FROM GALAXY CLUSTERING TO DARK MATTER CLUSTERING

DISSERTATION

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By

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Galaxy clustering measurement has been one of the leading tools in cosmology for estimating a more fundamental quantity, the clustering of the underlying dark matter distribution. With the recent advances in galaxy redshift surveys, and hence dramatic improvement in observational data, the main obstacle to achieving this goal has become the theoretical uncertainty of galaxy bias, the difference between the galaxy and the matter distributions. The halo occupation distribution (HOD) program has emerged as a powerful tool to overcome the difficulty in inferring dark matter clustering by providing a theoretical framework that describes statistical properties of galaxy populations in individual dark matter halos. Moreover, gravitational lensing depends only on gravity, regardless of whether it is produced by dark or luminous matter, thus providing an observational method to break the degeneracy between the galaxy bias and underlying cosmology. In particular, weak gravitational lensing uses the subtle distortion of background galaxy shapes to measure how foreground lensing matter is statistically distributed, making its method well suited to the HOD description.
In this thesis, I describe three methods to quantify dark matter clustering based on the HOD framework, making full use of precision measurements of galaxy clustering and weak lensing from recent galaxy redshift surveys. First, using galaxy clustering measurements on small scales, I infer the scale-dependent bias function, which makes it possible to extend the recovery of the primordial matter power spectrum over a large dynamic range, and thereby tighten constraints on cosmological parameters obtainable from the galaxy samples of the Sloan Digital Sky Survey. Second, I develop an analytic model for combining galaxy-galaxy lensing and galaxy clustering to constrain the matter density parameter \( \Omega_m \) and the matter fluctuation amplitude \( \sigma_8 \). Finally, I present a novel method to constrain dark energy models using cluster-galaxy weak lensing and apply our method to the planned Dark Energy Survey (DES), forecasting our ability to measure cosmological parameters. Comprehensive analysis of galaxy clustering measurements with these complementary approaches will provide a unique opportunity for a complete description of dark matter clustering.
dedicated to my parents and little sister
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Chapter 1

Galaxy Clustering and Dark Matter Clustering

Galaxies have been used to map out how the universe looks and to understand how the universe evolves by looking at more distant galaxies and hence in further past. The grand structure of the universe has been gradually revealed with the advances in galaxy redshift surveys and precise measurements of galaxy clustering. Notably in the past few years, the first large-scale survey (CfA Redshift Survey; Huchra et al. 1983) has given way to the two state-of-the-art surveys the Two Degree Field Galaxy Redshift Survey (2dFGRS; Colless et al. 2001; Hawkins et al. 2003) and the Sloan Digital Sky Survey (SDSS; York et al. 2000; Stoughton et al. 2002; Adelman-McCarthy et al. 2006). However, the structure observed from these surveys shows only the tips of icebergs: the more fundamental underlying structure is composed of invisible dark matter that provides most of the mass of the universe, and its distribution is characterized by dark matter clustering. The critical uncertainty in cosmological interpretation therefore lies in how we understand galaxy bias, the relation between observable galaxy clustering and invisible dark matter clustering.
As the universe evolves, tiny overdensities in the matter distribution, as inferred from cosmic microwave background measurements (e.g., Spergel et al. 2003), gravitationally attract nearby matter and eventually overcome the damping dilution by the expansion of the universe, collapsing in their own gravitational potential wells and forming self-bound objects, known as dark matter halos (e.g., Gunn & Gott 1972). When it reaches its approximate equilibrium, a dark matter halo has average density of 200 times the mean background density of the universe, providing a natural habitat where gas falls into the gravitational potential of dark matter halos and forms stars, making observable galaxies. The dark matter halo populations therefore yield a starting point for understanding the structure of the universe due to the distinctive nature in overdensity compared to the background.

With large N-body simulations, the statistical properties of dark matter halos have been extensively studied, such as mass function, density profile, and clustering (e.g., Press & Schechter 1974; Sheth & Tormen 1999; Jenkins et al. 2001; Navarro, Frenk & White 1997; Sheth et al. 2001b). To predict the clustering of galaxies, it is critical to understand how galaxies populate dark matter halos.

The recent development to study this relation is summarized as the halo occupation distribution (HOD) program, which provides a theoretical framework for describing the statistical properties of galaxy populations in individual dark matter halos (Jing et al. 1998; Seljak 2000; Ma & Fry 2000; Peacock & Smith 2000; Scoccimarro et al. 2001; Berlind & Weinberg 2002). The HOD framework
connects dark matter halo populations to galaxy populations by specifying the mean occupation number of galaxies in a halo of mass $M$ and the probability distribution around the mean. Galaxy bias can be fully described by specifying HOD parameters given a cosmological model, facilitating theoretical predictions of galaxy clustering. However, the degeneracy between galaxy bias and cosmology requires additional information for a full specification of the both.

Gravitational lensing distorts background images of galaxies and stars as their light passes by foreground lensing matter, depending only on gravity regardless of whether it is produced by luminous or dark matter, and hence it makes one of the best tools to break the degeneracy. Especially with the recent theoretical developments in weak gravitational lensing (Blandford et al. 1991; Miralda-Escudé 1991b; Kaiser 1992), weak distortions produced by individual galaxies or diffuse dark matter can be statistically measured with many background source galaxies (e.g., Bacon, Refregier, & Ellis 2000; Wittman et al. 2000; Van Waerbeke et al. 2000), and its application to large-scale structure has become one of the most powerful tools to probe cosmology. In this thesis, I describe three methods to understand dark matter clustering based on the HOD framework, making full use of galaxy clustering and weak lensing measurements.
1.1. Primordial Matter Power Spectrum

The universe starts with near perfect homogeneity. However, inhomogeneity arises from the quantum fluctuations boosted by inflationary expansion at a very early epoch. Since the fluctuations are described by a Gaussian random field, the power spectrum of primordial matter fluctuation retains complete information on the initial conditions of the early universe, which can be used to infer the detailed mechanism of inflation and the matter and energy composition of the universe (e.g., Yu & Peebles 1969; Baumgart & Fry 1991; Feldman et al. 1994; Park et al. 1994; Lin et al. 1996; Sutherland et al. 1999; Cole et al. 2005; Padmanabhan et al. 2007; Blake et al. 2007; Percival et al. 2007; Tegmark et al. 2006). In the linear regime, the fluctuations evolve only in amplitude and different modes do not mix with each other, which makes it easy to estimate the primordial matter power spectrum up to an unknown normalization from the matter power spectrum evolved until the present. However, as the fluctuations evolve, nonlinearity of matter clustering amplifies its growth and different modes are mixed, making its interpretation complicated. With large $N$-body simulations, this problem is relatively mitigated and now we have good machinery to map the evolved matter power spectrum into

\[\text{The power spectrum is a statistical description of the strength of these fluctuations on different physical scales, specifying the mean-squared amplitude of Fourier modes as a function of their wavelength.}\]
the primordial matter power spectrum (e.g., Hamilton et al. 1991; Peacock & Dodds 1996; Smith et al. 2003).

