determined both by the status and distribution of the hydrogen and its properties.

The transition between the ground and first excited state of hydrogen, the Ly\(\alpha\) transition (1215.67\(A\)), is resonant. Therefore when a photon of appropriate energy encounter a neutral hydrogen, it will invariably be absorbed. If the high redshift source is a quasar, the blueward spectrum at different frequency will be redshifted to the hydrogen resonant transition frequency at different redshifts. Hence, absorption features corresponding to different redshifts will show up in the quasar continuum spectra corresponding to different redshifts. The absorption is strong, no photons will arrive at the earth if all the hydrogen atoms are neutral. While the observation of high redshift quasar suggests that the hydrogen is highly photoionized or the universe between us and the high redshift quasar is empty. It turns out that the high redshift (e.g. z=3) universe or more specifically, the intergalactic medium (IGM) is highly photoionized by quasars and galaxies (especially Lyman break galaxies), with \(n_{HI}/n_H \leq 10^{-4}\) and temperature around \(3 \times 10^4 K\) (Sagent 1980).
If the hydrogen in the IGM is uniformly distributed, the quasar continuum spectra will be absorbed evenly at each wavelength, and there will be no high density troughs turning out. This dilemma could be solved by the cold dark matter (CDM) simulation naturally. In the simulation, the majority of mass is in the form of dark matter, which only interact via gravity. Small fluctuations in the mass field in early universe continue to grow and collapses under their own self-gravity. These collapsed objects are termed as “halos” and span a large range of mass scales. Here, the small mass-scale objects, dubbed “mini halos”, are not massive enough to host galaxies, but can retain hydrogen. The retained hydrogen clouds have larger rate of incidence \((dN/dZ)\) and higher velocity dispersion than galaxies because of the small mass. Therefore, the retained hydrogen is the most common absorption in the quasar spectra. A further breakthrough is achieved by the SPH simulations where both the dark matter and gas particles are simulated simultaneously. The small fluctuation of the hydrogen naturally generates the detailed Ly\(\alpha\) spectra known as Ly\(\alpha\) forest. This continuous description of the Ly\(\alpha\) forest is known as the Fluctuating Gunn-Peterson Approximation. In chapter 2, I will use the simulated Ly\(\alpha\) spectra to analyze the transverse proximity effect in quasar pairs.

The proximity effect is the reduction of the absorption when light passes close to the quasar. If the density profile nearby QSOs does not change too much compared with the general IGM, this effect can be used to detect the strength of UV background, since the effect is constrained both by the quasar luminosity and
the UV background intensity. When quasar pairs are observed, the decrement of the absorption in the spectra of the background quasar passing by the foreground quasar is called “transverse proximity effect”. While, in observation, no obvious transverse proximity effect is observed. The reason could be that the density profile around the foreground quasar is different from that in the general IGM, or the anisotropy or the time variation of the QSO luminosity. It means that the distribution of the baryons are not even. SPH simulations can naturally generate the distribution of the gas material in different locations. Therefore, I use the SPH simulated quasar lines of sight to analyze the transverse proximity effect statistically. I thus determine whether the changes (typically enhancement) of the hydrogen density could explain the scarcity of the transverse proximity effect in the quasar pairs.

1.2. Gravitational Lensing

Gravitational Lensing arises from the deflection of the light originating by masses between the source and the observer. The deflection angle of the light is influenced by the mass all the way back to the source. While, since the mass distribution in large scale (e.g. $10h^{-1}\text{Mpc}$ scale) is assumed homogeneous, typically only the objects with high density, which could be treated as a point mass or mass stacked in a thin plane, influence the deflection. Thus, gravitational lensing is a powerful and unique to detect the underlying mass of the lenses since the other
observations can only be related to the light directly. The lenses could be stars, black holes, galaxies or even galaxy clusters. There are different phenomena of lensing, for example, multiple images stretches of the image, time delay of the source light variation, all of which are the results of light deflection. Here, I mainly used the lens mass estimated from the multiple images.

As reviewed by Blandford & Narayan (1992), if the lenses are point like objects, such as stars and black holes, the point mass approximation of the lenses could be applied. Images of the lensing system are determined by the source, the mass distribution of the lens, and especially the relative position between the source and lens. A source on the optic axis forms an Einstein ring with the angular radius

\[ \Theta_E = \left( \frac{4GM}{c^2 D} \right)^{1/2} \]

where \( D = D_d \ast \frac{D_s}{D_{ds}} \), with \( M \) the lens mass, \( D_{ds} \) the distance between the source and lens, \( D_d \) the lens and observer distance, and \( D_s \) the source and observer distance. In the source of the optic axis case, two images are produced with the magnified one outside the Einstein ring and diminished one inside the Einstein ring, and they are in the opposite direction of the lens. Even when the lenses are extended objects like galaxies, if the image angular separation is larger than the extension of the lens, the point estimate method can still be used to estimate the mass “enclosed” by the images. If the length scale of the lens is much larger than the image, the influence of the lens on the source can be divided into two components, with convergence \( \kappa \) measuring the isotropic part of the magnification and shear \( \gamma \) (caused by matter lying outside the light beam) measuring
the anisotropic stretching of the images. When the image separation is comparable to the size of the lens, the density profile is needed to analyze the lens. As pointed out by some authors (e.g. Kochanek et al. (2000); Rusin et al. (2003); Treu et al. (2006)), the isothermal sphere could be a good approximation to the density profile of the lens, and the deflection angle is given by $\alpha = 4\pi \sigma^2 / c^2$, where, $\sigma$ is the 1D velocity dispersion.

Despite the power in determining the underlying mass, the lensing phenomenon is rare, affecting only about a percent of distant sources. The number of lenses observed is increasing rapidly, with the lens surveys (e.g. CfA-Arizona Space Telescope LEns Survey (CASTLES), Sloan Lenses ACS Survey (SLACS), and Lens Structure and Dynamics Survey (LSD)). In this dissertation, I used the results of lens mass estimation based on multiple images, and combine them with the measured velocity dispersion to determine the baryon fraction and stellar mass-to-light ratio in the early type lens galaxies. Although, the lens alone can only detect the dark and luminous mass together, combined with the velocity dispersion data, it is powerful enough to break the degeneracy.

1.3. LARGE SCALE CLUSTERING

Galaxies map out the luminous mass in the universe, and the distribution of galaxies at different redshifts gives us a hint on the evolution of clustering of the
universe. Hence, galaxy surveys are good tools for astronomers to study the luminous matter clustering in the sky, which has big improvement with the development of the Two Degree Field Galaxy Redshift Survey (2dFGRS; e.g. Colless et al. (2001); Hawkins et al. (2003)) and the Sloan Digital Sky Survey (SDSS; e.g. York et al. (2000); Stoughton et al. (2002); Adelman-McCarthy (2006)). However, when we look closer to the velocity dispersion of gas in galaxies or that of galaxies in clusters, the luminous matter is not massive enough to hold the galaxies or cluster together. Therefore, only small fraction of total mass is luminous in the universe, and most of the mass is invisible called dark matter.

In the early age of the universe, the baryons and dark matter are evenly distributed in the universe, until the random fluctuation breaks down the homogeneity. The tiny peaks is the mass fluctuations expand slower than the overall universe. In some point, the mass around the peaks overcomes the expansion of the universe and begins to collapse, dragging more mass into the gravitational potential well, with the average density inside the boundary about 200 times the background density of the universe (Gunn & Gott 1972). In this way, dark matter halos are formed, and meanwhile, the luminous matter or the baryons are also pulled into the potential well. With the increment of the baryon temperature, the energy inside the baryons is dissipated. In consequence, the luminous matter condenses more into the gravitational potential and influences the distribution of the central dark matter in some degree.
Dark matter halos are continuous and less concentrated than the galaxies. Therefore, halos and galaxies are not perfectly matched to each other and the bias has been intensively studied by halo occupation distribution (HOD) method (e.g. Jing et al. (1998); Seljak (2000); Ma & Fry (2000); Peacock & Smith (2000); Scoccimarro et al. (2001); Berlind & Weinberg (2002)). But, they are highly correlated to each other and the lognormal relationship between the cluster luminosity (or the galaxy richness inside clusters) and dark matter halo mass are well established. Hence, galaxies provide a good tool to understand the underlying dark matter. However, the scatter of halo mass around certain observables is difficult to determine, which gives the uncertainties in the halo mass and therefore the cosmology parameter estimation if the clustering method is used.

Since, in most circumstances, the halo masses are unknown, simulation is the usual way to get the halo mass. By matching the most massive dark matter halos from simulation with the most luminous observed clusters, the halo mass and the observables relation could be established. However, because of the scatter of both the observables and halo masses, there is inevitable bias in the relation, and the scatter could not be recovered by this method. In this sense, it is difficult to obtain the scatter of halo mass at the given observables. Therefore, it is necessary for us to find alternative ways to obtain the scatter of the halo mass around the given observables. In such a way, we can find the observable as an indicator of the halo mass, giving the minimum scatter of halo mass around this observable.
Observationally, the halo masses can be obtained by x-ray luminosity and weak lensing measurement. Combining the cluster luminosity and richness with the observed cluster mass, the scatter of the halo mass in the given observables could be achieved (e.g. Stanek et al. (2007); Reyes et al. (2008)). The observables and the halo mass can be connected directly by simulation as well. However, the complex physical mechanisms in the galaxy formation process, such as star formation, supernova and AGN feedback inside the clusters, prevent the accurate description of the galaxy formation in the simulations. The relatively less computational expensive method of semi-analytic model makes tentative simulations compared with the observations possible. Therefore, we still can get useful results on the scatter. Especially, the high resolution, 10 billion particle Millennium Run simulation (MRS) carried out by VIRGO consortium is available now (Springel et al. 2005; Gao et al. 2005; Croton et al. 2006; Springel 2005). Based on this platform, different semi-analytic galaxy formation models were rigorously explored. It is also possible for us to investigate the relation and scatter between the observables and halo mass.
Chapter 2

The Transverse Proximity Effect: Impact of the High-Density QSO Environment

2.1. Introduction

The proximity effect in the mean Ly$\alpha$ absorption caused by the intervening intergalactic medium (known as Ly$\alpha$ forest), measured in the spectra of QSOs (Carswell et al. 1982), is the reduction in Ly$\alpha$ absorption as the redshift of the QSO is approached. Normally, the degree of absorption increases with redshift owing to the increasing density of the intergalactic gas with redshift, but near the QSO the intensity of the ionizing flux increases because the QSO flux is added to that of the general background. This excess ionization rate from the QSO reduces the neutral fraction in the intervening gas (Murdoch et al. (1986)). The strength of the proximity effect measured in a quasar of known luminosity has been used to determine the intensity of the ionizing background at high redshift (e.g., Bajtlik et al. (1988); Scott et al. (2000)). This measurement is, however, subject to several systematic errors related to determining the QSO redshift, magnification bias due to gravitational lensing, and QSO variability over the photoionization timescale.
of $\sim 10^5$ years. In addition, the cosmological environment of the QSO where the proximity effect is seen has a higher than average density, which can increase the amount of absorption and compensate in part for the effect of the higher ionizing flux.

The transverse proximity effect denotes the same systematic reduction in the $\text{Ly}\alpha$ absorption when observed in the spectrum of a background QSO at redshift $z_b$ which is affected by the ionizing radiation of a foreground QSO at a small angular separation, and at redshift $z_f < z_b$. However, no reduction in $\text{Ly}\alpha$ absorption has actually been observed in most close pairs of QSOs where the foreground QSO flux is estimated to be substantially larger than the background intensity, implying that the effect of the enhanced ionization rate from the foreground QSO ought to have been apparent (see Crotts (1989); Møller & Kjærgaard (1992); Fernández-Soto et al. (1995); Liske & Williger (2001); Schirber et al. (2004)). A clear void in $\text{Ly}\alpha$ absorption associated with the presence of a foreground QSO was found only in the case of Q0302-003 (Dobrzycki & Bechtold (1991)). Croft (2004) searched for the transverse proximity effect at larger angular separations by averaging over many QSO pairs from the Sloan Digital Sky Survey (hereafter SDSS), and again found no detectable effect. The absence of the transverse proximity effect can be explained by either a greater density of matter near the QSO (which may compensate for or even overwhelm the ionizing flux effect), or a reduced flux from the foreground QSO in the line of sight to the background QSO due to anisotropic emission or
strong variability of the luminosity over a timescale of Myr (the typical time-delay associated with the separation between QSO pairs).

The objective of this chapter is to estimate the plausible modification of the transverse proximity effect by the high gas density expected in the QSO environment. The closest QSO pairs where this effect can be measured are typically separated by several arc minutes, corresponding to distances of several comoving Mpc. This distance is usually larger than the virial radii of even the most massive halos that could be likely QSO hosts at typical redshifts $z \sim 2$. The enhancement of the density is therefore not very large. The principal uncertainty in estimating the density distribution near the foreground QSO is the mass of the QSO host halo: the larger the halo mass, the higher the expected density at a fixed distance. We shall use cosmological simulations to select halos of different masses as candidate QSO hosts, and examine the Ly$\alpha$ absorption profiles that are predicted on lines of sight over a range of impact parameters. This should allow an estimate of the maximum enhancement of Ly$\alpha$ absorption that could be produced by the gas over density near QSOs. We also can use this effect to determine whether the lack of an observed transverse proximity effect in most QSO pairs can be explained by this over density.

In §3.2, we describe the method used in this chapter, then the result from the simulation and model of QSO luminosity in §3.3; and in §2.4, we pointed out a possible problem for our QSO luminosity model.
2.2. Method

There might be several reasons responsible for the proximity effect around QSOs: excess photoionization from QSOs, the galactic winds, material infalling, the variability of QSOs, etc. In this chapter, we explore the competitive effect to the Lyα forest from excess photoionization of the QSOs and from the enhanced environment density around halos where the QSOs live. Hence, in this chapter, we compare the transmitted flux close to halos with that from randomly located pixels, and also the flux with and without excess photoionization.

2.2.1. Simulation

In this calculation, the data are based on a smoothed particle hydrodynamics (SPH) simulation with a periodic cube of comoving size $h^{-1}50\text{Mpc}$ at redshifts $z=2$, $z=3$, and $z=4$. The simulation uses a parallel implementation of TreeSPH (Davé et al. 1997; Hernquist, & Katz 1989), incorporating radiative cooling and star formation as described in Katz et al. (1996) (hereafter KWH). There are $288^3$ dark matter particles and $288^3$ gas particles in the simulation. It assumes a $\Lambda CDM$ model. In the simulation, $\Omega_m = 0.25$, $\Omega_\Lambda = 0.75$, $\Omega_b = 0.044$, $h = 0.7$, the inflationary spectral index $n = 0.95$, and the power spectrum normalization $\sigma_8 = 0.80$. The resolution of the simulation is high enough to include the background photoionization, heating and cooling, therefore the temperature and ionization of the pixels can be more accurate.
than previous values obtained by the approximation described in Katz et al. (1996). There are 450 halos from the simulation at each of the redshifts. It is obvious from the simulation that halos grow with redshifts. The largest halo mass increases from $6.92 \times 10^{12} M_\odot$ in $z=4$, to $1.3 \times 10^{13} M_\odot$ in $z=3$, and to $6.92 \times 10^{13} M_\odot$ till redshift 2. In all of the three redshifts, the low mass halos are more common than the massive ones. Around each halo, fifteen lines of sight are extracted randomly from x, y and z directions respectively with five angular separations evenly distributed from 0.2' to 4.0' in logarithmic space. Each of the lines of sight are divided into 4000 cells in the simulation. Density, temperature and peculiar velocity information are given in each cell. The sizes of each cell are 1.14 km s$^{-1}$, 1.28 km s$^{-1}$, and 1.41 km s$^{-1}$ at redshifts 2, 3, and 4 respectively, which are much smaller than the typical thermal broadening velocity $\simeq 10$ km s$^{-1}$. Therefore, the resolutions of the cell are high enough to resolve the thermal broadening of the Ly$\alpha$ forest.