In practice, observables that trace the invisible dark matter distribution should be used to estimate the primordial matter power spectrum. Galaxies are the basic building block of cosmological structure and can be observed at a fairly large distance. Therefore, power spectrum measurements have been carried out using galaxy redshift surveys. The power spectrum of galaxies can be biased relative to the matter power spectrum (Kaiser 1984; Bardeen et al. 1986), but general theoretical arguments show that the galaxy bias only changes the amplitude of the matter fluctuations on large scales, and cosmological interpretation is therefore straightforward (Coles 1993; Fry & Gaztanaga 1994; Mann, Peacock & Heavens 1998; Scherrer & Weinberg 1998; Narayanan et al. 2000). However, on small scales, galaxies cluster differently from dark matter, making it difficult to estimate the primordial matter power spectrum from galaxy power spectrum measurements, even with the best measurement precision on quasi-linear scales. In chapter 2, I develop a method to map galaxy power spectrum measurements into the primordial matter power spectrum over a large dynamic range based on the HOD framework and complementary information of galaxy clustering measurements on small scales, overcoming the difficulty in complex galaxy bias and taking full advantage of precision measurements of galaxy power spectrum.
1.2. **Galaxy Clustering and Galaxy-Galaxy Lensing**

With the advent of large galaxy redshift surveys, galaxy clustering measurements have become highly precise over a large dynamic range and have been carried out for galaxy subsamples with various thresholds in luminosity and color, providing ample information on how different galaxy samples cluster or how galaxy bias depends on the properties of galaxies (e.g., Zehavi et al. 2004, 2005a). Cosmological parameter estimation is therefore the next step that one wants to achieve with precise galaxy clustering measurements. However, there exists degeneracy between galaxy clustering and cosmology, because low matter fluctuation amplitude can be masked out by high galaxy bias or vice versa. It is therefore difficult to break the degeneracy with galaxy clustering measurements alone.

While galaxy clustering can be nearly identical in certain cosmological models where galaxy populations are tuned to mask out the difference in cosmology, other properties of galaxy populations can be significantly different, such as mass around galaxies and galaxy velocity dispersion. Gravitational lensing has emerged as a powerful tool to probe the mass distribution regardless of whether it is luminous or dark, thus providing a way to untangle this degeneracy. In particular, galaxy-galaxy lensing uses weak distortion of background galaxy shapes to measure how matter is distributed around a sample of foreground lensing galaxies (e.g., Brainerd et al. 1996; Fischer et al. 2000; McKay et al. 2001; Hoekstra et al. 2002; Sheldon...
et al. 2004; Mandelbaum et al. 2006 for its recent measurements), and it has the advantage that the same galaxy sample can be used for both measurements of galaxy clustering and galaxy-galaxy lensing. In chapter 3, I develop a method for combining galaxy-galaxy lensing and galaxy clustering to tell the mass difference of galaxies in cosmological models where galaxy clustering is indistinguishable from measurements. This method can therefore constrain the matter density parameter and the matter fluctuation amplitude.

1.3. Dark Energy and Cluster-Galaxy Weak Lensing

In the late 1990s, observations showed with high confidence that the apparent brightness of high redshift supernovae, believed to be standard candles, is dimmer than expected in a matter-dominated universe, indicating that the distance to these high redshifts is larger than expected in such a model, and that cosmic expansion must therefore be accelerating instead of decelerating (Riess et al. 1998; Perlmutter et al. 1999). The general theory of relativity allows energy density and pressure as a source of gravity, and therefore a field with sufficiently negative pressure can act as a source of repulsive gravity, accelerating the expansion of the universe. Subsequent observations find that this mysterious substance has to be omnipresent over the entire universe with (negative) pressure close in magnitude to its own energy density. This substance is generally referred to as dark energy. The nature
of dark energy is one of the most outstanding questions that we need to solve to advance our understanding of nature. The simplest model of dark energy is a “cosmological constant,” invented by Einstein to balance the gravity produced by ordinary matter and make the universe static forever (Einstein 1917). Even without convincing arguments for a cosmological constant, it is the null hypothesis that provides the best candidate for dark energy, and we should aim to verify or falsify it with observations.

Counting massive galaxy clusters is one of the most sensitive tools to probe dark energy models, since the number of massive clusters depends strongly on the expansion history of the universe and the growth rate of structure, both of which are affected by the presence of dark energy (e.g., Wang & Steinhardt 1998; Haiman, Hohr & Holder 2001). However, since cluster masses are estimated using mass-observable relations such as optical or X-ray luminosity and galaxy velocity dispersion, the difficulty in accurate mass estimates for individual clusters prevents the full use of the constraining power that the cluster counting method can provide. In chapter 4, I develop a method to overcome the difficulty in mass estimates and take full advantage of the exponential sensitivity of massive galaxy clusters to dark energy models based on a mass-observable relation and weak lensing measurements. This method uses a mass-observable relation just to select a sample of clusters for weak lensing measurements, minimizing the dependence on the accuracy of a
mass-observable relation and thereby obtaining more robust constraints on dark energy models than obtainable from direct cluster counting methods.
Chapter 2

Extending Recovery of the Primordial Matter Power Spectrum

2.1. Introduction

In the linear regime, the power spectrum of matter fluctuations encodes information about the physics of early universe (e.g., the potential of the field that drives inflation) and about the matter and energy contents of the cosmos. The power spectrum of galaxies can be biased relative to the power spectrum of matter (Kaiser 1984; Bardeen et al. 1986), but fairly general theoretical arguments imply that the shape of galaxy power spectrum should approach the shape of the linear matter power spectrum $P_{\text{lin}}(k)$ at sufficiently large scales, i.e.,

$$P_{R}(k) = b_0^2 P_{\text{lin}}(k) + N_0, \quad (2.1)$$

where $b_0$ is a constant galaxy bias factor and $P_{R}(k)$ denotes the real-space galaxy power spectrum (Coles 1993; Fry & Gaztanaga 1994; Weinberg 1995; Mann, Peacock & Heavens 1998; Scherrer & Weinberg 1998; Narayanan et al. 2000). The additive
“shot noise” term $N_0$ reflects both galaxy discreteness and small scale clustering
(Scherrer & Weinberg 1998; Smith, Scoccimarro & Sheth 2007; McDonald 2006); in
general, it can differ from a simple Poisson sampling correction, but we will ignore
this complication here. In the linear regime, distortions of redshift-space structure
by peculiar velocities also alter the amplitude but not the shape of galaxy power
spectrum (Kaiser 1987). There have therefore been great efforts to measure the
galaxy power spectrum on large scales from angular catalogs and redshift surveys
and use the results to test cosmological models; early measurements (Yu & Peebles
1969; Baumgart & Fry 1991; Feldman et al. 1994; Park et al. 1994) have given
way to recent high precision measurements such as Las Campanas Redshift Survey
(Lin et al. 1996) and Point Source Catalog redshift survey (Sutherland et al. 1999).
Especially, these measurements are reaching the state of the art by the advent of the
Two Degree Field Galaxy Redshift Survey (2dFGRS; Colless et al. 2001; Hawkins et
al. 2003) and the Sloan Digital Sky Survey (SDSS; York et al. 2000; Stoughton et al.
2002; Adelman-McCarthy et al. 2006), such as Cole et al. (2005) analysis of power
spectrum in 2dFGRS, Padmanabhan et al. (2007), and Blake et al. (2007) analyses
of luminous red galaxies (LRGs) with photometric redshifts in SDSS, and Percival
et al. (2007) and Tegmark et al. (2006) measurements from SDSS redshift survey
of main sample galaxies and LRGs. This paper investigates the problem of going
from the galaxy power spectrum to the linear matter power spectrum, and hence to
cosmological conclusions.
The latest observational analyses yield impressive statistical precision on scales near transition from linear to non-linear regime, e.g., typical $1 - \sigma$ errors of 5–10% on $P(k)$ at $k \simeq 0.15h\text{Mpc}^{-1}$. The critical uncertainty in cosmological interpretation is therefore the accuracy of equation (2.1) on these scales. The effects of non-linearity and redshift-space distortions on the matter power spectrum can be computed using numerical simulations or tuned analytic models (Smith et al. 2003, and references therein), but details of galaxy formation physics can influence the relation between galaxy and matter power spectra in this regime. Percival et al. (2007) find that linear theory fits imply different cosmological parameters if applied up to measurements at $k = 0.06h\text{Mpc}^{-1}$ or $k = 0.15h\text{Mpc}^{-1}$. Furthermore, Cole, Sánchez & Wilkins (2006) analyze the SDSS and 2dFGRS galaxy samples and find that the measured shapes of galaxy power spectra are different and inexplicable by expected cosmic variance. They show that the likely source of the discrepancy is different scale-dependence of galaxy bias, originating from the different color distributions in the SDSS and 2dFGRS galaxy samples.

Cole et al. (2005), Padmanabhan et al. (2007), and Tegmark et al. (2006) approach this problem by fitting a parametrized model of scale-dependent bias,

$$P_{\text{gal}}(k) = b_0^2 P_{\text{lin}}(k) \frac{1 + Qk^2}{1 + Ak}.$$ (2.2)

The functional form is devised for convenience to approximate the scale-dependent bias of galaxy samples obtained by populating the Hubble volume simulation.
(Evrard et al. 2002) using a semi-analytic model of galaxy formation (Benson et al. 2000). Here $A = 1.4h^{-1}\text{Mpc}$ or $1.7h^{-1}\text{Mpc}$ for real-space power spectrum $P_R(k)$ or redshift-space power spectrum $P_0(k)$ measurements, respectively, and $Q$ is treated as a free parameter that is marginalized over in deriving cosmological parameter constraints. This approach is adequate if equation (2.2) is a sufficiently accurate description of scale-dependent bias for some value of $Q$, but it could yield biased parameter estimates or incorrect error bars if the actual scale-dependence is different, and it gives up on extracting cosmological information from scales where bias might be mildly scale-dependent. For example, Tegmark et al. (2006) find that cosmological parameters remain unaffected for changes in power spectrum measurements at $k \geq 0.1h\text{Mpc}^{-1}$ if $Q$-value is marginalized over. This implies that the statistical constraining power is lost at $k \geq 0.1h\text{Mpc}^{-1}$ by the marginalization process.

Different approach is adopted to develop an analytic model to study the scale-dependence of galaxy bias by using higher-order perturbation theory (e.g., Smith, Scoccimarro & Sheth 2007; McDonald 2006). This approach is elegant and transparent in nature, since it is based on linear theory and its extension to higher-order, while our approach is less ab initio in the sense of incorporating elements calibrated by numerical $N$-body simulations in our analytic model. However, the critical uncertainty as to this approach based on higher-order perturbation theory is its applicability on quasi-linear scales ($\gtrsim 0.1h\text{Mpc}^{-1}$), where first-order linear theory
is known to be inaccurate, and yet measurement precision is highest in practice. In contrast, our approach is fully nonlinear and phenomenological in nature, and hence its applicability is unlimited, within the limit of systematic uncertainty.

In this paper, we present an alternative approach to recovering the shape of the linear matter power spectrum, both more aggressive and more robust than “marginalizing over $Q$”, based on the halo occupation distribution (HOD) framework. The HOD describes a nonlinear relation between galaxies and matter by specifying the probability $P(N|M)$ that a halo of mass $M$ hosts $N$ number of galaxies of a given type, together with specification of the relative spatial and velocity distributions of galaxies within halos.\footnote{It has emerged as a powerful method of modeling galaxy bias (Seljak 2000; Ma & Fry 2000; Peacock & Smith 2000; Scoccimarro et al. 2001; Berlind & Weinberg 2002), because the dynamics of dark matter halos can be accurately calculated and the effects of galaxy formation physics can be at the same time separately parametrized.}

Our strategy for extending recovery of the primordial matter power spectrum is that we make use of complementary information from the measurements of projected correlation function $w_p(r_p)$ as a constraint to obtain HOD parameters given a
cosmological model. We then predict the galaxy power spectrum and calculate the scale-dependent bias

\[ b^2(k) \equiv P_R(k)/P_{\text{lin}}(k). \] (2.3)

In principle, our predicted galaxy power spectrum can be used to calculate the likelihood of a given cosmological model by comparing to the measurements. However, we present our results in terms of bias function \( b^2(k) \), since it captures the essence of the power spectrum recovery. In contrast, \( Q \)-model uses a fixed functional form fit to a specific cosmology and galaxy formation model and constrains its free parameter from \( P(k) \) measurement. Uncertainties in HOD modeling and HOD parameters can affect our \( b^2(k) \) calculations, but these uncertainties can be accurately computed and marginalized over. Therefore, we can extend the wavenumber range over which \( P(k) \) measurements are used for cosmological parameter constraints, taking full advantage of precision measurements on quasi-linear scales.

Analyses of redshift surveys typically estimate angle-averaged power spectrum \( P_0(k) \), the monopole of redshift-space power spectrum (e.g., Cole et al. 2005; Percival et al. 2007). Redshift-space distortions do not alter the shape in linear theory, but do change the shape in trans-linear regime (e.g., Cole et al. 1994), and finger-of-god (FoG) effects have impact out to large scales (e.g., Scoccimarro et al. 2001). Padmanabhan et al. (2007); Blake et al. (2007) deproject the angular clustering measurement of the SDSS LRG sample using photo-z catalogs to obtain
the real-space power spectrum, independent of redshift-space distortions. Tegmark et al. (2004, 2006) use a linear combination of the redshift-space monopole, quadrupole, and hexadecapole that recovers the real-space power spectrum in the linear regime, so called, pseudo real-space power spectrum $P_{Z\rightarrow R}(k)$. Moreover, these redshift-space estimators can be applied directly to galaxy redshift data or applied after compressing FoG effects. We will investigate $b^2(k)$ for all of these cases.

To this end, we develop an analytic model for calculating real-space and redshift-space galaxy power spectrum given $P_{\text{lin}}(k)$ and galaxy HOD, drawing on the Tinker (2007) model for redshift-space distortion, an improvement on previous work (e.g., Seljak 2001; White 2001; Kang et al. 2002; Cooray 2004). Tinker (2007) test the model for computing redshift-space correlation against a series of populated $N$-body simulations. Here we extend the model and present additional test of its applicability to modeling redshift-space power spectrum.

In this paper, we use HOD parameters for volume-limited galaxy samples that have well defined class of galaxies, mainly focusing on SDSS galaxy samples with absolute-magnitude limits $M_r \leq -20$ and $M_r \leq -21$ (Zehavi et al. 2005b) for application of our method. More complete modeling of conditional luminosity function might allow use of flux-limited galaxy catalogs, though it requires more free parameters to provide complete descriptions of the galaxy samples. Here we only consider volume-limited galaxy samples, whose results can be combined to improve

---

2For brevity, we quote the absolute magnitude thresholds $M_r - 5 \log h$ for $h \equiv 1$. 16
statistical precision. In \S 2.5, we also present the results on the LRG samples, which are approximately volume-limited, providing best current constraints.