### 2.2.2. Information Contained in Ly$\alpha$ Spectra

Typically, the Ly$\alpha$ forest system is fitted by the Voigt profile which includes both Doppler broadening and intrinsic broadening of the line profile. However, the effect of intrinsic broadening is not so obvious in low density areas. Therefore, we use Gaussian profiles to fit the simulated spectra and assume that the hydrogen abundance is 0.76. The density, temperature and peculiar velocity in all the cells
along each line of sight, which are needed for the spectra extraction, can be obtained from the simulation result.

According to Press et al. (1993), the mean optical depth evolves with redshift in the form of \( \bar{\tau}(z) = A(1 + z)^{\gamma+1} \), with \( \gamma = 2.46 \pm 0.37 \) and \( A = 0.0175 - 0.0056\gamma \pm 0.0002 \). The mean optical depth gives the mean fluxes 0.38, 0.64, and 0.85 at redshifts 2, 3, and 4 respectively. The mean transmitted flux is very similar to the result from Bernardi et al. (2003), if the bump of the transmitted flux at the redshift 2.9 – 3.1 is neglected and only a smoothed power-law fit is considered. The transmitted flux of the background QSOs is determined by the neutral hydrogen density, which is related to both the total hydrogen density and the photoionization rate. Compared with the sight lines that are far away from foreground QSOs, those passing by the foreground QSOs travel through regions of the higher density where the neutral hydrogen also experiences excess photoionization from QSO photons. The effect of the higher density can be seen directly from the simulation result, if the absorption around these halos is stronger. How the competitive effects interact is not so straightforward to determine from the simulation. If we fix the UV background luminosity, the proximity effect is proportional to the luminosity of the foreground QSO and the distance between the observed point and the QSO. In order to calculate the proximity effect from the foreground QSO, for each line of sight, we define \( \omega \) as the ratio of the foreground QSO ionization rate to the diffuse background ionization rate (Schirber et al. 2004). From the simulation result, we do know the mass of
each halo, but whether a given halo hosts a QSO and what is the halo mass-QSO luminosity relation are subject to considerable uncertainty. Hence, it is good for us to loose the photoionization rate in some larger range, say $\omega \sim 1 - 1000$, then narrow it down by some reasonable assumptions. Then the proximity effect will change with angular separation between the line of sight and halo, the relative redshifts of pixels and halos, the mass of the halos and the value of $\omega$. To narrow down the influence of the redshift deviation of pixels from QSOs, along each sight line, we only choose pixels that are close to the halos in redshift space to get the average transmitted flux.

2.2.3. An Example of the Spectra Extraction

Before averaging the transmitted flux around halos, we extract a spectra along a random line of sight to see the properties of neutral hydrogen density (NHD), temperature and spectra of the simulations as shown in Figure 2.1 (Here only a z=3 spectrum is shown as an example). The NHD is calculated from the total density in real space by assuming photoionization equilibrium. Although collisional ionization is unimportant for typical temperatures and densities, it is nonetheless considered according to the Katz et al. (1996) prescription, to avoid the deviation in the high density situation. The randomly chosen line of sight is close to a $4.53 \times 10^{11} M_\odot$ mass halo, with an angular separation 3.5'. The center of the shaded rectangle is where the halo lies and the width of the rectangle corresponds to 400km s$^{-1}$. The
temperature is obtained from the simulation directly, and the spectrum is displayed in the smoothed redshift space along the line of sight. If the extra heating from the foreground photoionization is neglected, the temperature does not change when the foreground QSO flux is considered (Figure 2.1). The three lines in the NHDs and spectrum are for different QSO-flux to background-flux ratios: the black line is without foreground QSO flux, the green line is with $1 + \omega = 10$, and the red line is for $1 + \omega = 100$. It is obvious that the NHD, temperature and transmitted flux are tightly correlated. The higher the NHD and the temperature are, the lower the transmitted flux is. Pixels around the dark matter halo have higher NHD and temperature. Hence if the QSOs trace the dark matter halos well, the QSOs will also reside in high density environment. Consequently, the sight lines passing by the foreground QSOs go through higher neutral hydrogen density regions and have more flux absorbed, compared with sight lines that are randomly located. Meanwhile, the flux from the QSO makes more hydrogen ionized and thus allows more background QSO flux to be transmitted. Then the absorption and photoionization effects compete with each other when sight lines to background QSOs pass close to the foreground QSOs. In this case, the more or less absorption effect upon the transmitted flux is uncertain.
2.2.4. Probability Distribution Function of Flux Near QSOs

The expected transmitted flux close to a QSO is related to the mass of its halo, the angular separation between the line of sight and the halo, and the excess photoionization from the QSO. Therefore, in our calculations, we extracted synthetic spectra by considering the following factors. First of all, according to the simulated halo mass ranges, we divide the halos into four groups at each redshifts. At redshift 2, the mass ranges are $7.62 \times 10^{11} - 1.96 \times 10^{12} M_\odot$, $1.96 \times 10^{12} - 5.04 \times 10^{12} M_\odot$, $5.04 \times 10^{12} - 1.30 \times 10^{13} M_\odot$ and $1.30 \times 10^{13} - 3.33 \times 10^{13} M_\odot$. The redshift 3 halos are divided into the following mass bins: $4.53 \times 10^{11} - 1.04 \times 10^{12} M_\odot$, $1.04 \times 10^{12} - 2.42 \times 10^{12} M_\odot$, $2.42 \times 10^{12} - 5.61 \times 10^{12} M_\odot$ and $5.61 \times 10^{12} - 1.30 \times 10^{13} M_\odot$. Similarly, the redshift 4 halos are binned in the mass ranges $2.32 \times 10^{11} - 5.43 \times 10^{11} M_\odot$, $5.43 \times 10^{11} - 1.27 \times 10^{12} M_\odot$, $1.27 \times 10^{12} - 2.96 \times 10^{12} M_\odot$ and $2.96 \times 10^{12} - 6.92 \times 10^{12} M_\odot$. Based on the above mass bins, we fix the angular separation $\theta$ into different ranges, which are $0.20 - 0.42\text{arcmin}$, $0.42 - 0.89\text{arcmin}$, $0.89 - 1.89\text{arcmin}$, and $1.89 - 4.00\text{arcmin}$. Then, the NHD is calculated for each of the $1 + \omega$ from 1 to 1000. We assume that each cell in the simulation with neutral hydrogen will absorb with a Gaussian distribution in velocity and thus absorption at the centroid velocity of a cell will include contributions from neighboring cells as well. Next, for each of the mass and angular separation ranges, we smoothed the spectra in redshift space.
with velocity bins of 50km s$^{-1}$, 100km s$^{-1}$, 200km s$^{-1}$, and 400km s$^{-1}$. We then take the smoothed spectra (divided up into groups as given above) at the same $1 + \omega$ and determine the fraction (5, 20, 50 and 80%) that have a given quantity of transmission or less (Figure 2.4 etc.).

2.3. Result

2.3.1. The General Effect and Velocity Bin Width

In Figure 3, we plot the probability distribution function (PDF) of the randomly located pixels with $1 + \omega$ ranging from 1 to 1000, with the panels corresponding to $z=2$, $z=3$, and $z=4$ from top to bottom. The triangles from bottom to top are the 5%, 20%, 50%, and 80% percentiles of the transmitted flux from random pixels without excess photoionization ($\omega = 0$). Figure 4 shows the PDF of the mean transmitted flux in pixels around halos within the angular separation fixed at the ranges from $0.20 - 0.42$arcmin to $1.89 - 4.00$arcmin and mass ranges in $7.62 \times 10^{11} M_\odot - 1.96 \times 10^{12} M_\odot$ and $1.30 \times 10^{12} M_\odot - 3.33 \times 10^{13} M_\odot$ at reshift $z=2$. The solid line traces the points where 5% of all sight lines have a given flux or lower at particular values of $\omega$. The dotted, dashed and long-dashed lines corresponds to 20%, 50% and 80% respectively. Similar plots show the percentiles of the transmitted flux in the above given mass ranges and within the assigned angular separation at redshifts $z=2$, and $z=4$ in Figure 5 and Figure 6.
The fluctuation of the density and temperature in redshift space causes noise in the spectra along each line of sight. In order to smooth the spectra around the halos, a certain velocity bin width is needed. We tested several velocity bin widths around the halos: 50km s$^{-1}$, 100km s$^{-1}$, 200km s$^{-1}$, and 400km s$^{-1}$. The influence of the velocity bin widths on the PDF is related to the angular separation, the mass of the halo etc. At the same time, it is also different for different percentile lines. Typically, the larger the probability to which the line corresponds, the less obvious the PDF changing with the velocity bin widths. This results from smaller fluctuations from counting more pixels. The transmitted flux increases with the velocity bin width, which indicates the decrement of the NHD with the increasing distance between the halo and cells in redshift space.

From Figure 2.2, it is clear that the dispersion is larger for smaller velocity bin width. Hence, the larger velocity bin width smooths the spectra more and makes the flux closer to median. The small velocity bin widths leave more room for noise, which can also be seen from Figure 2.2. The skew of the lines with small velocity bin width indicates that the velocity bin width is not large enough to cancel the effect from peculiar velocity. However, for large velocity bin width, the signal of high absorption around the halo is weakened because it includes too many pixels with large distances from the halo. For this reason, in the following discussion, we choose 200km s$^{-1}$ as our velocity bin width.
2.3.2. The Gravitational Effect

Figures 4 to 6 show the spectra with $1 + \omega$ from 1 to 1000 at the assigned mass ranges and angular separation bins at redshifts 2, 3, and 4. If we focus on the $1 + \omega = 1$ point for each of the PDF lines (without QSO excess photoionization), we can see the net influence of the overdensity around the QSO by comparing the random transmitted flux with that close to halos. In general, the mean flux of randomly located pixels is higher than that in the pixels close to halos. However there is an exception for the 5% probability case, and even 20% probability case in redshift 4. This might arise from the methods that we calculate the transmitted flux in the randomly located and close-to-halo pixels. For each of the random pixels, we sort all the transmitted flux and select the require flux percentile from all the pixels. While, for the pixels around the halos, to smooth the flux, we averaged the transmitted flux inside 200 km s$^{-1}$ velocity bin along the line of sight. Then, the extreme fluxes are smoothed out. Then, in this sense, the 50% percentile is the most reliable comparison, we overestimate the 5% flux percentile around the halos compared to random flux. For each of the figures, the larger the angular separation is, the higher the flux becomes. It reflects the decrement of the NHD with larger angular separation. Combining the increment of the flux with the larger velocity bins, we can conclude that the hydrogen density increases when pixels approach the halo no matter whether along the line of sight or transversely. The flux does not change too much if the sight line is close to higher mass halos at fixed angular
separation. The reason might be although the density around the higher mass halos increases, which brings more hydrogen around. The temperature of each of the pixels also increases, which could ionize more neutral hydrogen. At the end, the obvious influence of the overdensity around different halos are washed out.

### 2.3.3. The Photoionization Effect

In Figure 2.3, we plot the PDF of randomly located pixels with $1 + \omega$ from 1 to 1000. For each of the line with different probability, the transmitted flux increases rapidly with $\omega$. This comes from the fact that the more the flux that comes from a QSO, the more hydrogen is photo ionized by the QSO’s flux and the less neutral hydrogen is left. Therefore, when a sight line approaches a QSO, the competition between the higher density environment and the excess flux from the QSO determines whether less or more photons are absorbed compared to those passing through the randomly located pixels. The simulation directly provides the information about the density enhancement but does not itself contain the photoionization effects of QSOs, so we provide that as $\omega$, a free parameter.

To compare the effect of excess photoionization and the enhanced environment density by gravitation, we follow the halo mass QSO luminosity relation from Figure 2 in Alam & Miralda-Escudé (2002), hereafter AM. If we assume that the massive halos host luminous QSOs, and once we know both of the QSO luminosity function
and halo mass function, we can obtain $M(L)$ by assuming that only a certain percent of halos hold QSOs. Here Press-Schechter halo mass function is adopted (Press & Schechter 1974b), and QSO luminosity function is from Pei (1995). From Figure 2 in AM, the luminosity of QSO increases dramatically at fixed halo mass if the percentage of halos host QSOs decreases. This can be easily understood since more massive halos hold more luminous QSOs. Then, to calculate our excess ionization, we need to fix what percentage of halos host QSOs. The QSO luminosity function and the growth time of the mass of black hole provide hints as to which percentage to select. The Salpeter time for black hole growth is $M c^2/(\epsilon L_{Edd}) \gtrsim 4 \times 10^8 \epsilon$ years. Typically, the QSO luminosity function evolves in $10^9$ years. If we assume that the QSO luminosity doesn’t change dramatically in its lifetime and the QSO doesn’t revive in the same halo, then the percentage of the halos host QSOs $P_0 < 0.04$ for $\epsilon < 0.1$. From all the above assumptions, it is reasonable for us to assume that about 1% of the halos host QSOs. We applied the background photoionization as $5 \times 10^{-22} \text{erg} \text{s}^{-1} \text{sr}^{-1} \text{Hz}^{-1}$ at redshift 3 (e.g. Bajtlik et al. (1988); Scott et al. (2000); Tytler et al. (2007)). The revolution of the UV background with redshifts is assumed as proportional to $(1 + z)^{-1}$ (e.g. Katz et al. (1996)). We list the luminosity of the QSOs hosted by the corresponding mass halos from Alam & Miralda-Escudé (2002), and the $\omega$ in different angular separation ranges in Table 2.1. Similarly, the halo mass and the hosted QSO luminosity at redshifts $z=2$ and $z=4$ are also listed
in Table 1. Then, we can quantify the excess ionization in Figure 4, Figure 5, and Figure 6 by putting a vertical line which corresponds to a certain value of $\omega$.

### 2.3.4. General Result

In Figure 4, each of the line denotes the PDF of transmitted flux varying with $\omega$ at redshift $z=2$, the $\omega + 1 = 1$ corresponding to the flux without excess photoionization. The vertical lines are where $\omega$ under each of the situations should be. If we focus on the probability of 50%, we can see that the transmitted flux is smaller than that of the randomly located pixels when the sight line passes by a QSO. The transmitted flux increases when the sight line goes away from the QSO. Furthermore, in order to see the tendency clearly, we extracted the transmitted flux from 50% probability lines by combining the corresponding $\omega$ value in each of the mass ranges: $7.62 \times 10^{11} - 1.96 \times 10^{12} M_\odot$, $1.96 \times 10^{12} - 5.04 \times 10^{12} M_\odot$, $5.04 \times 10^{12} - 1.30 \times 10^{13} M_\odot$ and $1.30 \times 10^{13} - 3.33 \times 10^{13} M_\odot$, and angular separation bins: $0.20 - 0.42$arcmin, $0.42 - 0.89$arcmin, $0.89 - 1.89$arcmin, and $1.89 - 4.00$arcmin. Then, in Figure 7, we compare the flux with the mean transmitted flux. It is obvious that the transmitted flux around QSOs has different trends compared to the randomly distributed sight lines. The transmitted flux is smaller than that from randomly located pixels if the lines of sight pass by a low luminosity QSO (a low mass halo). However, the transmitted flux increases if the sight line passes by a higher
luminosity QSO. The more luminous the QSO is, the more relative influence of the excess ionization displays compared to that of the enhanced environment density.