2.2. Calculational Methods

2.2.1. Numerical Model

We use $N$-body simulations to test our analytic model calculations of correlation function and power spectrum in both real-space and redshift-space. We have carried out five simulations of a flat LCDM universe using the publicly available tree-code GADGET (Springel et al. 2001), and all the simulations are performed with identical cosmological parameters except for the random seed numbers used to generate initial conditions. The initial scale-invariant ($n_s = 1$) power spectrum is modified by the transfer function of Efstathiou et al. (1992) with shape parameter $\Gamma = 0.2$. The simulation was evolved from an expansion factor $a = 0.01$ to $a = 1.0$ with $\Omega_m = 0.1$, $\Omega_\Lambda = 0.9$ and $\sigma_8 = 0.95$ at $z = 0$. To cover a range of parameter space spanned by $\Omega_m$ and $\sigma_8$, we use earlier outputs to represent different cosmological models from the simulations. Our choices for the earlier expansion factors are $a_{\text{out}} = 0.84$, 0.64, 0.49, and 0.40. These outputs correspond respectively to simulations with different parameter combinations ($\Omega_m, \sigma_8$) = (0.16, 0.90), (0.30, 0.80), (0.48, 0.69), and (0.63, 0.60) with the identical power spectrum shape ($\Gamma = 0.2$) but evolved beginning at expansion factor $a = 0.01/a_{\text{out}}$. Note that this procedure correctly provides
dark matter particle mass and density contrast from a separate simulation with the corresponding parameter combination of $\Omega_m$ and $\sigma_8$ (Zheng et al. 2002). We evolve $360^3$ particles in a volume of comoving side length $253 \, h^{-1}\text{Mpc}$ to take into consideration that the lowest mass halos that host galaxies with $M_r \leq -20$ contain at least 32 particles. Dark matter halos are identified by using the friends-of-friends algorithm (FoF; Davis et al. 1985) with a linking length of 0.2 times the mean interparticle separation, i.e., $140\, h^{-1}\text{kpc}$.

To populate dark matter halos with galaxies in $N$-body simulations, we use HOD parameters listed in Table 2.6 that are chosen to match the mean number density $\bar{n}_g$ and projected correlation functions $w_p(r_p)$ of the SDSS galaxy samples with absolute-magnitude limits $M_r \leq -20$ and $M_r \leq -21$ (Zehavi et al. 2005b). In our standard HOD parameterization, the number of central galaxies is a step function changing from zero to one at a minimum halo mass $M_{\text{min}}$. Therefore, halos of mass $M < M_{\text{min}}$ lack galaxies. We assume $\langle N_{\text{sat}} \rangle \propto M$ at high masses with smooth cutoff at low mass. Therefore, the number of satellite galaxies is,

$$
\langle N_{\text{sat}} \rangle_M = \left( \frac{M}{M_{\text{c}}(M)} \right) \exp \left( - \frac{M_{\text{cut}}}{M - M_{\text{min}}} \right),
$$

for a halo of mass $M \geq M_{\text{min}}$, and the distribution of satellite galaxy number $P(N_{\text{sat}}|\langle N_{\text{sat}} \rangle)$ is assumed to be Poisson (Kravtsov et al. 2004; Zheng et al. 2005). This parametrization is well suited to our purposes, but we also investigate the effect of adopting a more flexible HOD parametrization in §2.4.
We replace halos identified by the FoF algorithm by spherical NFW halos (Navarro, Frenk & White 1997) with identical mass, truncated at virial radius \(R_{\text{vir}}\), within which the mean density is 200 times the mean matter density. The concentration parameter \(c_{\text{dm}}\) of dark matter halos are computed using the relation of Bullock et al. (2001) and are scaled to account for the different definition of halo overdensity adopted here. This method reduces numerical artifacts caused by finite force resolution in our simulations. We place a central galaxy at the center of mass of halos. Assuming that satellite galaxies trace the dark matter distribution within halos, we place satellite galaxies following the NFW profile of halos.

In redshift-space, galaxies are displaced because of peculiar velocity. Central galaxies are assumed to be at rest relative to the halo center; no velocity bias is assumed for central galaxies. For satellite galaxies, we add line-of-sight velocities drawn from a Gaussian distribution with zero mean and dispersion,

\[
\sigma_v(M) = \left(\frac{GM}{2R_{\text{vir}}}\right)^{1/2},
\]

(2.5)

to the velocity of the halo center of mass. This procedure is exact for isotropic singular isothermal halos and is reasonably accurate for NFW profiles (see Tinker et al. 2006 for detailed tests).

Finally, we compute the density contrast field by cloud-in-cell weighting the particle distribution onto \(360^3\) grids on a side and use the publicly available fast
Fourier transform (FFT) code, 
\texttt{fftw}, to obtain the Fourier components in units of
the fundamental mode of our simulation box, \( \Delta k = 0.02 \, h \text{Mpc}^{-1} \). We deconvolve
the cloud-in-cell weighting function \( W_{\text{CIC}}(k) \), and subtract shot-noise contributions
\( 1/N_{\text{gal}} \) to compute the power spectrum at each \( k \):

\[
P_{\text{gal}}(k) = P_{\text{FFT}}(k)/W_{\text{CIC}}^2(k) - \frac{1}{N_{\text{gal}}},
\]

and the weighting function is

\[
W_{\text{CIC}}(k) = \left[ \prod_{i=1}^{3} \frac{\sin(\pi k_i/2k_N)}{\pi k_i/2k_N} \right]^2,
\]

where \( N_{\text{gal}} \) is the number of galaxies, \( k_i \) is the \( i \)-th component of wavenumber \( k \), and
\( k_N = 4.5h \text{Mpc}^{-1} \) is the Nyquist wavenumber of our simulations. For computations
in redshift-space, we simply displace particles using the \( z \)-component of the peculiar
velocity scaled by Hubble constant as they would appear to a distant observer at
\( z = -\infty \). This process satisfies the distant observer approximation we adopt here,
and ensures periodic radial velocity fields in the simulation volume appropriate for
FFT. Redshift-space multipoles are extracted by least-squares fitting to the Legendre
polynomial coefficients (eq.[2.13]). We repeat the procedure for the \( x \)- and \( y \)- axes,
treating each axis as the line-of-sight, and we average the resulting power spectra
over the three line-of-sight directions.
2.2.2. Analytic Model

Our analytic calculation of the real-space galaxy auto-correlation function $\xi_R(r)$ follows Tinker et al. (2005) with accurate treatments of scale-dependent halo bias and halo exclusion, improving the method by Zheng (2004). For a given galaxy sample with its projected correlation function measurements $w_p(r_p)$, we obtain HOD parameters by fitting the mean space density $\bar{n}_g$ and $w_p(r_p)$, computed by

$$
\bar{n}_g = \int_{M_{\text{min}}}^{\infty} dM \frac{dn}{dM} \langle N \rangle_M, \quad (2.8)
$$

$$
w_p(r_p) = 2 \int_0^{z_{\text{max}}} dz \xi_R (r_p^2 + z^2)^{1/2}, \quad (2.9)
$$

where we use $z_{\text{max}} = 40h^{-1}\text{Mpc}$ adopted in SDSS clustering measurements (e.g., Zehavi et al. 2004, 2005a,b) and $dn/dM$ is the halo mass function (Sheth & Tormen 1999; Jenkins et al. 2001). The real-space galaxy power spectrum $P_R(k)$ is computed by taking Fourier transform of the correlation $\xi_{\text{R}}^{1h}(r)$ from a single halo and adding to the contribution from two distinct halos, which is computed already in Fourier space (see for details, Zheng 2004; Tinker et al. 2005).

We compute the redshift-space correlation function $\xi(r_{\parallel}, r_{\perp})$ using the probability distribution of galaxy pairwise velocities $f(v_z, r)$,

$$
1 + \xi(r_{\parallel}, r_{\perp}) = \int_{-\infty}^{\infty} d\mathbf{v}_z [1 + \xi(r)] f(\mathbf{v}_z, r), \quad (2.10)
$$
where $r_\perp$ is the projected separation, $r_\|\|$ is the line-of-sight separation in redshift-space, and $v_z = 100 \text{ km s}^{-1}(r_\|\| - z)/h^{-1}\text{Mpc}$ is the pairwise velocity of galaxies separated by $r = (r_\perp^2 + z^2)^{1/2}$. Equation (2.10) is called the streaming model and has been extensively used to measure the mean matter density (Peacock et al. 2001; Cole et al. 2005) and to model redshift-space correlations (White 2001; Seljak 2001). It is valid in the linear and nonlinear regime (Fisher 1995; Scoccimarro 2004), provided that the correct $f(v_z, r)$ is used. We adopt the probability distribution function of Tinker (2007) for the galaxy pairwise velocities, which is an analytic model with some elements calibrated on $N$-body simulations. We refer the reader to the work by Tinker (2007) for more extensive and detailed discussion and tests.

### 2.2.3. REDSHIFT-SPACE MULTIPOLES

Using the analytic model, we compute the redshift-space correlation function $\xi(r_\|, r_\perp)$ given $P_{\text{lin}}(k)$ and a set of HOD parameters, and we expand $\xi(r_\|, r_\perp)$ with Legendre polynomials,

$$\xi(r, \mu) = \sum_{l=0}^{\infty} L_l(\mu)\xi_l(r),$$

(2.11)

where $r = (r_\perp^2 + r_\|^2)^{1/2}$, $\mu = r_\|/r$ is the direction cosine of the separation and line-of-sight vectors. The redshift-space multipole component is then,

$$\xi_l(r) = \frac{2l + 1}{2} \int_{-1}^{1} d\mu L_l(\mu)\xi(r, \mu).$$

(2.12)
We use $L_l(\mu)$ for Legendre polynomial to avoid the confusion with redshift-space multipole power spectrum $P_l(k)$ defined below. Similarly, the redshift-space power spectrum $P(k_\parallel, k_\perp)$ can be expanded using Legendre polynomials,

$$P(k, \mu) = \sum_{l=0}^{\infty} L_l(\mu) P_l(k),$$  \hspace{1cm} (2.13)$$

where $k = (k_\perp^2 + k_\parallel^2)^{1/2}$ and $\mu = k_\parallel/k$, in analogy to quantities in real-space. The isotropy of the correlation function and power spectrum ensures that multipoles with odd $l$ vanish. Making use of the fact that $\xi(r_\parallel, r_\perp)$ and $P(k_\parallel, k_\perp)$ are Fourier counterparts, each redshift-space multipole component is computed by

$$P_l(k) = 4\pi i^l \int_0^{\infty} r^2 \xi_l(r) j_l(kr) dr,$$  \hspace{1cm} (2.14)$$

where $j_l(x)$ are spherical Bessel functions (Cole et al. 1994; Hamilton 1998). Note that the quadrupole ($l = 2$) has opposite sign in Fourier space.

We note that equation (2.14) requires knowledge of $\xi_l(r)$ on large scales, while our analytic model is only tested at $r \leq 40h^{-1}\text{Mpc}$. Therefore, we compute $\xi_l(r)$ at $r \geq 40h^{-1}\text{Mpc}$ with the linear approximation for redshift-space distortion. In the linear regime, the multipole expansion of $\xi(r_\parallel, r_\perp)$ has only three nonzero multipoles: monopole $\xi_0$, quadrupole $\xi_2$, and hexadecapole $\xi_4$ (Kaiser 1987) that are in turn related to $\xi_R(r)$,

$$\xi_0(r) = C_0 \xi_R(r),$$
\[ \xi_2(r) = C_2 \left( \xi_R(r) - \bar{\xi}(r) \right), \]
\[ \xi_4(r) = C_4 \left( \xi_R(r) + 2.5 \bar{\xi}(r) - 3.5 \bar{\bar{\xi}}(r) \right), \]  \tag{2.15}

where \( C_0 = 1 + \frac{2}{3} \beta + \frac{1}{5} \beta^2, \ C_2 = \frac{4}{3} \beta + \frac{4}{7} \beta^2, \ C_4 = \frac{8}{35} \beta^2, \ \beta = \Omega_m^{0.6}/b_0, \ b_0 \) is the asymptotic galaxy bias factor, and the barred correlations are

\[ \bar{\xi}(r) = \frac{3}{r^3} \int_0^r s^2 \xi_R(s) ds, \]  \tag{2.16}

\[ \bar{\bar{\xi}}(r) = \frac{5}{r^5} \int_0^r s^4 \xi_R(s) ds, \]  \tag{2.17}

(Hamilton 1992). We compute the three multipoles at \( r \geq 40 h^{-1} \text{Mpc} \), rather than \( \xi(r_\parallel, r_\perp) \) itself. We first compute the values of \( C_l \) at \( r = 40 h^{-1} \text{Mpc} \), where the deviations from the linear theory predictions are less than 5%, then we smoothly transition \( C_l(r) \) values to \( r_{\text{lin}} \) at which the \( C_l(r) \) values exactly become the linear theory predictions. The redshift-space multipoles \( \xi_i(r) \) at \( r \geq 40 h^{-1} \text{Mpc} \) are then obtained by using equations (2.15) with \( C_l(r) \), in place of constant \( C_l \), the linear theory prediction. We simply set \( r_{\text{lin}} = 500 h^{-1} \text{Mpc} \), and \( P_l(k) \) values are insensitive to the choice of \( r_{\text{lin}} \) as long as \( r_{\text{lin}} > 100 h^{-1} \text{Mpc} \). We adopt the Smith et al. (2003) prescription for the matter power spectrum \( P_{\text{mm}}(k) \) and its correlation \( \xi_{\text{mm}}(r) \) to compute \( \xi_R(r) \) on large scales, and the asymptotic galaxy bias factor is computed by

\[ b_0 = \frac{1}{n_g} \int_0^\infty dM \frac{dn}{dM} \langle N \rangle_M b_h(M), \]  \tag{2.18}

where \( b_h(M) \) is the bias factor of halos of mass \( M \) (e.g., Sheth et al. 2001b).
2.3. Recovering the Real-Space Galaxy Power Spectrum

Before turning to the bias between the galaxy power spectrum and the linear power spectrum, we investigate how well the method used by Tegmark et al. (2004, 2006) recovers the true real-space galaxy power spectrum. Along the way, we test the accuracy of our analytic model prediction for the redshift-space power spectrum against the results obtained from the $N$-body galaxy catalogs described in §2.2.1. Our basic approach to predicting the galaxy power spectra is that we first determine HOD parameters given a cosmological model, and the galaxy power spectra are then calculated using the relation between galaxies and dark matter halos. We obtain HOD parameters by fitting the mean number densities $\bar{n}_g$ and projected correlation function measurements $w_p(r_p)$ of the SDSS $M_r \leq -20$ and $M_r \leq -21$ galaxy samples, taking into account the full covariance error matrix, estimated through jackknife resampling of the observational sample (Zehavi et al. 2005b).