Hence, the proximity effect around QSOs depends on the mass of the halo. If the relationship between the QSO luminosity and the halo mass holds, at small angular separation, pixels passing by lower luminosity QSOs have no proximity effect and even more absorption observed. With the increment of the mass of the halos, the excess flux from QSOs at small angular separation begins to win. Therefore, the more luminous the QSOs are, the more obvious the proximity effect around them. In our calculation, the transmitted flux is still smaller than average in the low mass halo case within angular separation $1.89 - 4.00\text{arcmin}$. In all of the redshifts, the QSO excess photoionization wins around massive halos at small angular separation, while the overdensity held by the halos is more powerful than the extra flux from QSO if the lines of sight passing by low mass halos. Comparing the results in different redshifts, the influence of the foreground QSOs are more important in higher redshifts. From Figure 7, we can see clearly that the transmitted fluxes in higher redshifts are farther away from that in the randomly located pixels no matter below or above it. The reason for that might because the UV background decreases with redshift which makes the relative QSO flux relatively more important, meanwhile the hosting halos are less massive and only can host relatively dimmer QSOs at high redshifts.
2.4.  Discussion

2.4.1.  How to Use the Calculation

In this chapter, we have combined SPH simulation results and a halo hosting QSO model to predict the transverse proximity effect in QSO pairs. By our model, the results are that the proximity effect is detectable around luminous QSOs and is difficult to observe around faint QSOs. The uncertainty mostly arises from our M(L) relation. In addition, the theoretical prediction can tell us how to classify the observations and to see if this prediction is right. We investigated 3 QSO pairs from Schirber et al. (2004). In our calculation, we focused on the statistical properties of photoionization from foreground QSO. The sample is too small to judge if the model in this chapter good enough. Therefore, the more interesting comparison with data is from a large number of QSO pairs.

2.4.2.  Compare with Data

Here, we compare the calculation with the data of QSO pairs in Schirber et al. (2004). The foreground QSOs for the 3 pairs are in different redshift 4.032, 3.620 and 2.245, which are not at the exactly same redshifts as our simulated foreground QSO. While, we still can approximate the sight lines into redshift 4 for the first two pairs and into redshift 2 for the third pair. The angular separation for first
pair in Schirber et al. (2004) is 1.89arcmin. Corrected by the UV background that we use here, redshift effect, and the angular separation, the first pair has excess photoionization around $\omega = 46.8$ and the second pair has $\omega = 6.8$. From (Schirber et al. 2004; Alam & Miralda-Escudé 2002) and our 1% halo hosting QSO assumption, the foreground QSO is hosted by a halo with mass as high as $6.9 \times 10^{12}M_\odot$. The transmitted flux in this pair is far below our 5% flux percentile given the value of excess photoionization. Then overdensity around QSO can not explain the so low transmitted flux for this pair. The angular separation of pair 2 is $\theta_3 = 3.95$arcmin, which is comparable to our largest computed angular separation $\theta = 4$.arcmin. It might be hosted by the halo in the most massive halo group or in the second most massive halo groups (We just put it in the most massive halo group). The transmitted flux of this pair is slightly below the 5% percentile of the transmitted flux with the excess photoionization included. This pair can only be marginally explained by the overdensity of the QSO environment. The foreground QSO in pair 3 is close to redshift 2, Apply the data $\omega_3 = 6.1$ and $\theta_3 = 2.57$ of pair 3, this QSO might be hosted by a halo in the mass ranges $5.04 \times 10^{12} - 1.30 \times 10^{13}$. The data point for pair 3 is displayed in Figure 4 by a error-barred point. It’s slightly lower than our 20% percentile prediction from our calculation.

Then from the above comparison, we can explain QSO pair 2 and 3 marginally, with better explanation on pair 1, while fail to explain the low transmitted flux in pair 1. Then it’s hard to get the definite conclusion if the overdensity of the
QSO environment can fully explain the scarcity of the transverse proximity effect or not, since there are so many uncertainties in our estimation. For example, the relation between the halo mass and QSO luminosity, which is influenced by the halo mass function and QSO luminosity and also the percentage of how many halos hosting QSOs. The other influence factor is the UV background, which is not so well quantified factor too. The other possible uncertainty comes from the small number of lines of sight close to the massive halos in our simulation. In order to prove that if the overdensity around QSOs can explain the scarcity of transverse proximity effect or not, we need more QSO pairs and more accurate luminosity and halo mass relation, and UV background knowledge.

In this chapter, we come to the conclusions that the transverse proximity effect is related to the hosting halo mass (QSO luminosity) and angular separation between the line sight and QSO. However, this conclusion needs to be proved by larger samples. Then, the most reliable way to detect if the prediction in the chapter right is to explore a large number of QSO pairs. SDSS offers a good resources to do this, although in this chapter, we haven’t investigated the large amount of pairs.
Fig. 2.1.— density, temperature and spectra for an randomly selected line of sight ($z=3$). The black, green and magenta lines are for the $1 + \omega = 1$, 10, and 100 respectively. The density and temperature are in the real space, the spectrum is in the redshift space.
Fig. 2.2.— The probability distribution function (PDF) of the mean transmitted flux around the halo (z=3), within mass range $2.4 \times 10^{12} - 5.6 \times 10^{12} M_\odot$, and with the angular separation from halos within $\Delta \theta 0.42 - 0.89$ arcmin. The solid line indicates the 5% percentile of the transmitted flux with given excess photoionization rates $1 + \omega$. Dotted, dashed, and long-dashed lines are 20%, 50% and 80% percentile respectively. Each of the panel represents the probabilities which are smoothed with different velocity bin withs 50km/s, 100km/s, 200km/s and 400km/s around the halos.
Fig. 2.3.— The PDF of randomly located pixels changes with $1 + \omega$ at redshift $z=2$, $z=3$, and $z=4$. The triangles are the points without excess photoionization. From the bottom to top, the 5%, 20%, 50% and 80% percentiles of the transmitted flux are shown in each panel.
Fig. 2.4.— The 5%, 20%, 50%, and 80% percentiles of the close-to-halo transmitted flux at $z=2$ are shown. Same angular separation data are in the same row, and same mass bin data are in the same column, with the values as marked. The "n's" indicate the number of lines of sight at given mass and angular separation bins. The triangles from bottom to top are the 5%, 20%, 50%, and 80% percentiles of transmitted flux of the random pixels without excess photoionization. The vertical lines are located in the estimated excess photoionization in each specific conditions. The point with error bar is the observed data from Schirber et al. (2004), but with the revised excess photoionization according to the angular separation and our UV background.
Fig. 2.5.— Similar figure as in Fig. 2.4, but for the redshift $z=3$ simulation.
Fig. 2.6.— Similar as in Fig. 2.4, but the simulation and data at redshift $z=4$. 
Fig. 2.7.— This figure shows the close-by 50% percentile of the transmitted flux over that in the randomly located pixels ratios changing with angular separations. Each connected lines corresponds to different halo mass bins. The upper two panels show $z=2$, and $z=3$ results, and the lower panel shows $z=4$ simulation results. The dashed horizontal line of value ”1” represents 50% percentile of the flux in randomly located pixels without excess photoionization. Lines above ”1” are the proximity effect detectable cases in the 50% percentile; otherwise, proximity effect is undetectable in 50% percentile.
Table 2.1. Halo Mass, QSO Luminosity and Excess Ionization

<table>
<thead>
<tr>
<th>mass (m/M_\odot)</th>
<th>luminosity (L/L_\odot)</th>
<th>(1 + \omega) ((\theta) in arcmin)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20–0.42</td>
<td>0.42–0.89</td>
<td>0.89–1.89</td>
</tr>
</tbody>
</table>

| \(z = 2\) |
|-------------------|--------------------------|---------------------------------|
| \(1.36 \times 10^{12}\)  | \(4.5 \times 10^{11}\)  | 31.6                            | 7.1   | 1.6   | 0.4   |
| \(3.50 \times 10^{12}\)  | \(2.7 \times 10^{12}\)  | 188.1                           | 42.1  | 9.4   | 2.1   |
| \(9.00 \times 10^{12}\)  | \(8.1 \times 10^{12}\)  | 561.5                           | 125.5 | 28.1  | 6.3   |
| \(2.31 \times 10^{13}\)  | \(3.0 \times 10^{13}\)  | 2106.1                          | 470.8 | 105.3 | 23.5  |

| \(z = 3\) |
|-------------------|--------------------------|---------------------------------|
| \(0.75 \times 10^{12}\)  | \(1.1 \times 10^{11}\)  | 12.0                            | 2.7   | 0.6   | 0.1   |
| \(1.73 \times 10^{12}\)  | \(7.5 \times 10^{11}\)  | 81.7                            | 18.3  | 4.1   | 0.9   |
| \(4.01 \times 10^{12}\)  | \(3.0 \times 10^{12}\)  | 324.1                           | 72.4  | 16.2  | 3.6   |
| \(9.30 \times 10^{12}\)  | \(8.5 \times 10^{12}\)  | 925.9                           | 208.9 | 46.3  | 10.4  |

(cont’d)
<table>
<thead>
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<th>mass $m/M_\odot$</th>
<th>luminosity $L/L_\odot$</th>
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<th>$0.42-0.89$</th>
<th>$0.89-1.89$</th>
<th>$1.89-4.00$</th>
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<tr>
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<td>$1.1$</td>
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<td>$1.2\times10^{12}$</td>
<td>$195.1$</td>
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<td>$148.1$</td>
<td>$33.1$</td>
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</table>

Note. — The halo mass is in the middle of the mass bin, luminosity is obtained by assuming that the most massive halos host the brightest QSOs and only 1% halos host QSOs.
The presence of dark matter in early type galaxies is clear on large scales, based on both weak lensing (e.g. Kleinheinrich et al. 2006, Mandelbaum et al. 2006), X-ray (e.g. Humphrey et al. 2006) studies and dynamics analysis of the satellite galaxies (e.g. Conroy et al. 2007, van den Bosch et al. 2004, and Prada et al. 2003). The distribution of the dark matter and the mass fraction represented by the stars are less well-determined because of the difficulties in measuring early-type galaxy structure in the transition region between the stars and the dark matter. Stellar kinematic studies of the central regions, when compared to estimates of stellar mass-to-light ratios, have argued either that there is little dark matter inside an effective radius (e.g. Gerhard et al. 2001) or that there is a substantial dark matter fraction (e.g. Padmanabhan et al. 2004). The significance of these differences depends on the reliability of estimating the stellar mass from combinations of photometry, spectroscopy and population synthesis models. Studies on larger scales
using planetary nebulae, have found examples of galaxies with falling rotation curves (Romanowsky et al. 2003), while the globular clusters in one of these systems show a flat rotation curve (Pierce et al. 2006). Surveys of structure with gravitational lenses (e.g. Rusin & Kochanek 2005, Treu et al. 2006) indicate that the typical lens has a flat rotation curve on scales of $1–2R_e$, but the interpretation of the scatter around the mean structure has been used to argue for both inhomogeneity (e.g. Treu & Koopmans 2002a, Kochanek et al. 2006) and homogeneity (e.g. Rusin & Kochanek 2005, Koopmans et al. 2006) in the mass distributions.

In this paper we reanalyze a sample of 15 lenses from the Sloan Lens ACS Survey (SLACS, Bolton et al. 2006) and 7 lenses from the Lens Structure and Dynamics Survey (LSD, Koopmans & Treu 2003) that have both mass estimates from the lens geometry and central velocity dispersion measurements. Koopmans et al. (2006) analyzed the sample using a simple, global power law model, $\rho \propto r^{-\gamma}$, for the mass distribution to find a mean slope of $\gamma = 2.01^{+0.02}_{-0.03}$ where $\gamma = 2$ corresponds to a flat rotation curve. With a nominal scatter in the slope of only 0.07, Koopmans et al. (2006) argue that the halo structures appear to be fairly homogeneous. It is difficult, however, to relate these power law models to theoretical models of halos or to evaluate the significance of the scatter in the slope. Additionally, the models are somewhat unphysical because they allow mass distributions more compact that the luminous galaxy.
Here we reanalyze the SLACS and LSD lens samples using a more physical mass model that combines a Hernquist (1990) profile for the stars with a Navarro, Frenk & White (1996, NFW) model for the dark matter. By comparing the mass inside the Einstein ring of the galaxies ($M_E(<R_E)$) with the mass needed to produce the observed velocity dispersion, we can estimate the stellar mass fraction and the stellar mass-to-light ratio explicitly. We use a Bayesian formalism that allows us to quantitatively address the homogeneity of the sample. We review the data and describe our mass models and analysis techniques in §3.2. In §3.3, we discuss the models of the individual systems (§3.3.1), the homogeneity of the sample (§3.3.2), and finally, the stellar mass fraction, the mean stellar mass-to-light ratio and the evolution of the mass-to-light ratios (§3.3.3). We summarize our results in §3.4.

3.2. Data and Method

3.2.1. Data

In this paper we reanalyze the data for 15 lenses from the Sloan Lenses ACS Survey (SLACS) and 7 lenses from the Lens Structure and Dynamics Survey (LSD). We neglect two lenses, Q0957+561 and Q2237+030, from the LSD, since Q0957+561 is in a cluster and Q2237+030 is a barred spiral galaxy, to leave us with a sample of 22 galaxies with measured velocity dispersions, effective radii, Einstein radii, enclosed masses and rest frame B band magnitudes taken from the original
analyses. For convenience, we summarize the data in Table 3.1, particularly since the equivalent table in Treu et al. (2006) and Treu et al. (2006) contains ordering errors. For consistency we have adjusted all the data to a flat ΛCDM cosmological model with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $H_0 = 70 \text{ km s}^{-1} \text{ Myr}^{-1}$.