Figure 2.1 shows the projected correlation functions of the two SDSS galaxy samples, where the solid lines are the analytic model predictions for the five different cosmological models (listed in Table 2.6), obtained by fitting the Zehavi et al. (2005b) measurements shown as symbols. The error bars only show the diagonal elements of the covariance matrix of the $w_p(r_p)$ measurements, and the analytic model fits to the measurements are acceptable over the wide range of parameter combination ($\Omega_m, \sigma_8$) when the full covariance matrix is considered, since the errors
between data points are strongly correlated. However, there is strong degeneracy between the shape $\Gamma$ and spectral index $n_s$ of the power spectrum, and the best-fit HOD parameters are insensitive to $\Gamma$ and $n_s$, which we discuss in § 2.4.2. The change to the adopted value of $z_{\text{max}}$ has little effect on the HOD parameters inferred by fitting $w_p(r_p)$, though it affects the $\chi^2$ values of these fits slightly. We use $z_{\text{max}} = 40h^{-1}\text{Mpc}$ for further analyses. The shaded regions represent the statistical uncertainty in the mean value of $w_p(r_p)$, computed from the dispersion among the five independent populated $N$-body simulations, for the central model ($\Omega_m = 0.3$, $\sigma_8 = 0.8$; see Table 2.6). Our analytic model predictions for $w_p(r_p)$ agree with the $N$-body results within 5% fractional differences.

Using the analytic model and assuming the central model, we illustrate the dimensionless power spectra $\Delta^2(k) = k^3 P(k)/2\pi^2$ of the $M_r \leq -20$ galaxy sample in Figure 2.2, where the thick solid line represents the real-space galaxy power spectrum $P_R(k)$ and the shaded region is the statistical uncertainty in the mean value of $P_R(k)$. The finite box size of the simulations puts a limit on Fourier modes $k \geq k_{\text{box}} \equiv 2\pi/L_{\text{box}} = 0.025h\text{Mpc}^{-1}$ that we can measure from the populated halo catalogs. We test our analytic model predictions for galaxy power spectra below. The two thick dashed lines are the matter power spectra of the assumed cosmological model; the long-dashed line is the linear matter power spectrum $P_{\text{lin}}(k)$ and the short-dashed line is the nonlinear matter power spectrum $P_{\text{nl}}(k)$, computed by using the Smith et al. (2003) prescription. On large scales, $P_R(k)$ has the identical shape of
$P_{\text{lin}}(k)$ with normalization differing by a galaxy bias factor squared $b^2$, as predicted by the linear bias approximation (eq.[2.1]). However, on small scales $k \gtrsim 0.2h\text{Mpc}^{-1}$, where $P_{\text{nl}}(k)$ departures from $P_{\text{lin}}(k)$ or $\Delta_{\text{lin}}^2(k) \simeq 1$, $P_R(k)$ follows more closely the $P_{\text{nl}}(k)$ shape than the $P_{\text{lin}}(k)$ shape.

We also present the analytic model predictions for the redshift-space multipole power spectra $\Delta^2_l(k) = k^3 P_l(k)/2\pi^2$. The dotted, long-dashed, and dot-dashed lines in Figure 2.2 represent the redshift-space monopole $P_0(k)$, quadrupole $P_2(k)$, and hexadecapole $P_4(k)$, respectively. On large scales, coherent peculiar infall along the overdense region produces the redshift-space distortion, where the overdense region shrinks and the underdense region inflates in redshift-space. Therefore, when averaged over angle, $P_0(k)$ is larger than $P_R(k)$. However, on small scales, non-linear collapse and random motions in virialized objects stretch systems along the line-of-sight, giving rise to the Finger-of-God (FoG) effect, which inflates overdense regions and depresses their density contrast. Therefore, $P_0(k)$ is smaller than $P_R(k)$ on small scales.

In the linear regime, the redshift-space power spectrum can be written as

$$P(k, \mu_k) = (1 + \beta \mu_k^2)^2 P_R(k),$$

and the real-space power spectrum can be reconstructed by using a linear combination of the three redshift-space multipoles to remove the unknown variable $\beta$,

$$P_{Z\rightarrow R}(k) \equiv P_0(k) - \frac{1}{2} P_2(k) + \frac{3}{8} P_4(k),$$

(2.19)
which exactly reduces to $P_R(k)$, if the linear theory approximation holds (Kaiser 1987). The upper panel of Figure 2.3 tests the ability of equation (2.19) to recover the true $P_R(k)$. $P_{Z-R}(k)$ recovers $P_R(k)$ to 5% at $k \leq 0.4h\text{Mpc}^{-1}$, while it substantially underestimates $P_R(k)$ at $k > 0.4h\text{Mpc}^{-1}$. This pseudo real-space power spectrum $P_{Z-R}(k)$ can be used to correct for the effect of redshift-space distortions and to estimate $P_R(k)$. This procedure is called disentanglement approach in Tegmark et al. (2004, 2006) and our $P_{Z-R}(k)$ corresponds to $P_{gg}(k)$ in their notation. The alternative, modeling approach, is to construct two more power spectra, namely $P_{gv}(k)$ and $P_{vv}(k)$ from redshift-space multipole measurements and to solve for $P_R(k)$ with the linear theory approximation (Tegmark et al. 2004, 2006). The former method is more robust due to the cancellation of deviations in the multipoles from the linear theory approximation on nonlinear scales, but overly conservative by essentially marginalizing over the other two power spectra. In contrast, the latter method provides decorrelated estimates of $P_R(k)$ with smaller error bars at the cost of dealing with complicated window functions, but is subject to systematic error in the linear theory approximation when modeling the other two power spectra. Our primary interest lies in power spectrum estimate itself, rather than in error estimate, and hence we adopt the disentanglement approach for further analyses. Cole et al. (2005) use the Fourier-based method of Percival, Verde & Peacock (2004), an extension of the Feldman et al. (1994) method to include luminosity dependent
galaxy bias, corresponding to $P_0(k)$ in our notation. We also take $P_0(k)$ as one of our principal estimates of $P_R(k)$.

A significant part of the nonlinearity in redshift-space distortions arises from the random motion of galaxies in virialized objects, which can be identified by finding groups or clusters, and of which the FoG effects can be subsequently compressed. In practice, clusters of galaxies are identified by applying the friends-of-friends algorithm to galaxy positions in redshift-space. Two galaxies are assumed to belong to the same cluster if the density windowed through an ellipse (usually several times longer in the radial than transverse direction) is higher than an overdensity threshold, and their radial dispersion of member galaxy positions is set equal to the transverse dispersion to compress the FoG effect, if the former exceeds the latter (Tegmark et al. 2004, 2006). Here we assume a perfect process of compressing the FoG effects; all the halos with velocity dispersion $\sigma_h$ greater than a threshold are compressed by setting $\sigma_h = 100$ km s$^{-1}$ and none of the halos with $\sigma_h$ smaller than the threshold are affected. The pseudo real-space power spectrum $P_{Z \rightarrow R}^{750}(k)$ with FoG compression threshold $\sigma_h = 750$ km s$^{-1}$ agrees with $P_R(k)$ to 5% at $k \leq 0.5h$Mpc$^{-1}$, and these halos are easily identifiable in practice. With more aggressive threshold $\sigma_h = 400$ km s$^{-1}$, $P_{Z \rightarrow R}^{400}(k)$ recovers $P_R(k)$ to 5% at $k \leq 1h$Mpc$^{-1}$. We also compare the $P_0(k)$ shape to $P_R(k)$ after scaling the constant factor $C_0$ in §2.2.3. The scaled $P_0(k)$ is depressed by 5% at $k = 0.1h$Mpc$^{-1}$ compared to $P_R(k)$. FoG compression
helps at \( k \approx 0.1h\text{Mpc}^{-1} \), but the difference between \( P_0(k) \) and \( P_R(k) \) reaches 5% at \( k \approx 0.2h\text{Mpc}^{-1} \) and 10% at \( k \approx 0.3h\text{Mpc}^{-1} \).