3.2.2. Method of Analysis

We model the lenses with the two component mass model for lenses introduced by Keeton (2001), which was also used for early-type galaxies in the SDSS by Padmanabhan et al. (2004). It consists of a Hernquist (1990) model for the luminous galaxy and a Navarro, Frenk & White (1996, NFW) profile for the dark matter halo. By using this physically motivated model rather than the simple power law normally used by the LSD/SLACS studies, we both better connect the results to theoretical halo models, and avoid models in which the dark matter can be more centrally concentrated than the stars. In essence, we use the mass enclosed by the Einstein radius of the lens to set the virial mass $M_{\text{vir}}$, the stellar velocity dispersion to determine the stellar mass fraction $f_\ast$, and theoretical halo models to constrain the halo concentration $c$. Finally, by comparing the derived stellar mass to the observed luminosity, we can estimate the rest frame B-band mass-to-light ratio $M_\ast/L$ and its evolution.
The Hernquist (1990) model used for the luminous lens galaxy is defined by

\[ \rho_H(r) = \frac{M_\ast}{2\pi} \frac{r_H}{r(r + r_H)^3}, \tag{3.1} \]

where the scale length \( r_H = 0.551R_e \) is matched to the measured de Vaucouleurs profile effective radius \( R_e \) and the stellar mass \( M_\ast = f_\ast M_{\text{vir}} \) is related to the virial mass by the cold baryon/stellar mass fraction \( f_\ast \). The NFW profile (Navarro, Frenk & White 1996) used to model the initial dark matter halo is defined by

\[ \rho_N(r) = \frac{M_{\text{dm}}}{4\pi f(c)} \frac{1}{r(r + r_s)^2}, \tag{3.2} \]

where the scale length \( r_s \) is related to the virial radius by \( c = r_{\text{vir}}/r_s \), \( f(c) = \ln(1 + c) - c/(1 + c) \), and \( M_{\text{dm}} = (1 - f_\ast)M_{\text{vir}} \) is the mass in dark matter. The average concentration was modeled by

\[ c = \frac{9}{1 + z} \left( \frac{M_{\text{vir}}}{8.12 \times 10^{12} \ hM_{\odot}} \right)^{-0.14}, \tag{3.3} \]

and the individual halos have a log-normal dispersion in their concentrations of \( \sigma_c = 0.18 \) (base 10) around the average (Bullock et al. 2001). These initial models neglect the compression of the dark matter density profile by the more concentrated baryons. We estimated the changes in the dark matter distribution using the adiabatic compression model of Blumenthal et al. (1986). This approximation may exaggerate the compression (Gnedin et al. 2004), so we should regard our compressed and uncompressed results as bounding the possible effects of adiabatic compression.
The observations provide two constraints, the mass inside the Einstein radius, and the stellar velocity dispersion. For any value of $c$ and $f_*$, we use the projected mass inside the Einstein radius to determine $M_{\text{vir}}$ (which also determines $r_{\text{vir}}$), then use the spherical Jeans equation and a constant orbital isotropy $\beta$ to compute the velocity dispersion expected for the measurement aperture. The effects of seeing were modeled using a Gaussian PSF with the observed FWHM of the observations. Given the estimated dispersion $\sigma_{\text{i,model}}$, the measured dispersion $\sigma_i$ and its uncertainties $e_{\sigma_i}$ for galaxy $i$, we estimate a goodness of fit $\chi_i^2(\sigma_i) = (\sigma_{\text{i,model}} - \sigma_i)^2 / e_{\sigma_i}^2$. We model the mass-to-light ratios of the stars using the standard power law (e.g. van Dokkum & Franx (1996); Treu (2001); Rusin & Kochanek (2005); Koopmans et al. (2006)),

$$\log \left( \frac{M_*}{L} \right) = \log \left( \frac{M_*}{L} \right)_0 + z \left( \frac{d \log (M_*/L_i)}{dz} \right)$$

(3.4)

where $(M_*/L)_0$ is the value today and $d \log (M_*/L)/dz$ is the rate at which it changes with redshift $z$. This in turn defines a goodness of fit $\chi_i^2((M/L)_i)$ with which the model fits the logarithm of the mass-to-light ratio $(M/L)_i$ of galaxy $i$, defined by the ratio of the estimated stellar mass (a model parameter) to the observed luminosity, given its uncertainties $e_{L_i} = \Delta \log (M/L)_i = \Delta L_i / (\ln(10)L_i)$. These two terms define a probability of fitting the velocity dispersion $P(\sigma_i|\xi) = \exp(-\chi_i^2(\sigma_i)/2)/\sqrt{2\pi e_{\sigma_i}}$ and the mass-to-light ratio $P((M/L)_i|\xi) = \exp(-\chi_i^2((M/L)_i)/2)/\sqrt{2\pi e_{L_i}}$ given the model parameters $f_*$, $c$, $(M_*/L)_0$ and $d \log (M/L)/dz$ which we abbreviate as $\xi$. 

45
Combining the two terms, we have the probability of the model fitting the data $D_i$ for galaxy $i$

$$P_i (D_i|\xi) = P (\sigma_i|\xi) P ((M/L)_i|\xi). \quad (3.5)$$

In addition to the measurement errors listed in Table 3.1, we should also consider sources of systematic errors. The essence of the method is to compare the mass inside the Einstein ring $M(<R_e)$ to a virial mass estimate from the velocity dispersion $\sigma_v^2 R/G$. We can identify five sources of systematic errors. First, while there is little uncertainty in $M_E$, some of the mass may be projected surface density from either a parent group halo to which the lens belongs, or from another along the line of sight. The extra density, $\kappa = \Sigma/\Sigma_c$ in dimensionless units, modifies the mass inside the Einstein radius by $\pi \kappa R_E^2 \Sigma_c$, so we can think of its effects as a systematic error in interpreting $\sigma_v$ of $e_\sigma = \sigma_\kappa/2$. The full probability distribution of $\kappa$ is skewed to positive values (e.g. Takada & Hamana (2003)), but we will ignore this problem and assume $\sigma_\kappa \approx 0.05$ since the positive tail of the distribution is associated with detectable objects (galaxies and clusters). This systematic error also affects estimates of the mass-to-light ratios. Second, there are $1 - 10\%$ uncertainties in the galaxy effective radius measurements which contribute uncertainties of $0.5\%$ to $5\%$ to our interpretation of the velocity dispersion. Third, the measured velocity dispersion is a Gaussian fit to the spectrum, which is not identical to the rms velocity appearing in the Jeans equation (e.g. Binney & Tremaine (1987)). The difference
can be estimated from the typical Gaussian-Hermite coefficients $|h_4| \approx 0.02$ (Bender, Saglia & Gerhard 1994) as a fractional error in $\sigma_v$ of order $\sqrt{6}|h_4| \approx 0.05$ in the velocity dispersion (e.g. van der Marel, van Dokkum & Franx (2003)). Fourth, non-sphericity, (somewhat to our surprise) leads to negligible systematic errors provided we use the intermediate scale length (the geometric mean of the semi-major and minor axes), at least in the limit of the tensor virial theorem. It leads to large errors if any other scale length is used. Barnabe & Koopmans (2007) have taken the first steps towards removing these two dynamical problems, although they are restricted to oblate two-integral models which may not be appropriate for massive elliptical galaxies. Finally, calibration errors in the velocity dispersions contribute fractional errors of order 0.03 (see Bernardi et al. (2003a)). Combining all these contributions in quadrature, which corresponds to assuming a Gaussian model for each systematic error, we estimate that the typical systematic uncertainty to interpreting the velocity dispersions is approximately 8% with the exact value depending on the uncertainties in the effective radius.

Our statistical methods are chosen so that we can understand the homogeneity of the lens galaxies in either their evolution or their dynamical properties and estimate their average properties in the presence of inhomogeneities. We will analyze the results using two Bayesian methods. In the first method, we will fit the data while simultaneously estimating the systematic errors in the velocity dispersion and the mass-to-light ratio. When combined with the measurement errors, these define
new uncertainty estimates for the data which we will call the “bad” case errors in comparison to the original uncertainties (the “good” case). These broadened uncertainties can be representative of either true systematic uncertainties, such as the ones we discussed above for the dynamical measurements, or indicative of inhomogeneities in the structure or evolution of the galaxies. In the second method we will compare these two cases using the approach outlined in Press (1997) to determine the degree to which the sample homogeneous or heterogeneous. In this method, we assume that there are probabilities $p_{\sigma}$ and $p_{L}$ that the galaxies have homogeneous structures or evolutionary histories in the sense that the scatter in the measurements is simply determined by the “good” measurement errors. There are then probabilities $1 - p_{\sigma}$ and $1 - p_{L}$ that the galaxies are not a homogeneous group in either their structure or their evolution, where we characterize this by assuming that the uncertainties in the velocity dispersion and the mass-to-light ratio are significantly broadened to be the “bad” measurement errors. In essence, we are determining the relative probabilities of the stated measurement errors and our estimate of the true uncertainties from the first method. Both approaches provide uncertainties on the average properties of the sample that account for potential inhomogeneities, although the second method is a better formal approach since it can reject individual objects.

In the first approach, we will estimate the fractional systematic errors $e_{\sigma}$ and $e_{L}$ in the velocity dispersion and luminosity. The $\chi^2$ expressions are modified to use
uncertainties of $e_{\sigma i} \rightarrow \sqrt{e_{\sigma i}^2 + e_{\sigma i}^2 \sigma_i^2}$ and $e_{Li} \rightarrow \sqrt{e_{Li}^2 + e_{Li}^2}$ for the velocity dispersions and the logarithm of the mass-to-light ratios respectively. We assume logarithmic priors for $f_\star$, $(M_\star/L)_0$, and $(d(M/L)/dz)$, and the theoretical prior defined by Eqn. (3.3) for the concentration $c_i$. Note that we are forcing all galaxies to have the same concentration, which should have no significant impact given the scales we are studying. The priors for the systematic errors, $P(e_{\sigma}) = 1/\sqrt{\langle e_{\sigma i}^2 \rangle + e_{\sigma i}^2 \sigma_i^2}$ and $P(e_L) = 1/\sqrt{\langle e_{Li}^2 \rangle + e_{Li}^2}$, naturally switch between uniform priors for systematic errors small compared to the mean square measurement errors ($\langle e_{\sigma i}^2 \rangle$ and $\langle e_{Li}^2 \rangle$) and logarithmic priors for large systematic errors. The resulting probability distribution for the fractional errors is then

$$P(e_{\sigma}, e_L|D) \propto P(e_{\sigma})P(e_L) \int d\xi P(\xi) \prod_i P(D_i|e_{\sigma}, \xi) P(D_i|e_L, \xi)$$  \hspace{1cm} (3.6)$$

where $P(D_i|e_{\sigma}, \xi)$ and $P(D_i|e_L, \xi)$ are the probability distributions modified by the addition of the systematic errors $e_{\sigma}$ and $e_L$. We then use these systematic error estimates to define the uncertainties used for the “bad” case in our second formalism.

The second, Press (1997) approach properly weights all combinatorial possibilities of the individual systems being members of a homogeneous sample or not. Let $P_{Gi}(\sigma_i|\xi)$ and $P_{Gi}((M/L)_i|\xi)$ be the probabilities of the data given the parameters for galaxy $i$ if it is a member of a homogeneous group based on the measured, “good” uncertainties, and $P_{Bi}(\sigma_i|\xi)$ and $P_{Bi}((M/L)_i|\xi)$ be the probabilities if it is not and we should be using the “bad” uncertainties based on
the systematic error estimates derived from our first method. The Press (1997) provides estimates of the relative likelihoods describing either the full sample or the individual systems by the “good” or “bad” data model. If we want the Bayesian probability distribution for the parameters $\xi$ properly weighted over all possible group membership combinations, we find that

$$P(\xi|D) \propto P(\xi) \int dp_\sigma dp_L \prod_i F_i$$

(3.7)

where

$$F_i = [p_\sigma P_{G\alpha}(\sigma_i|\xi) + (1 - p_\sigma)P_{B\alpha}(\sigma_i|\xi)] [p_L P_{G\alpha}((M/L)_i|\xi)$$

$$+ (1 - p_L)P_{B\alpha}((M/L)_i|\xi)]$$

(3.8)

and where $P(\xi)$ sets the prior probability distributions for the parameters. We assume a uniform priors for $p_\sigma$ and $p_L$. We obtain the probability distribution for any parameter by marginalizing Eqn. (3.7) over all other variables and then normalizing the total probability to unity. We can also estimate the probability that the sample is homogeneous in either its structural or evolutionary properties as

$$P(p_\sigma, p_L|D_i) \propto \int d\xi P(\xi) \Pi_i F_i$$

(3.9)
and the probability that a particular galaxy is in the dynamically homogeneous class is

\[
P(\sigma_i \in \text{homogeneous}|D) = \frac{A_i}{A_i + B_i} \quad (3.10)
\]

where

\[
A_i = \int d\xi P(\xi) \int [dp_\sigma dp_L p_{Gi}(\sigma_i) \{p_L P_{Gi}((M/L)_i|\xi)\} \\
+ (1 - p_L) P_{Bi}((M/L)_i|\xi)] \Pi_{i \neq j} F_j \quad \text{and}
\]

\[
B_i = \int d\xi P(\xi) \int [dp_\sigma dp_L (1 - p_\sigma) P_{Bi}(\sigma_i) \{p_L P_{Gi}((M/L)_i|\xi)\} \\
+ (1 - p_L) P_{Bi}((M/L)_i|\xi)] \Pi_{i \neq j} F_j.
\]

A similar set of expressions gives the probability that the galaxy is in the set of galaxies with a homogeneous evolutionary history.

### 3.3. Results

We divide our discussion of the results into three subsections. First, we present the results for the individual galaxies. Next, we discuss the homogeneity of the structural properties of the galaxies. Finally, we estimate the stellar mass fraction, mass-to-light ratios and the rate of galaxy evolution.
3.3.1. Properties of Individual Galaxies

Figure 3.1 shows contours for the goodness of fit of the models to the velocity dispersion, measured for each galaxy as a function of the stellar mass fraction $f_*$ and the concentration $c$ once we have normalized the mass inside the Einstein radius. For these calculations, we have included our estimates of the systematic errors in the velocity dispersions but used the stated uncertainties in the luminosities. Note that the dispersion measurements cannot determine the halo concentrations but the goodness of fit contours always pass through the region set by our prior on the concentration. The permitted stellar mass fractions vary widely between objects. Three of the 22 objects, SDSS J0737 + 321, SDSS J1250 + 052 and PG1115 + 080, appear to require mass distributions that are more centrally concentrated than the stars, in the sense that the best fits for $f_* \leq 1$ have $\chi^2 > 2$. This is also seen in the LSD models for PG1115 + 080 (Treu & Koopmans 2002a), where the only models consistent with both the lensing constraint and the estimated velocity dispersion are more centrally concentrated than the stars. A fourth lens, SDSS J1627−005, is only marginally consistent with $f_* \leq 1$. Of the remaining 18 galaxies, eleven are consistent with $f_* = 1$ ($\Delta \chi^2 < 1$), and seven are not. Four of these eleven galaxies have enormous parameter uncertainties. One problem for many SLACS lenses is that the scales of the velocity dispersion aperture/effective radius differ little from the observed Einstein radius, which limits the leverage for constraining the mass profile.
Figure 3.3 shows the goodness of fit to the mass-to-light ratio of each galaxy given the best fit model for the average evolution of the sample. Most of the galaxies are consistent with this best fit model for the mass-to-light ratio and its evolution (§3.3.3). The mass-to-light ratios of the sample appear to be more uniform than the dynamical properties, probably for the same reasons that there is little scatter in the fundamental plane (see Bernardi et al. (2003b)). However, there are three 3σ outliers in the sample, SDSS J1420+602, SDSS J1250+052 and H1543+535, all of which have very low M/L ratios compared to the other galaxies. Note that only one of these, SDSS J1250+052, is also an outlier in the dynamical fits. This is not unique to our approach, since our mass-to-light ratio for H1543+535 is comparable to that in Treu & Koopmans (2004). In Figure 6 of Treu et al. (2006), they also find significantly lower mass-to-light ratios for SDSS J1420+602 and SDSS J1250+052 than they do for the other SLACS members. One possible solution is that the lens masses are significantly mis-estimated due to contamination from a group or cluster halo, but only H1543+535 has a neighboring, bright galaxy and it is sufficiently distant to only modestly perturb the estimated mass.

3.3.2. Homogeneity

The broad uncertainties and occasional outliers mean that it is important to have a quantitative approach to determining the homogeneity of the sample and to appropriately weight each object when determining mean properties. This is
why we introduced the Bayesian frameworks of §3.2. Fig. 3.6 shows our estimates of the fractional systematic errors from our first analysis method (Eqn. 3.6). The best fit estimates for the fractional systematic errors in the velocity dispersion and luminosity are $e_\sigma \simeq 0.1$ and $e_L \simeq 0.18$. The reported measurement errors lie well outside the 99.7% likelihood region. For the dynamical errors, the best fit systematic errors are quite consistent with our prior estimates based on simple considerations about the dynamical data.

Fig. 3.5 shows that the results for the homogeneity of the sample are very sensitive to the assumed uncertainties. If we simply used the stated measurement errors, then the probability that the sample is homogeneous in its dynamical properties (i.e. that the “good” uncertainty estimates are correct and the scatter is due only to measurement error) is $p_\sigma \leq 24\%$ and that it is homogeneous in mass-to-light ratio evolution is $p_L \leq 14\%$. Many objects have low probabilities of belonging to either a homogeneous dynamical subset (SDSS J1627–0053 with $p_\sigma = 0.005$, SDSS J1250+0523 with $p_\sigma = 0.010$, SDSS J0737+321 with $p_\sigma = 0.010$, PG1115+080 with $p_\sigma = 0.029$, and SDSS J1420+602 with $p_\sigma = 0.042$) or a homogeneous evolutionary subset (SDSS J1250+0523 $p_L \approx 0$, H1543+535 $p_L \approx 0$, SDSS J0912+002 $p_L = 0.001$, SDSS J1420+602 $p_L = 0.001$ and MG1549+305 $p_L = 0.001$). Not surprisingly, these objects are also outliers in the individual fits from the previous section. If we include our estimates of the systematic errors in interpreting the dynamical measurements, then the probability that the sample is homogeneous in its dynamical properties
rises to $p_\sigma \geq 40\%$, but the probability of a homogeneous evolutionary population remains small at $p_L \leq 14\%$. With the inclusion of the systematic error estimates, the objects with the lowest probabilities of belonging to the homogeneous dynamical subset are PG1115+080 (with $p_\sigma = 0.48$), SDSS J1250+0523 (with $p_\sigma = 0.49$), SDSS J1627–0053 (with $p_\sigma = 0.52$), SDSS J0737+321 (with $p_\sigma = 0.52$), and H1417+526 (with $p_\sigma = 0.55$). These estimates strongly indicate that either the SLACS/LDS lens populations are inhomogeneous or that the measurement errors underestimate the true uncertainties. In a few cases, these problematic lenses show some evidence for disks (SDSS J1420+602, MG1549+305).

In sum, the SLACS/LDS galaxies are homogeneous in neither their dynamical nor their evolutionary properties if we take the measurement errors at face value. It is likely that most of the problem for the dynamical measurements is that the systematic errors in interpreting velocity dispersions are significant and need to be included in any analysis of the dynamics of lenses. One of these systematic errors, surface density contributions from structures other than the lens galaxy, also produces systematic errors in the mass-to-light ratio, with $\sigma_M = \sigma_\kappa \simeq 0.05$, but this is much too small to explain the spread in the mass-to-light ratios. This problem is probably caused by a combination of underestimated uncertainties in the luminosities and true variance in the evolutionary history of early-type galaxies. Rusin & Kochanek (2005) and Treu et al. (2006) had found earlier that the lens sample was better fit by allowing a range for the mean redshift at which the stars
formed than by assuming a single value, and in this analysis a range of formation redshifts would lead to non-zero systematic errors in the mass-to-light ratio. To make sure that the scatter of the parameters are not model induced, we also did the calculation for a power-law mass distribution with $\rho \propto r^{-\gamma}$, still using a Hernquist model for the mass distribution. When we compute the corrected errors, we find that they must be broadened by the same factor as before. Like Treu et al, we find that the best fit solution is isothermal, $\gamma = 2.00 \pm 0.02$ with the modified error bars and the change is little for stated error bars. Fig. 3.10 shows the density distribution of $\gamma$ with the modified error bars. The joint effect constrains $\gamma$ in small ranges although the individuals scatter larger. This is not very surprising, since we know from previous studies (e.g. see Rusin & Kochanek 2005) that sums of Hernquist models with NFW halos that are compatible with lensing constraints have total mass distributions over the relevant scales that are similar to an isothermal distribution.

### 3.3.3. The Stellar Mass Fraction and Mass-to-Light Ratio

We can combine the galaxies to make joint estimates of the stellar mass fraction $f_*$, the mass-to-light ratio $M_*/L$ at $z = 0$ and its evolution. We considered both Bayesian frameworks so that either the uncertainties are broadened to make the results consistent with all the lenses (method 1, Eqn. 3.6) or the outliers in the
sample are properly down weighted in the analysis (method 2, Eqn. 3.7) The two methods give similar results, so we only present the detailed results from the second Bayesian method. Fig. 3.7, shows the estimated stellar mass fraction \( f_* \) for both the individual galaxies and the sample as a whole, and for orbital anisotropies of \( \beta = -1/3, 0 \) and 1/3, where \( \beta = 1 - \sigma_t^2/\sigma_r^2 \) is related to the ratio of the tangential \( \sigma_t \) and radial \( \sigma_r \) velocity dispersions. For isotropic, adiabatically compressed models, we find \( f_* = 0.056 \pm 0.011 \), and like Koopmans et al. (2006), we find that that the isotropy has little effect on the inferred mass distribution. The stellar mass fraction is significantly lower than the global baryon fraction of 0.176^{+0.006}_{-0.019} from the WMAP CMB anisotropy measurements (Spergel et al. 2006). If we do not include the adiabatic compression of the halo, the stellar mass fraction drops to 0.026 \pm 0.006, again with little dependence on the isotropy \( \beta \). While the uncompressed models have less dark matter in the central regions, the total halo mass is much larger than in the compressed models. To the extent that adiabatic compression occurs, but the Blumenthal et al. (1986) model exaggerates its degree (Gnedin et al. 2004), reality is intermediate to these two extremes.

We also fit the mass-to-light ratio as \( \log(M_*/L) = \log a + b z \), where \( a = (M_*/L)_0 \) is the mass-to-light ratio at \( z = 0 \) and \( b = d(\log(M_*/L))/dz \) is its evolution with redshift. Fig. 3.8, shows the likelihood contours for these two parameters for both compressed and uncompressed models. For the compressed isotropic models, we find \( a = (7.2 \pm 0.5) M_\odot/L_\odot \) and \( b = -0.72 \pm 0.08 \). This agrees with the local value
of \( a = (7.3 \pm 2.1) M_\odot / L_\odot \) from Gerhard et al. (2001) that was used by Treu et al. (2006). It also agrees with the Treu et al. (2006) estimate for the rate of the evolution \( b = -0.69 \pm 0.08 \). Changing the isotropy over the range \( \beta = -1/3, 0, 1/3 \) has little effect, while the model without adiabatic compression requires higher normalizations for the mass-to-light ratio \((10.0 \pm 0.3)\) and slightly slower rates of evolution. Our analysis includes 2 lenses (PG1115+080 and H1543+535) that were not used by Treu et al. (2006), but excluding them from the analysis has little effect on the mass-to-light ratios. There are no significant changes in \((M_*/L)_0\) and \(d\log(M_*/L)/dz\) if we neglect these two lenses.

3.4. DISCUSSION

We reanalyzed the data from the SLACS and LSD surveys of gravitational lenses with velocity dispersion measurements. Our mass distribution consists of a Hernquist model for the luminous galaxy embedded in a theoretically constrained NFW halo model. We investigated the homogeneity of the sample, the stellar mass fraction \( f_* \), the local \((z = 0)\) stellar mass-to-light ratio \((M_*/L)_0\) and its evolution \(d\log(M_*/L)/dz\). As in the earlier study by Koopmans et al. (2006), we found that the effects of orbital anisotropy on both the stellar mass fraction and the mass-to-light ratio are small.
In most cases, a central velocity dispersion measurement provides only weak a constraint on halo structure in the physically interesting region. Typical limits on the mass fraction represented by the stars have logarithmic errors of order 0.5 dex. While this appears to contradict the conclusions of (for example) Koopmans et al. (2006), this is not the case. Koopmans et al. (2006) fit mass models where $\rho \propto r^{-\gamma}$, and find values in the range $1.8 < \gamma < 2.3$. Fig. 3.9 shows the expected range of this slope for our models of SDSS J0037–0942, where we estimated the slope by fitting the $\rho \propto r^{-\gamma}$ power law model to the projected mass distribution over a radial baseline of $R_e/8$ to $R_E$ that approximates the leverage in using stellar dynamics combined with gravitational lensing to determine halo structure. For this typical lens, the variations in $\gamma$ of $1.6 \lesssim \gamma \lesssim 2.06$ are comparable to the system-to-system spread in $\gamma$ observed for the SLACS systems (Koopmans et al. (2006)). Thus, the spread in $\gamma$ observed for individual SLACS/LSD lenses is comparable to the range of values found in our halo models, so strong conclusions about halo structure from these system will depend on averages over the samples rather than the results for individual lenses.

The critical issue for determining the sample average properties is the degree to which the populations are homogeneous. A heterogeneous sample cannot easily be averaged to determine mean properties. We find the probability of homogeneity is very sensitive to the uncertainties in both the velocity dispersion and the luminosity. If take the measurement errors at face value, there is a low probability of homogeneity
in either dynamical structure \((p_{\sigma} \leq 20\%)\) or evolutionary history \((p_{L} \leq 15\%)\). Many lenses such as SDSS J1250+052, H1543+535, SDSS J1420+601, SDSS J0912+002, MG2016+112, and MG1549+305 have low \((< 10\%)\) likelihoods of belonging to a homogeneous sample. The primary problem is probably that there are significant systematic uncertainties that must be included with the measurement errors. Simple considerations show that typical systematic errors in interpreting the velocity dispersions should be large compared to the measurement errors, 8\% versus 5\%, and adding these estimated systematic uncertainties greatly increases the likelihood of dynamical homogeneity. Sources of systematic error in the mass-to-light ratio are less amenable to simple arguments, but should certainly include the dispersion in the average star formation epoch of early-type galaxies found in an earlier analyses of galaxy evolution with lenses by Rusin & Kochanek (2005) and Treu et al. (2006).

If we simply analyze the data to determine the most likely systematic errors, we find that we must include fractional systematic errors of approximately 10\% in the velocity dispersion estimates and 19\% in the mass-to-light ratio estimates in order to make the sample consistent with the hypothesis of homogeneity.

Once we account for the inhomogeneity or systematic errors in the sample, we can evaluate sample averages that properly account for these problems. We find that the halo mass fraction represented by the baryons in stars is \(f_* = 0.056 \pm 0.011\) if we adiabatically compress the dark matter and \(f_* = 0.026 \pm 0.006\) if we do not. These results are comparable to similar the range of estimates that relied on stellar
population models to estimate the stellar mass. For example, Lintott, Ferreras & Lahav (2006) obtained a stellar mass fraction of \( \approx 8\% \) by fitting monolithic collapse models to 2000 SDSS galaxies, Hoekstra et al. (2005) found \( f_* = 0.065^{+0.010}_{-0.008} \) using weak lensing, and Mandelbaum et al. (2006) found \( f_* = 0.03^{+0.020}_{-0.010} \) using weak lensing. The results in these studies depend on the assumed IMF – the Hoekstra et al. (2005) estimate drops to \( f_* = 0.035^{+0.005}_{-0.004} \) if the initial mass fraction of the stars is changed from a standard Salpeter IMF to a scaled Salpeter IMF. Our results probably bound the stellar mass fraction since the Blumenthal et al. (1986) model we used may overestimate the amount of adiabatic compression (Gnedin et al. 2004). In all our models, the stellar mass fraction is much smaller than the cosmological baryon mass fraction \( \Omega_b/\Omega_m = 0.176^{+0.006}_{-0.019} \) from WMAP (Spergel et al. 2006), which means that the star formation efficiency \( (f_*\Omega_m/\Omega_b) \) of early-type galaxies is only 15–30\%. The remaining baryons must remain as gas distributed on the scale of the halo or its parent (group) halo. This discrepancy appears to be a common problem for any baryon accounting for normal galaxies (e.g. Fukugita (2004)) and a significant constraint on star formation efficiency.

Analysis of the evolution of early-type galaxies with redshift, whether using the fundamental plane (e.g. Jørgensen et al. (1999); Franx et al. (2000); Treu et al. (2006)), dynamical mass estimates (e.g. van der Marel & van Dokkum (2006)), or gravitational lens data alone (e.g. Rusin & Kochanek (2005)), have consistently observed a steady brightening of early-type galaxies with look-back time, albeit
with modest disagreements as to the rate. Here we use a hybrid method, fitting
the stellar mass-to-light ratios inferred from mass models of gravitational lenses
with stellar dynamical data, to find that \((M_*/L)_0 = (7.2 \pm 0.5)M_\odot/L_\odot\) and
\(d(\log(M_*/L))/dz = -0.72 \pm 0.08\) for the compressed models. The mass-to-light
ratio is comparable to the local value of \((M_*/L)_0 = (7.3 \pm 2.1)M_\odot/L_\odot\) from
Gerhard et al. (2001) which was adopted by Treu et al. (2006) in their analysis
of this data. The mass-to-light ratio evolution rate is also close to the value
\(d\log(M_*/L)/dz = 0.69 \pm 0.08\) from Treu et al. (2006), and marginally larger than
the estimates by Rusin & Kochanek (2005) and van der Marel & van Dokkum
(2006). As pointed out by Rusin & Kochanek (2005), the differences in evolution
rates are partly due to different approaches to weighting the contribution of each
lens to the analysis, but at least our Bayesian approach carries out these weightings
in an objective fashion.

In our basic analysis we cannot distinguish between the adiabatically compressed
and uncompressed models. In essence, we can obtain the same mass distribution
either using a high stellar mass-to-light ratio and a more extended halo or the
reverse. If we impose the locally estimated mass-to-light ratio as a constraint,
then the adiabatically compressed model is favored (6 to 1). We can also use mass
measurements on much larger scales to distinguish the two models because the total
halo mass is larger in the uncompressed model. In particular, we can calculate the
weak lensing \(\Delta\Sigma\) and compare it to the measurement by Gavazzi et al. (2007, also
see Mandelbaum et al. 2006) for an overlapping sample of SLACS lenses where they found that $\Delta \Sigma = (100 \pm 30)hM_{\odot}pc^{-2}$ on scales of $94h^{-1}$ kpc. With this constraint the adiabatically compressed models are again strongly favored (by 1000 to 1). In general, any third constraint that is dominated by the contribution from one mass component will break the degeneracy and lead to constraints on the degree of adiabatic compression or an additional structural variable such as the inner slope of the dark matter density distribution.