Having shown the agreement between the analytic model predictions and the \( N \)-body results in \( w_p(r_p) \), we now test the accuracy of the analytic model predictions for real-space and redshift-space power spectra against the \( N \)-body simulations. The bottom panel shows the fractional difference in galaxy power spectra between the analytic model and the \( N \)-body results, where the shaded region shows only the statistical uncertainty in the mean value of \( P_R(k) \) from the simulations. Note that the uncertainties on the other power spectra are larger and are not shown. The analytic model calculation of \( P_R(k) \) is accurate to better than a few percent at \( k > 0.08h\text{Mpc}^{-1} \), while it is difficult to assess the statistical significance at \( k < 0.08h\text{Mpc}^{-1} \), where the simulations only have few Fourier modes due to the finite box size. Since linear theory should become accurate on large scales, it would be surprising if the analytic model becomes less accurate on this regime. Our analytic model also provides accurate predictions for \( P_0(k) \) and \( P_{Z-R}(k) \), both with and without FoG compression, at \( k > 0.1h\text{Mpc}^{-1} \).

2.4. RECOVERING LINEAR MATTER POWER SPECTRUM

We now turn to our principal results, the scale-dependent bias relations 
\[
b^2(k) = \frac{P_{\text{obs}}(k)}{P_{\text{lin}}(k)}
\]
between observable galaxy power spectra \( P_{\text{obs}}(k) \) and the
linear matter power spectra $P_{\text{lin}}(k)$. As potentially observable power spectra, we consider $P_R(k)$ (inferred from the angular clustering power spectrum), $P_{Z-R}(k)$, and $P_0(k)$, with varying levels of FoG compression for the latter two. Here we use HOD constraints for the $M_r \leq -20$ and $M_r \leq -21$ samples of Zehavi et al. (2005b) based on their $w_p(r_p)$ measurements. These could be further improved with $w_p(r_p)$ measurements from the SDSS Data Release 5 and with group multiplicity constraints. Then we investigate the uncertainties in the power spectrum recovery associated with our HOD modeling and adopted fiducial cosmological model. We make one change in our application of the analytic model to enhance the accuracy of our analytic model predictions, by using a CMBFAST transfer function (Seljak & Zaldarriaga 1996) to compute $P_{\text{lin}}(k)$ for a given cosmology, in place of the Efstathiou et al. (1992) parametrization used to test our analytic model against the $N$-body simulations (which used these initial conditions).

2.4.1. Scale-Dependent Bias of the $M_r \leq -20$ and $M_r \leq -21$ Galaxy Samples

Figure 2.4 plots the scale-dependent bias functions $b^2(k)/b_0^2$ (eq.[2.1]) of the galaxy sample with $M_r \leq -20$, in which $b_0$ is the asymptotic galaxy bias factor in equation (2.18). Figure 2.4a shows the real-space $P_R(k)$ (solid) and the nonlinear matter power spectrum $P_{\text{nl}}(k)$ (circle), which follow each other remarkably closely.
The physical origin of $P_{nl}(k)$ shape is discussed by Smith, Scoccimarro & Sheth (2007). Note that we compute the nonlinear matter power spectrum $P_{nl}(k)$ using the Smith et al. (2003) prescription, modified to utilize a CMBFAST transfer function. $P_{lin}(k)$ retains its baryonic acoustic oscillations (BAO) imprinted by the baryon-photon plasma before the recombination, whereas most of the oscillation features are washed out in $P_{nl}(k)$ and hence $P_{\text{obs}}(k)$ due to the nonlinear evolution at low $z$. The bias functions $b^2(k)/b^2_0$ in Figure 2.4 are therefore smooth at $k < 0.1h\text{Mpc}^{-1}$, while there exist small oscillatory feature at $k > 0.1h\text{Mpc}^{-1}$, arising from the degradation of the BAO in $P_{\text{obs}}(k)$ and $P_{nl}(k)$.\footnote{The amplitude of oscillations is small since we divide by $P_{\text{lin}}(k)$, instead of a smooth fit to the overall shape of $P_{\text{obs}}(k)$, as is done in BAO studies to emphasize the contrast of the oscillations.} Note that it is beyond our scope to investigate how accurate the Smith et al. (2003) prescription is regarding the degradation of acoustic oscillation features in $P_{nl}(k)$ due to the nonlinearity, while we suspect that the overall trend of the bias functions can be well represented by our approach to modeling $P_{nl}(k)$.

The accuracy of recovering the $P_{lin}(k)$ shape is $\sim 5\%$ at $k \leq 0.1h\text{Mpc}^{-1}$, climbs above by $5\%$ at $k \simeq 0.22h\text{Mpc}^{-1}$, and rises rapidly thereafter. However, for this galaxy sample, the assumption that the real-space galaxy power spectrum traces the nonlinear matter power spectrum remains quite accurate, to $1\%$ at $k \leq 0.2h\text{Mpc}^{-1}$. The Dotted curve shows the $Q$-model prediction of equation (2.2) with $Q = 10.3$, which gives a least-squares fit to our predicted bias curve over range
$0.01h\text{Mpc}^{-1} < k < 0.3h\text{Mpc}^{-1}$. The largest difference is at $k \simeq 0.04h\text{Mpc}^{-1}$, where the $Q$-model predicts 5% deviation from $P_{\text{lin}}(k)$ while we find 3%. The $Q$-model traces the predicted bias shape well beyond $k = 0.1h\text{Mpc}^{-1}$. However, if curves are normalized to match at $k \simeq 0.04h\text{Mpc}^{-1}$, as might well be the case in practice with large statistical uncertainties at low $k$, then deviations in the bias shape would be smaller at $k < 0.1h\text{Mpc}^{-1}$ and larger at $k > 0.1h\text{Mpc}^{-1}$.

Figure 2.4b plots $P_{Z\rightarrow R}(k)$ (solid) and $P_0(k)$ (dashed), with no FoG compression. The bias shape for $P_{Z\rightarrow R}(k)$ is qualitatively similar to the true real-space $P_R(k)$, while it follows the $P_{\text{lin}}(k)$ shape more closely than the $P_{\text{nl}}(k)$ shape at $k \leq 0.05h\text{Mpc}^{-1}$. $P_{Z\rightarrow R}(k)$ is poorly described by the $Q$-model prescription, with differences of 6% at $k \simeq 0.05h\text{Mpc}^{-1}$. The bias shape for $P_0(k)$ is completely different from those for $P_R(k)$ and $P_{Z\rightarrow R}(k)$. By $k = 0.1h\text{Mpc}^{-1}$, the $P_0(k)$ shape is below $P_{\text{lin}}(k)$ by 10% and $P_{\text{nl}}(k)$ by 5%, and remains below $P_{\text{lin}}(k)$ by 10% to $k = 0.5h\text{Mpc}^{-1}$. The best-fit $Q$-model also fails to describe our $P_0(k)$ calculation with the few percent level differences at $k \leq 0.1h\text{Mpc}^{-1}$.

Figures 2.4c and 2.4d illustrate the effects of FoG compression on $P_{Z\rightarrow R}(k)$ and $P_0(k)$. Minor changes arise to $P_{Z\rightarrow R}(k)$, though compressing FoG effects significantly helps $P_{Z\rightarrow R}(k)$ recover the true real-space $P_R(k)$ at $k \geq 0.25h\text{Mpc}^{-1}$. Compared to $P_{Z\rightarrow R}(k)$, FoG compression gives rise to substantial changes to $P_0(k)$ at $k \geq 0.25h\text{Mpc}^{-1}$, since the nonlinear effect arising from the virial motion of satellites adds up in $P_0(k)$, while it tends to cancel out in $P_{Z\rightarrow R}(k)$. Notice, however, that the
$P_0(k)$ shape is quite far from the true real-space $P_R(k)$, even with the aggressive FoG compression with $\sigma_h \geq 400$ km s$^{-1}$.