The sample of lenses available for such analyses will continue to grow and can include lenses with time delay measurements (which constrain the halo structure by measuring the surface density near the lensed images, Kochanek 2002) as well as those with velocity dispersions. With larger samples it should be possible to explore additional correlations such as the scaling of the stellar mass fraction and mass-to-light ratios with halo mass and the dependence of the evolution rate on halo mass. In SAURON project, Cappellari et al. (2006) investigated the mass-to-light (I band) ratios of the elliptical and lenticular galaxies, and they found the ratios increases with the velocity dispersions of the systems by $M/L = (3.80 \pm 0.14) \times (\sigma_e/200\text{km s}^{-1})^{0.84\pm0.07}$. Although the luminosity band is different from what we use here, it might imply that the mass-to-light ratios for the early-type galaxies are not uniformed everywhere. In the Mandelbaum et al. (2006) and Padmanabhan (2004) analyses of early-type galaxies in the SDSS, the changes in the mass-to-light ratio with halo mass are due to an increasing dark matter fraction.
with mass rather than changes in the stellar populations, but their results depend on population synthesis models to correctly estimate the stellar masses. In a larger sample of lenses, this could be tested directly. van der Marel & van Dokkum (2006) and Treu et al. (2006) see some evidence for differential evolution with mass, but significantly larger samples will be needed to test this given the sensitivity of even the present results to sample weighting.
Fig. 3.1.— The goodness of fit to the velocity dispersion as a function of stellar mass fraction $f_*$ and concentration $c$. The solid curves are drawn at $\Delta \chi^2 = 1(1\sigma)$ for the fit to the velocity dispersion and the dotted lines are drawn at $\Delta \chi^2 = 4, 9$ and $16 (2, 3, \text{and} 4 \sigma)$. When there is no region with $\Delta \chi^2 < 1$, we label the lowest contour present. The roughly vertical pair of solid lines indicate the $1\sigma$ range of concentrations given the halo mass at each point. The inset text identifies the object, the measured velocity dispersion and the $\chi^2$ of the best fit. These are the isotropic ($\beta = 0$) adiabatically compressed models that include our estimate of the systematic uncertainties in the stellar dynamical measurements.
Fig. 3.2.— The goodness of fit to the velocity dispersion as a function of stellar mass fraction $f_*$ and concentration $c$ (continued). See Figure 3.1 for details.
Fig. 3.3.— The goodness of fit to the mean trend in the stellar mass-to-light ratio as a function of the stellar mass fraction and the concentration $c$. The solid curves are drawn at $\Delta \chi^2 = 1$ (1$\sigma$) for the fit to the mass-to-light ratio and the dotted lines are drawn at $\Delta \chi^2 = 4, 9$ and 16 (2, 3, and 4 $\sigma$). The inset text identifies the object. These are the isotropic ($\beta = 0$) adiabatically compressed models that include our estimates of the systematic uncertainties in the stellar dynamical measurements. For these figures, we use the best fit evolution model – the fits would improve if we included the measurement errors in the evolution model.
Fig. 3.4.— The goodness of fit to the mean trend in the stellar mass-to-light ratio as a function of the stellar mass fraction and the concentration c. See Figure 3.3 for details.
Fig. 3.5.— The probability distribution of the fractional systematic errors $e_\sigma$ and $e_L$ in the velocity dispersion and mass-to-light ratio. The contours encompass 68%, 95% and 99.7% of the probability starting from the maximum likelihood solution indicated by the triangle. The crosses indicate the measurement errors from from Treu et al. (2006,?); Koopmans et al. (2006) and the rectangles are the modified errors. This figure changes little if we use the power law distributions.
Fig. 3.6.— The likelihood distributions for the probability that the galaxy sample is homogeneous in its dynamics ($p_{\sigma}$) or its evolution ($p_{L}$). The contours encompass 68%, 95% and 99.7% of the probability. The solid contours use the measurement errors for the dynamical uncertainties while the dashed contours include our estimate of the systematic uncertainties in the dynamical measurements. In both cases we used an adiabatically compressed, isotropic ($\beta = 0$) model.
Fig. 3.7.— The probability distribution for the stellar mass fraction $f_*$ for tangentially anisotropic ($\beta = -0.33$, top panel), isotropic ($\beta = 0$, middle), and radially anisotropic ($\beta = 0.33$, bottom) dynamical models. The thin lines in each panel show the weak constraints found for the individual galaxies, and the thick solid line corresponds to the joint probability from combining the sample. These models are adiabatically compressed using the modified errors and include the fit to the mass-to-light ratios. The thick dashed line in the middle panel shows the effect of not including the adiabatic compressions.
Fig. 3.8.— The probability distribution for density slope $\gamma$ under the power law mass density model with the modified error bars. The thin lines show the slope distribution for each of the individual galaxies and the thick line corresponds to the joint density distribution of $\gamma$. 
Fig. 3.9.— The probability distributions for the local mass-to-light ratio \((M_*/L)_0\) and its evolution \(d \log (M_*/L)/dz\) in the adiabatically compressed (top) and uncompressed (bottom) models. The contours show the 68%, 95%, and 99.7% enclosed probability contours for the isotropic models. The estimated evolution rate is marginally inconsistent with the estimated of \(d \log (M_*/L)/dz = -0.50 \pm 0.19\) from Rusin & Kochanek (2005) which is shown by the horizontal band of solid and dashed lines. The three triangles in each panel show the effect of changing the isotropy on the likelihood peak, with \(\beta = -0.33\), \(\beta = 0\), and \(\beta = 0.33\) as we move from upper left to lower right. The squares with error bars are the results from Treu et al. (2006,?) the same galaxies.
Fig. 3.10.— The range for the 3 dimensional density slope exponent $\gamma$, where $\rho \propto r^{-\gamma}$, for the typical lens SDSS J0037–0942. We estimated $\gamma$ by fitting the projected mass distribution as a power law between $R_e/8$ and $R_E$. Note that the variation of $\gamma$ over the physically interesting regime is comparable to the scatter observed by Koopmans et al. (2006) of $1.8 \lesssim \gamma \lesssim 2.3$. 
<table>
<thead>
<tr>
<th>Objects</th>
<th>$z_l$</th>
<th>$z_s$</th>
<th>$R_e$</th>
<th>$R_E$</th>
<th>$M_E$</th>
<th>$\sigma_{ap}$</th>
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<tr>
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(cont’d)

Table 3.1. Lens property data I
Table 3.1—Continued

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<th>$R_E$</th>
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<tr>
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<tr>
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<td>0.31</td>
<td>13.70</td>
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<td>304±27(47)</td>
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</table>

Note. — The observational properties of the lenses. $z_l$ and $z_s$ are the lens and source redshifts, $R_e$ is the lens effective radius (arc minute), $R_E$ (Kpc) and $M_E$ ($10^{10}M_\odot$) are the lens Einstein radius and Einstein mass, $\sigma_{ap}$ (kms$^{-1}$) is the measured velocity dispersion. We include both the measurement errors inside the listed aperture and our estimated systematic uncertainties are in (brackets). We lacked the seeing FWHM for SDSS J1627−0053 and SDSS J2300+0022 and simply used the average value for the other SDSS objects.
<table>
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<th>seeing</th>
<th>$f_{dmw}$</th>
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Table 3.2. Lens property data II
Table 3.2—Continued

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<th>$f_{dm\text{n}}$</th>
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<td>0.84</td>
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<tr>
<td>MG1549+305</td>
<td>0.17±0.02</td>
<td>1.0×4.3</td>
<td>0.65</td>
<td>0.60</td>
<td>0.46</td>
<td>11,12</td>
</tr>
<tr>
<td>MG2016+112</td>
<td>1.60±0.08</td>
<td>0.65</td>
<td>0.7</td>
<td>0.44</td>
<td>0.26</td>
<td>7</td>
</tr>
</tbody>
</table>

Note. — The observational properties of the lenses, the telescope parameters of the observations, and the references. $f_{dm\text{c}}$ is the projected dark matter fraction inside the Einstein radius for adiabatically compressed ("dm\text{c}") or not compressed ("dm\text{n}") models. $L_B$ is in unit of $10^{10}L_{B,\odot}$, and "Aperture" and "seeing" sizes are in units of arc minute.

Treu et al. (2006)(1), Koopmans et al. (2006)(2), Treu et al. (2003)(3)
Keeton et al. (1998)(10), Lehár et al. (1993)(11), Lehár et al. (1996)(12)
Chapter 4

Optical Cluster Observables as Indicators of Halo Mass

4.1. Introduction

The mass function of rich galaxy clusters plays an important role in constraining cosmological parameters, in part because the mass structure of dark matter halos is dominated by gravitational processes, and insensitive to the complex physics of galaxy formation such as star formation, and feedback. Hence, the understanding of the halo mass function is more robust than the understanding of the galaxy distribution and properties. Strong constraints on the density fluctuation normalization $\sigma_8$ and the mass density parameter $\Omega_m$ can be achieved by comparing the theoretical mass function (e.g. Press & Schechter (1974); Jenkins et al. (2001)) with the observations. The reason that the halo mass function is sensitive to the current $\sigma_8$ value is that it determines the mass scale of fluctuations that are collapsing today (White et al. 1993). Data from high redshift supernovae (e.g. Riess et al. (1998); Perlmutter et al. (1999)) implies that the universe is accelerating, indicating either a breakdown of General Relativity in cosmological scales or an energy component (known as “dark
energy”) that exerts repulsive gravity. The evolution of the halo mass function is potentially a sensitive diagnostic for testing theories of dark energy. (e.g. Huterer & Turner (2001); Haiman et al. (2001); Levine et al. (2002)).

However, halo masses are not directly observed, and the inference of the halo mass function comes from measuring observable mass indicators. Observables include optical richness or luminosity (e.g. Bahcall et al. (2003)), galaxy velocity dispersion (e.g. Becker et al. (2007)), X-ray luminosity or temperature (e.g. Stanek et al. (2007)), Sunyaev-Zel’dovich (S-Z) decrement (e.g. Sunyaev & Zeldovich (1972); Annis et al. (2005)), and weak lensing strength (e.g. Johnston et al. (2007); Sheldon et al. (2007)). Each of them has observational error and physical scatter, and uncertainty in calibration. Optical richness or luminosity depends on our definition of clusters. The members which are loosely connected to the cluster or the galaxies behind or in front of the cluster might be miscounted. X-ray luminosity is obtained. Dynamical equilibrium is assumed from X-ray luminosity to derive cluster masses. Equilibrium may not be fully achieved in the massive cluster gas. The measurement of galaxy velocity dispersion depends on the location and types of the measured galaxies. S-Z effect assumes the electrons in the cluster are in thermal equilibrium and ignores the peculiar velocity of the cluster itself. The influence of radio sources behind or inside the clusters might also disturb the Cosmic Background Radiation (CBR) used to measure the S-Z signal. The signal from weak lensing is weak and it may be contaminated by the background galaxies and clusters.
An emerging strategy is to take a sample selected based on one mass indicator such as optical richness, then measure average mass profiles with weak lensing (Sheldon et al. 2007). For this purpose, an indicator with small and predictable scatter is wanted, but a prior calibration is not needed because this will come from weak lensing. This chapter examines scatter between optical observables and mass predicted by semi-analytic galaxy formation models applied to the Millennium Run Simulation (MRS) of Springel et al. (2005), assuming halo finding itself is perfect. This approach isolates physical scatter in the models from observational scatter introduced by cluster finding itself.

In this chapter, we used the galaxies obtained by semi-analytic (SA) models of Bower et al. (2006) and Delucia & Blaizot (2007) based on MRS halo merger trees (Springel et al. 2005; Springel 2005; Lemson et al. 2006) to investigate the relation between the observables and halo mass. The exploration could be used in future cosmological estimation and shed light on the galaxy formation, star formation, and feedback processes when compared with observed data.

From a theoretical perspective, the physical mechanism of galaxy formation are poorly understood. Smoothed particle hydrodynamical (SPH) simulations can predict both the distributions of galaxies and dark matter halos, but they are computationally expensive, making it difficult to test the effects of different physics input in the simulation. SPH simulations also cannot simulate the large volumes needed to produce cluster populations with the resolution needed for galaxy
formation. SA modeling is an alternative based on the ΛCDM simulated halos, the formation of which is dominated by gravitational processes and insensitive to galaxy formation physics. Recently, the halo merger trees of the MRS (Lemson et al. 2006) have become a good platform to apply the SA method with different galaxy formation models. MRS combines a large simulation volume with high resolution. It was carried out by the collaboration of Max-Planck-Institute for Astrophysics and the international Virgo consortium. The simulation was computed in a periodic box of 500 $h^{-1}$Mpc on one side, and there are a total of 10 billion particles with mass of $8.6 \times 10^8 h^{-1} M_\odot$. The resolution is enough to represent dwarf galaxies by a hundred particles, and the box is large enough to generate the richest observed clusters of galaxies with several million particles. Assuming a flat universe, Millennium Run Simulation simulated the halo forming history with $\Omega_b = 0.045$, $\Omega_m = \Omega_{CDM} + \Omega_b = 0.25$, and $\Omega_\Lambda = 0.75$. The Hubble parameter $H_0 = 73 \text{km s}^{-1} \text{Mpc}^{-1}$ and the power spectrum normalization $\sigma_8 = 0.9$ were chosen.

In §4.2, I explained how we selected galaxies and clusters from the catalogs and compare the data from Bower et al. (2006) with that from Delucia & Blaizot (2007). In §4.3, I obtained the relation between different observables and halo mass, supplement from central galaxies. §4.4 investigated the changes of the relations with redshift and different galaxy magnitude cut-off. Finally, in §4.5, I concluded our investigation of the project.
4.2. Galaxy and halo catalogs

The ΛCDM cosmological model has many successes in reproducing cosmological observations, including microwave background fluctuations seen at $z \approx 1000$, the power spectrum of the low-redshift galaxy distribution, and the Hubble diagram of Type Ia supernovae. However, numerical or SA models of galaxy formation in ΛCDM still show discrepancies with observations, especially the results before Croton et al. (2006). First of all, the shape of the simulated luminosity function is different from that of the observation, predicting too many galaxies at the faint and bright ends of the luminosity function. To solve these problems, cooling inefficiencies in massive systems and star formation, supernova feedback in low-mass systems were introduced (White & Rees 1978; White & Frenk 1991). The scarcity of galaxies at very low mass could be explained by photoionization heating (Efstathiou 1992), while the bright-end exponential cut-off is hard to be suppress by cooling effects alone (Thoul & Weinberg 1995). Secondly, the model predictions cannot match the observed high redshift and local galaxies simultaneously (Baugh et al. 1998, 2005; Somerville et al. 2001). The other closely related problems are the observed central galaxy tending to be red and old, and the apparent absence of cooling flows at the centers of the rich clusters (Tamura et al. 2001; Peterson et al. 2001). Motivated by the observation (Burns et al. 1981) that every cluster with a strong cooling flow contains a massive and active central radio galaxy, Tabor & Binney (1993) pointed out that radio galaxies might regulate the the cooling flows. Croton et al. (2006)
insert a semi-empirical model describing the feedback from AGN in their SA model, which efficiently suppresses the star formation in the massive halo systems.

Bower et al. (2006) and Delucia & Blaizot (2007) carried out the similar SA simulations by inserting AGN feedback with varying prescriptions and generated different galaxy catalogs. Although Bower et al. (2006) focus on the explanation of the evolution of the galaxy luminosity function and Delucia & Blaizot (2007) emphasize the formation and evolution of brightest cluster galaxies (BCGs), both of them produced an exponential cut-off of the number of galaxies in the high luminosity end. While both of the SA models are based on the dark matter distribution of the Millennium simulation, neither the halos nor the SA models are the same. The halos in Bower et al. (2006) are found by the friends-of-friends (FoF) SUBFIND algorithm (Harker et al. 2006; Springel et al. 2001), using a linking length of $b = 0.2$. A halo is split into two or more pieces if the substructure is outside twice the half-mass radius of the main halo, and the subhalo has retained more than 75% of the mass it previously had when it was an independent object in the process of constructing merger tree. The resulting halo catalog is larger than the initial FoF halo catalog. The Delucia & Blaizot (2007) halos are found by the same FoF SUBFIND algorithm but with a different criterion about “first progenitors”. The total number of halos in Delucia & Blaizot (2007) is only about 55% of that in Bower et al. (2006), for halos more massive than $10^{13.5} h^{-1} M_\odot$ and hosting at least one galaxy brighter than $r$-band absolute magnitude $M_r = -19$. However, the
shapes of the mass functions of the two catalogs are the similar as is displayed in Fig. 4.1. At higher redshifts, the numbers of halos in the two catalogs are closer.

In terms of SA modeling, the two calculations used different star formation rates, different stellar initial mass function (IMF), and different models for AGN feedback. Especially in the treatment of AGN feedback, Bower et al. (2006) assume the feedback flow can balance the heating and cooling automatically, whenever the AGN Eddington luminosity is large enough, whereas Croton et al. (2006) and Delucia & Blaizot (2007) use a black hole related empirical model describing the AGN feedback. The above differences in SA models of the two catalogs lead to different color-magnitude and halo mass-observables relations, which will be investigated later. Comparing the two catalogs gives a good sense of which aspects of our results are robust to the treatment of halo finding and galaxy formation physics and which aspects are sensitive to these details.

In this chapter, we would like to investigate the observable-halo mass relations in rich clusters. Therefore, we only choose the halos more massive than $10^{13.5}h^{-1}M_\odot$ in both catalogs. To explore the influence of the luminosity cut-off of the galaxy sample, I chose samples with absolute magnitude thresholds $M_r=-19$, -20, and -21 in the rest-frame SDSS $r$-band. I am also interested in the evolution of the relations, so different sets of data at redshift $z = 0$, $z = 0.3$, and $z = 0.5$ were chosen. Thus, for the data sets drawn from each catalogs, there are nine sets of data to be investigated. We focus most of our analysis and discussion on the $z = 0.3$ and
Mr < −20 catalogs and discuss other results in relation to these. I chose $z = 0.3$
other than $z = 0$ because most large cluster samples are centered at moderate
redshifts. In the following, “Durham catalog” refers to the data from Bower et al.
(2006) and MPA catalog stands for the data from Delucia & Blaizot (2007).

Before doing the exploration, we investigated the color-magnitude relations
for both the catalogs. In Fig. 4.2, the left panels, from top to bottom, show the
central galaxy, satellite, and all galaxy r magnitude and $g - r$ color relation from
Durham catalog, while the right panels show the same information for the MPA
catalog. When not specified otherwise, the analysis redshift is $z = 0.3$. In the
upper four panels, only 0.5% of the satellites and 3% central galaxies are randomly
selected and plotted, and the median color at each given magnitude is displayed by
the lines. The bottom two panels show the color-magnitude relations for all the
galaxies, with the lines corresponding to 5%, 10%, 20%, 50%, 80% and 90% color
quantiles of the galaxies. Only the 10% reddest and 5% bluest galaxies are shown
individually for galaxies fainter than $Mr = -22$. The color in the Durham catalog
slightly increases with galaxy brightness and goes down at the very bright end of the
galaxies, while the MPA data shows a relatively flat color-magnitude relation, with
a very slight blueward dip at the bright end. Despite the differences, neither catalog
show an obvious division between the blue and red galaxies because in halos more
massive than $10^{13.5} M_\odot$ both simulations only generate red galaxies. The blue galaxy
population (dominant by number overall) resides almost entirely in lower mass halos.
In many cluster finding algorithms, clusters and clusters members are identified using just the red galaxy population, but these models predict that these massive halos contain only red galaxies in any case. In the following analysis, therefore, we will not divide the galaxies into blue and red, just look at them as one group. We sum the luminosities of all galaxies in the cluster to compute the cluster luminosity. There are 11,862 clusters in the Durham catalog and 8212 clusters in the MPA catalog.

4.3. The Relations between Optical Observables and Halo Mass

4.3.1. Galaxy Richness, Cluster Luminosity and Halo Mass

For both of the catalogs, scatter plots of the total cluster luminosity $L_{\text{tot}}$, total cluster stellar mass $M_g$, galaxy richness $N_g$ and average galaxy luminosity $< L >$ in the cluster are shown in Fig. 4.3. To avoid saturation, we plot points for individual halos only above $M_h = 3 \times 10^{14} M_\odot$ and for the highest and lowest 10% of the halos at fixed mass below. Lines show the median relations in halo mass bins. The cluster luminosity is in units of $L_\odot$. It is clear that the median trends of all the relations are well described by power laws, although the scatter about the median changes with
halo mass. Weighted by the number of clusters in each halo mass bin, a least square fit of the observables given the halo masses is carried out. The observables in the Durham catalog can be explained by the halo mass in the following manner,

\[
\log \left( \frac{L}{L^*} \right) = (1.711 \pm 0.040) + (0.939 \pm 0.003)(\log M_h/M_\odot - 14.3),
\]

\[
\log (N_g) = (1.521 \pm 0.041) + (0.963 \pm 0.003)\left[ \log M_h/M_\odot - 14.3 \right].
\] (4.1)  

In the MPA catalog, the observables changes more slowly with the halo mass,

\[
\log \left( \frac{L}{L^*} \right) = (1.784 \pm 0.071) + (0.899 \pm 0.005)(\log M_h/M_\odot - 14.3),
\]

\[
\log (N_g) = (1.642 \pm 0.077) + (0.936 \pm 0.006)(\log M_h/M_\odot - 14.3).
\] (4.2)  

We do not give the inverse fit of halo mass given the cluster observables, because our samples are cut off at \( M_h = 10^{13.5} M_\odot \) and miss lower mass clusters at fixed cluster luminosity or galaxy richness. If we restrict span of luminosity or richness so that the cluster population is complete above the threshold, we get inverse fits consistent with those above.

The normalizations of the fits have a good agreement with, although the slopes are only roughly consistent with Johnston et al. (2007). Both of the slopes in the two catalogs, whether for cluster luminosity and halo mass or galaxy richness and halo mass, are steeper than those given by Reyes et al. (2008). They obtained slopes of 0.83 and 0.65 for cluster richness and luminosity, respectively, as a function of halo
mass. The discrepancy might arise from inaccuracies of the SA galaxy formation models or from observational errors in the halo mass and cluster luminosity. However, the analysis method might also contribute to the discrepancy: when Reyes et al. (2008) calibrated the relations, they did not use the observables and halo mass for each cluster, but binned the data in only six bins. Our slopes are also much shallower than that found from X-ray data, e.g. 1.65 given by Rykoff et al. (2008). This is not surprising, since the X-ray luminosity represents the luminosity from the hot gas in the cluster center, which is more sensitive to the depth and shape of the potential well.

We also show the average galaxy luminosity trend with halo mass in rich clusters in last two panels of Fig. 4.3. As shown by the median, which is the line in the bottom panels, the mean galaxy luminosity does not change much with halo mass. It implies that the high mass halos tend to have more galaxies rather than systematically more luminous galaxies. We examine this point more systematically in §4.3.3

4.3.2. Scatter in Mass-\(L\) and Mass-\(N\) Relations

The scatter of the observables given halo mass is a diagnostic of the “irregularities” associated with galaxy formation. Fig. 4.3 shows similar scatter of stellar mass and cluster luminosity in both the Durham and MPA catalogs. If we
decompose the scatter of the cluster luminosity into two parts, with one counted by stellar mass scatter given halo mass and the other by the luminosity scatter given stellar mass, we find that the luminosity scatter comes almost entirely from the stellar mass scatter; the scatter in stellar mass-to-light ratio at fixed halo mass contributes negligibly. This result is expected when the galaxy star formation is relatively independent of the stellar dynamics of the galaxies. In these models, therefore, the galaxy star formation processes are more homogeneous than the galaxy dynamical process.

The scatter of the halo mass given the observables determines how well we can constrain the halo mass. From Fig. 4.3, we see that the scatter of halo mass given the observables is not constant at different cluster luminosity or galaxy richness, and that the MPA model predicts larger scatter than the Durham model. To estimate the scatter of halo mass, we divide the data into 100 cluster luminosity or galaxy richness bins, with each bin having the same size. As seen in the upper panels in Fig. 4.3, the cluster catalog is incomplete at low mass end, because of the mass cut-off when we selected the halos. We therefore cut out the low luminosity or low galaxy richness clusters and only estimate the halo mass scatter where the data are complete. In each of the cluster luminosity or galaxy richness bins, we estimate the scatter of the halo mass, and we drop all the bins with fewer than five members. As shown in Fig. 4.4, the scatter of halo mass in the Durham and MPA catalog exhibit different patterns. The scatter of the halo mass in the Durham catalog decreases
with the cluster luminosity or galaxy richness, while the scatter of halo mass even slightly increases with these observables that in the MPA data. The scatter in the Durham catalog is smaller than in the MPA catalog. Both of the catalogs show a slightly narrower scatter of halo mass at given galaxy richness than that at given cluster luminosity. We also did a quadratic fit the scatter of halo mass as the function of the observables, which are shown as the solid lines in each of the panels in Fig. 4.3. For the Durham data we find,

\[ \sigma_{\log M_h}(L) = 0.394 - 0.278 \log(L/L_*) + 0.057 \log(L/L_*)^2, \]  
\[ \sigma_{\log M_h}(N_g) = 0.508 - 0.473 \log(N_g) + 0.123 \log(N_g), \]  

where \( \sigma_{\log M_h}(L) \) and \( \sigma_{\log M_h}(N_g) \) denote the scatter (index) of the logarithm of halo mass at fixed luminosity or richness and all logarithms are base 10. For the MPA data we find,

\[ \sigma_{\log M_h}(L) = -0.079 + 0.241 \log(L/L_*) - 0.062 \log(L/L_*)^2, \]  
\[ \sigma_{\log M_h}(N_g) = 1.366 - 1.532 \log(N_g) + 0.482 \log(N_g). \]  

The bin-to-bin scatter of the dispersion at the high luminosity and galaxy richness end is large because of the scarcity of high mass clusters in the finite simulation volume.

Since we start from a “true” halo catalog identified in three dimensions, these results show just the physical scatter expected from the varying histories of halo
assembly and associated galaxy formation. Compared with the scatter from Johnston et al. (2007), the scatter from the Durham data is a little bit small, and the MPA data has a larger scatter than the observation. The observations show a decreasing trend of the scatter in the observables with increasing cluster mass consistent with the information obtained from the Durham data as shown in Fig. 4.4.

The scatter from both the catalogs is smaller than the 0.43 dex, which is obtained by the X-ray data by Stanek et al. (2007). There might be several possible reasons to account for the discrepancy. First of all, the optical observation is related to the galaxies, while the X-ray data is closer to the hot gas in the clusters. Secondly, Stanek et al. (2007) determined the scatter of the halo mass based on the relationship between the halo mass and cluster temperature, which is itself poorly constrained, and the scatter is sensitive to the adopted relation. Furthermore, constant variance of the halo mass at given cluster luminosity or galaxy richness is assumed, which might not be true. In this sense, weak lensing and optical observable combination might be a good solution to the scatter problem.

It’s also interesting to estimate the observable scatter at given halo mass, as shown in Fig 4.5. The upper two panels show the scatter of cluster luminosity at given halo mass, and the lower two panels show that of galaxy richness at given halo mass. The solid lines are the quadratic fit of the scatter, and the dotted lines in the two bottom panels are the Poisson scatter with the galaxy richness estimated from the fixed halo mass. Left panels are for the Durham model and right panels are for
the MPA model. The values and patterns of the scatter of luminosity and galaxy richness in either of the models are similar. However, the scatter decreases with halo mass more sharply in the Durham model than in MPA model. The Durham model obtained a close to Poisson scatter, with the scatter in low mass halo is slightly lower. However, the MPA model results in a larger scatter from Poisson distribution.

4.3.3. Distributions of the Observables

A log-normal relation between observables and halo mass is usually assumed (e.g. Kaiser 1986). Here, we would like to check the distributions of cluster luminosity and galaxy richness around each given halo mass, to test the accuracy of this this assumption. Fig. 4.6 and Fig. 4.7 show the distribution of the cluster luminosity and galaxy richness at fixed halo mass respectively. In both the catalogs, the clusters are divided into three halo mass ranges $10^{13.5} M_{\odot}$, $10^{13.9} M_{\odot}$, $10^{14.3} M_{\odot}$ and more massive than $10^{14.3} M_{\odot}$. The histograms are the distributions of the observables around the fitted values and standardized by the fitted scatter, and the lines in each of the panels correspond to normal distributions. The distribution with r-band magnitude cut at -19, -20 and -21 are investigated. The upper three panels are from Durham data, and lower three panels are for MPA data. Strictly speaking, neither of the observables is distributed normally at fixed halo mass in either of the data sets. However, the distributions are not very far from log-normal, and the largest deviations in the histogram are in cases where the number of clusters is limited.
Because of the scatter of the halo mass at a given luminosity or richness, the average mass of the N richest/most luminous clusters in a given volume is lower than that of the N most massive clusters (which can be predicted directly from the halo mass function). To evaluate the bias, we chose four mass thresholds, $10^{13.8} M_\odot$, $10^{14.0} M_\odot$, $10^{14.2} M_\odot$ and $10^{14.5} M_\odot$. For dark energy analysis with clusters, Yoo (PhD Thesis) advocates measuring (via weak lensing) the mean mass of clusters above thresholds in richness or luminosity, similar to the approach of Sheldon et al. (2008), though they examine richness/luminosity bins instead of thresholded samples.

Fig. 4.8 shows the average halo mass ratio of the most luminous clusters over that of the most massive halos versus average halo mass of the most massive halos, where the luminosity threshold is chosen to give an equal number of clusters. If the ratio is 1, there is no bias. The triangles are estimated from Durham model and solid points are from MPA model. The left, middle and right columns correspond to absolute $r$ magnitude $M_r$ less than -19, -20, and -21 respectively. From top to bottom the data are for redshift $z = 0$, $z = 0.3$, and $z = 0.5$. In most of the cases, the bias is less than 1% for both the Durham and MPA catalogs, as shown in Fig. 4.8. The bias in the Durham model is smaller than that in the MPA model, because of the smaller scatter. Also as expected, the bias obtained from the high mass end in Durham data is smaller than that obtained in the low mass end, while MPA data behaves oppositely. The bias increases with redshift. Furthermore, the bias increases with only brighter galaxies selected, especially in Durham catalog. All the above results
are consistent with the mass scatter trend at given the given cluster luminosity in the both of the catalogs. The bias estimated by matching the galaxy richest clusters to the most massive halos have the similar results, as shown in Fig. 4.9.

4.3.4. Galaxy Luminosity and Halo Mass-to-light Ratio

Fig. 4.10 shows the average luminosity of galaxies as a function of richness in narrow (±0.1dex) bins of halo mass centered at $\log M_h/M_\odot = 13.8, 14.4, \text{ and } 14.8$. The left panels show Durham data and the right three panels MPA data. As already seen in Fig. 4.3, the MPA clusters have significantly higher richness at fixed halo mass, but the average luminosity of galaxies (above the $M_r = -20$ threshold) is similar. The striking feature of Fig. 4.8 is the insensitivity of $\langle L \rangle$ to $N_g$ for massive clusters; for the two upper mass bins, $\langle L \rangle$ is essentially flat over a range of two or more in richness at fixed halo mass. The low richness clusters are not a consequence of galaxy mergers that pack similar numbers of baryons into a smaller number of galaxies. At the low richness end of the $10^{13.8}M_\odot$ bin, there is a trend of higher mean luminosity at low richness, but it largely disappears above $N_g \approx 10$.

In Fig. 4.11, the top, middle, and bottom panels show central galaxy luminosity, satellite galaxy luminosity, and mass-to-light ratio as a function of halo mass. In the $L_c$ vs. $M_h$ panels, the lines, from bottom to top, correspond to 10%, 20%, 50%, 80% and 90% percentiles of $L_c$ at each given halo mass. In the middle panels, the dotted
"lines" show the same percentile levels for the luminosities of all satellite galaxies in each mass bin. The left panels are for the Durham data and the right ones are for the MPA data. As expected, more massive halos host more luminous central galaxies on average. The correlation is considerably tighter for the MPA data, though even here the 10%-90% range in $L_c$ at fixed $M_h$ is roughly a factor of three.

In contrast to central galaxies, the typical luminosity of satellites is strikingly insensitive to halo mass at all percentile levels, for both the Durham and MPA models. The explains why the average galaxy luminosity is almost independent of halo mass (Fig. 4.3, bottom), since satellites dominate the total luminosity and galaxy count. Except for the central galaxy itself, the ratio of the bright and dim galaxies is almost independent of halo mass. This is also the reason why the cluster luminosity and galaxy richness are correlated very tightly and neither of them is a much better indicator of the halo mass. The bottom two panels in Fig. 4.3 show the halo mass-to-light ratios in the two simulations. The Durham data has a slightly higher mass-to-light ratio averaged around $243 M_\odot / L_\odot h^{-1}$, compared to $195 M_\odot / L_\odot h^{-1}$ from MPA data. Although they are not consistent with each other, both of the results are consistent with those obtained by other authors (e.g. Carlberg et al. 1996; Tinker et al. 2005), but smaller than the $362 M_\odot / L_\odot h^{-1}$ obtained by Sheldon et al. (2007), if $h = 0.7$ assumed. The mass-to-light ratios do, however, depend on bandpass and on the absolute magnitude threshold, so these comparisons are only approximate.
Fig. 4.12 repeats the analysis of Fig. 4.10, but it shows central galaxy luminosity vs. richness in fixed mass bins instead of average galaxy luminosity. The central galaxy luminosity is remarkably insensitive to richness at fixed $M_h$; even in the lowest $M_h$ bin, any trend is weak compared to the scatter. Physically, this implies that in these models the halos with lower richness do not arise because of increased mergers of satellites with central galaxy, which would boost $L_c$ at low $N_g$. Observationally, it implies that the central galaxy luminosity provides almost no additional information about halo mass once the total richness and/or luminosity is known.

To further investigate this last points, we calculated the correlation matrix of halo mass, cluster luminosity, galaxy richness and central galaxy luminosity in the Durham data. We find a correlation matrix,

$$
\begin{pmatrix}
1 & 0.910 & 0.911 & 0.423 \\
0.910 & 1.0 & 0.950 & 0.485 \\
0.911 & 0.95 & 1 & 0.322 \\
0.423 & 0.485 & 0.322 & 1.0
\end{pmatrix}
$$

(4.9)

where the rows and columns are ordered as stated above. As expected, the correlation matrix shows that the halo mass is well correlated (and equally correlated) with both richness and luminosity, and that these two quantities are correlated with each other.
about as well as they are correlated with halo mass. The correlation of all three quantities with $L_c$ is certainly present, but it is much weaker. The MPA data is

$$
\begin{pmatrix}
1. & 0.861 & 0.841 & 0.566 \\
0.861 & 1.0 & 0.961 & 0.533 \\
0.841 & 0.961 & 1 & 0.417 \\
0.566 & 0.533 & 0.417 & 1.0
\end{pmatrix},
$$

(4.10)

These theoretical results appear somewhat at odds with those of Reyes et al. (2008), based on weak lensing analysis of SDSS clusters. They find a best-fit dex evidence of halo mass on $N_g$ and $L_{tot}$ that includes a substantial dependence on $L_c$. The fitting procedure is different from that used here, so it is not clear that there is a conflict of results. However, this difference could indicate that these models predict too much scatter in the $L_c - M_h$ relation. Alternatively, it could be that the observationally estimated richness is a noisy indicator of the true richness, allowing central galaxy luminosity to provide useful additional information about halo mass.
4.4. Redshift and Magnitude Cut-off Influence

Fig. 4.13 and Fig. 4.14 show how the relationship between the cluster observables and halo mass changes with r-band galaxy magnitude cut and redshift for Durham and MPA data respectively. All the solid lines are the best fit of the data at redshift $z = 0.3$, dotted line at $z = 0$, dashed line at $z = 0.5$. From top to bottom, the panel groups are the best log-normal fit between cluster luminosity and halo mass, the scatter of halo mass at given cluster luminosity, best log-normal fit between galaxy richness and halo mass, and the scatter of halo mass at given galaxy richness. As shown in the upper three panels of Fig. 4.13 and Fig. 4.14, the cluster luminosity especially the slope of the fit, does not change much over this range. Thus the relative mass-to-light ratio in the more and less massive clusters is almost stable since $z = 0.5$. The intercept of the cluster luminosity increases with redshift, which implies that the galaxies at $z = 0.5$ are more luminous than current galaxies, while the relations for galaxy richness are closer to each other at different redshifts. This result is consistent with the idea that much of the luminosity evolution is driven by passive evolution of stellar populations. There are fewer galaxies above brighter luminosity thresholds, as expected. The slopes of the relations are almost independent of luminosity cutoff, consistent with the almost flat satellite percentiles with halo mass as shown in Fig. 4.11.
The second row of panels shows the relation between halo mass errors and cluster luminosity, where the errors increase with brighter galaxies selected in each cluster. Such an increase is expected in part just because there are fewer bright galaxies, increasing the Poisson scatter. The halo mass errors grow most significantly at the low luminosity end, while the bright end is not influenced too much. The galaxy richness exhibits even less evolution than luminosity, in both intercept and slope. The slope of log$N_g$ vs. log$M_h$ is much shallower for the $M_r = -21$ cutoff, perhaps because the influence of central galaxies at low $M_h$ is more important for this brighter sample. The halo mass errors at each galaxy richness are slightly smaller than those at given cluster luminosity at each magnitude cut-off, and the evolution with redshift is also less obvious.

Fig. 4.14 conveys similar information from MPA data. The slope between cluster luminosity and halo mass does not change too much either with redshift or with different $r$ magnitude cut-off, but the intercepts separate more at different redshifts than those in Durham data. The scatter of halo mass at given cluster luminosity and galaxy richness, and the relation between galaxy richness and halo mass have the tendency of wider separation in MPA simulation. Redshift evolution of the properties of the clusters might be a good test to detect which simulation is more realistic. For scatter, the trend seen already in Fig. 4.4 holds at all redshifts and magnitude cutoffs: the scatter in the MPA catalogs is larger and less mass-dependent than in the Durham catalogs.
4.5. Conclusion

In this chapter, our analysis was based on the semi-analytic simulations given by Bower et al. (2006) and Delucia & Blaizot (2007). Both of the catalogs are based on the Millennium Run Simulation (MRS) carried out by (Springel et al. 2005; Springel 2005; Lemson et al. 2006). To investigate the relationship between halo mass and cluster observables, the halos more massive than $10^{13.5} M_\odot$ and the hosted galaxies brighter than -19, -20 and -21 in r-band absolute magnitude are selected. All galaxies in both of the catalogs are red galaxies. The halo mass and the cluster observable relationship might be approximated by a log-normal fit. The intercepts and the slopes of the fit based on Durham and MPA catalogs agree with each other with minor differences. The Durham data show no obvious evolution of the relationships from $z = 0.5$ to $z = 0$, while the MPA data show slightly more obvious evolution of the fits with redshift.

In both the Durham and MPA catalogs, the scatter of observables at fixed halo mass (and vice versa) depends on cluster luminosity and galaxy richness, but with different trends. Scatter in the Durham catalog decreases significantly with increasing halo mass, while the MPA’s increases very slightly. The scatter of halo mass at given cluster luminosity in MPA data is in the range of 10% to 20%, larger than 8% to 12% from the Durham data. The scatter for luminosity and richness is similar. The scatter becomes larger when only the brightest galaxies in the clusters
are selected. The evolution of the scatter is small since redshift $z = 0.5$. The cluster luminosity and galaxy richness are correlated with each other tightly, which is why neither of them has much narrower scatter than the other. While, the central galaxy luminosity is also positively correlated with the halo mass, the correlation coefficient is much smaller. At fixed halo mass, the central galaxy luminosity is almost independent of richness, so it does not provide useful additional information about halo mass. Our scatter estimates are close to the results from optical analysis by Reyes et al. (2008) and Johnston et al. (2007), but much smaller than that from the X-ray analysis of Stanek et al. (2007).

The bias between the average halo mass of halos above thresholds $10^{13.8} M_\odot$, $10^{14.0} M_\odot$, and $10^{14.5} M_\odot$. Compared to luminosity thresholds with the same number of clusters is small. The bias decreases with selected halo mass threshold in Durham data, and increases in MPA data, with no obvious redshift evolution, but it is influenced by the galaxy magnitude cut. All of these properties of the bias are as expected from the halo mass and observable scatter.

In general, these simulations present a rather simple picture of the cluster population. The scatter in luminosity is dominated by the scatter in the number of galaxies, with no significant trends of average luminosity vs. richness at fixed halo mass. If cluster finding and richness estimation is sufficiently accurate, then the scatter between cluster mass and optical observables can be as low as $0.05 - 0.15$ dex, making optically selected cluster samples a useful tool for cosmological investigation.
Fig. 4.1.— Halo mass functions, with the dashed line from MPA catalog and solid line from Durham catalog. The upper panel includes all halos, while the lower panel only includes the halo hosting central galaxies.
Fig. 4.2.— Color-Magnitude relations from Durham catalog (left three panels) and MPA catalog (right three panels). From top to bottom are central galaxy, satellite, and total galaxy color-magnitude relations, where only 0.3% of satellites and 1% of central galaxies are randomly selected for each of the catalogs. For the total galaxy data, only the upper 10% and lower 5% color percentile data at each given magnitude are displayed. The lines in each of the upper four panels correspond to the median colors at given magnitude for each data set. The lines in the last two panels are the 5%, 10%, 20%, 50%, 80%, and 90% percentile colors for each catalog total galaxies.
Fig. 4.3.— The relation between four optical observables and halo mass, with the left four panels for Durham data and the right four panels for MPA data. From top to bottom, the relations are for cluster luminosity, cluster stellar mass, galaxy richness, and average galaxy luminosity vs. halo mass. Lines show the median relation and points are plotted above 90% percentile threshold and below the 10% percentile threshold, with the full distribution shown above $M_h = 10^{14.5} M_\odot$. Galaxy luminosities, stellar masses, and number counts for populations with $M_r < -20$. 
Fig. 4.4.— The logarithmic scatter in halo mass at given cluster luminosity (upper panels) and given galaxy richness (bottom panels). The left two panels are for Durham data and right two panels for MPA data. All the four lines are best quadratic fit of the corresponding scatter at the given observable.
Fig. 4.5.— The upper two panels show the cluster luminosity scatter at given halo mass and the lower two represent that at given galaxy richness. The left two panels are from the Durham model and right panels are from the MPA model.
Fig. 4.6.— The distribution of the cluster total luminosity within different halo mass bins. From left to right, the distribution of the cluster luminosity with absolute galaxy magnitude $M_r$ cut-off at -19, -20, and -21. The upper nine panels are for the data from Durham catalog and the bottom nine panels are for the data from MPA catalog. In each magnitude cut-off and data catalog, the halo mass bins are in $(10^{13.5} \, M_\odot, 10^{13.9} \, M_\odot)$, $(10^{13.9} \, M_\odot, 10^{14.3} \, M_\odot)$, and greater than $10^{14.3} \, M_\odot$, as marked in each panels. To maintain visibility, we have used “$M_h$” to designate $\log M_h/M_\odot$ in the panel legends.
Fig. 4.7.— The distribution of the galaxy richness within different halo mass bins. The detail layout of this figure is same as Figure 4.6.
Fig. 4.8.— Halo mass bias estimation if the luminous clusters are matched to the massive halos. The triangles are estimated from Durham model, and solid points from MPA model. The points should have the estimated mass ratio value 1, if there is no bias between the two estimations. The left, middle, and right columns correspond to the data at $r$-band absolute magnitude cut-off $M_r < -19$, $M_r < -20$, and $M_r < -21$. The rows, from top to bottom, represent the data at redshifts $z = 0$, $z = 0.3$, and $z = 0.5$. 
Fig. 4.9. — Similar figure as 4.8, but the bias arises by matching the richest cluster to the massive halo.
Fig. 4.10.— Average galaxy luminosity vs. galaxy richness in individual clusters in three halo mass bins. The three left panels are for Durham data and the three right panels for MPA data. From top to bottom, only \((10^{13.7} M_\odot, 10^{13.9} M_\odot), (10^{14.3} M_\odot, 10^{14.5} M_\odot),\) and \((10^{14.7} M_\odot, 10^{14.9} M_\odot)\) mass bins are selected. In the to upper and left middle panels, solid lines show the median in each richness bin, and in those bins points are plotted only above the 90% percentile and below 10% percentile.
Fig. 4.11.— From top to bottom panels, the figure shows the central galaxy luminosity as a function of halo mass, the satellite luminosity 10%, 20%, 50%, 80%, and 90%, and mass-to-light ratio and halo mass. The left three panels are from Durham data and right three panels from MPA data. In the central luminosity vs. halo mass panel, only upper and lower 10% percentile data in luminosity at each given halo mass are displayed as points, and with the lines correspond to 10%, 20%, 50%, 80%, and 90% percentile of the central galaxy luminosity.
Fig. 4.12.— The central galaxy luminosity changes with galaxy richness within each selected halo mass bins. The detail layout of the figure is the same as in Figure 4.10.
Fig. 4.13.— Dependence of the observable-mass relations on redshift and $r$-band magnitude cutoff for the Durham model. In each panel, dotted, solid and dashed lines are for redshifts $z = 0$, $z = 0.3$, and 0.5 respectively. Left, middle and right columns are for thresholds $M_r = -19$, -20, and -21. Top panels show total luminosity vs. halo mass, and the second row shows the dispersion of $\log M_h$ at fixed $L/L_\star$. The lower two rows show corresponding results for richness. All the points are from $z = 0.3$ data.
Fig. 4.14.— Like Figure 4.13, but for MPA models.
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117
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