Figure [2.5] plots the scale-dependent bias functions of the galaxy sample with $M_r \leq -21$, in the same format as Figure [2.4]. Compared to the $M_r \leq -20$ galaxy sample, this galaxy sample is more biased by 20% and hence their power spectra differ by nearly 50%. However, note that the scale-dependent bias shapes are almost identical for the two galaxy samples over a large dynamic range $k \leq 0.3h$Mpc$^{-1}$.

The $Q$-model prescription also provides descriptions of our predictions for $P_R(k)$, $P_{Z-R}(k)$, and $P_0(k)$ at the 5% level at $k \leq 0.3h$Mpc$^{-1}$.

2.4.2. Uncertainties in Recovering $P_{\text{lin}}(k)$

Our method for calculating $b^2(k)$ is that we first determine HOD parameters by fitting $w_p(r_p)$ measurements given a cosmological model. With perfect knowledge of HOD and cosmological parameters, it should be possible in principle that we can calculate $b^2(k)$ exactly and correct observed galaxy power spectrum to recover $P_{\text{lin}}(k)$ up to high $k$. However, uncertainties in the parameters result in uncertainty in our $b^2(k)$ calculations. Therefore, the dependence of $b^2(k)$ on our adopted fiducial HOD and cosmological model will increase uncertainty in cosmological parameter estimation relative to measurement errors alone. We define a recovered linear matter power spectrum $P_{\text{rec}}(k)$, constructed by our scale-dependent bias shapes $b^2(k)$ of the
fiducial model and observable galaxy power spectra, $P_{\text{rec}}(k) \equiv P_{\text{obs}}(k)/b^2(k)$. Note that $P_{\text{rec}}(k)$ recovers $P_{\text{lin}}(k)$ by construction on all scales if our assumed fiducial model is correct. Therefore, we investigate the dependence of recovering the $P_{\text{lin}}(k)$ shape on the assumptions built into the scale-dependent bias shapes, which will provide the robustness of our scale-dependent bias shapes.

We first investigate the uncertainties in the power spectrum recovery associated with our HOD modeling. We adopt a more flexible HOD parametrization from Zheng et al. (2005) to relax the constraints arising from the functional form of our adopted HOD parametrization in §2.2.1. Then we generate a Markov Chain Monte Carlo based on the Metropolis-Hastings algorithm with the new HOD parametrization by fitting the $w_p(r_p)$ measurements to estimate uncertainties in HOD parameter determination. The mean occupation function for central galaxies is represented by

$$
\langle N_{\text{cen}} \rangle_M = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\log M - \log M_{\text{min}}}{\sigma_{\log M}} \right) \right],
$$

(2.20)

where erf($x$) is the error function. This general parametrization includes the feature of a sharp cut-off at $M_{\text{min}}$ that we adopted in §2.2.1. The characteristic transition width $\sigma_{\log M}$ permits a smooth transition from empty halos to halos populated with a central galaxy, described by a Gaussian distribution of $\log M$ at fixed halo mass. The
distribution of $N_{cen}$ about the mean is a nearest-integer or Bernoulli distribution. The mean occupation function for satellite galaxies is

$$\langle N_{sat} \rangle_M = \left( \frac{M - M_0}{M'_{0}} \right)^{\alpha},$$  \hspace{1cm} (2.21) 

for $M > M_0$, and halos of $M \leq M_0$ are devoid of satellite galaxies. Note that we allow the power-law slope $\alpha$ to vary from one. Figure 2.6 shows 10 HOD models, randomly chosen from the Markov Chain Monte Carlo with $\chi^2 \leq 1$ relative to the best-fit HOD models of the $M_r \leq -20$ and $M_r \leq -21$ samples, respectively. These 10 HOD models represent 1-$\sigma$ errors in the $P_{lin}(k)$ shape recovery, arising from the parametrization and the determination of parameters.

The left panels show the mean occupation functions of the 10 HOD models. Even with more degrees of freedom in the HOD parametrization, the resulting HOD models show negligible variations in the mean occupation functions, while larger uncertainties in the $w_p(r_p)$ measurements of the $M_r \leq -20$ sample allow relatively larger variations than for the $M_r \leq -21$ sample. The right panels show $P_{rec}(k)$ of the 10 HOD models relative to the true $P_{lin}(k)$ using the bias shape of the best-fit HOD model. We only show $P_{rec}(k)$ using $P_R(k)$, noting that FoG effects are similar for the fixed cosmological parameters. $P_{rec}(k)$ for the $M_r \leq -20$ sample recovers $P_{lin}(k)$ at the 4% level at $k \geq 0.3h\text{Mpc}^{-1}$ and is perfectly accurate in practice on large scales. For the $M_r \leq -21$ sample, the accuracy is better by a factor of two due to the smaller measurement error in $w_p(r_p)$. 

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Current uncertainties in HOD parameters contribute less than 5% \((1-\sigma)\) uncertainty to \(P_{\text{rec}}(k)\) up to \(k = 0.5h\text{Mpc}^{-1}\) for the \(M_r < -20\) sample, and less than 3% for the \(M_r < -21\) sample. However, note that these HOD uncertainties can be reduced by better \(w_p(r_p)\) measurements and by additional constraints such as group multiplicity function. Therefore, they are unlikely to make major contribution to overall error budget in \(P_{\text{rec}}(k)\).

We have assumed no environmental dependence of our HOD model calculation. However, it is recently shown that there is a strong correlation for lower mass halos with older halos more strongly correlated \(\text{(e.g., Gao et al. 2005; Harker et al. 2006)\). However, Croton et al. (2006) use the Millennium simulation to investigate the environmental dependence of the halo occupation function and find that the environmental dependence changes galaxy bias factor for luminosity thresholded samples by a few percent, but with virtually no scale-dependence. Our HOD parameters are insensitive to large-scale amplitude of \(w_p(r_p)\), being driven mainly by shape of \(w_p(r_p)\) in a regime where contribution of galaxy pairs from a single halo and two distinct halos are comparable. Therefore, we suspect that the environment dependence at the level predicted by Croton et al. (2006) would have few percent impact on predicted asymptotic galaxy bias factor \(b^2_{\text{0}}\), but probably much smaller effect on scale-dependence of \(b^2(k)/b^2_{\text{0}}\). However, this issue will require further investigation in future work.}
Now we investigate the uncertainties in the power spectrum recovery associated with our adopted fiducial cosmological model. In Figure 2.7, we first find the best-fit HOD parameters to fit the $w_p(r_p)$ measurements given the true cosmological model listed in the legend and predict observable galaxy power spectra $P_{\text{obs}}(k)$. $P_{\text{rec}}(k)$ is then constructed by using $b^2(k)$ of our adopted fiducial cosmological model. Note that the ratio of $P_{\text{rec}}(k)$ to $P_{\text{lin}}(k)$ would be flat if our fiducial model were correct. In Figure 2.7a, we consider the $\sigma_8$ variations. While we need to assume a value of $\sigma_8$ in the fiducial model to obtain HOD parameters by fitting $w_p(r_p)$, measurements of the galaxy power spectrum and correlation function have little leverage to constrain $\sigma_8$, which is almost degenerate with galaxy bias. Therefore, the $\sigma_8$ values should be marginalized over in constructing $P_{\text{rec}}(k)$, which results in the maximum error of 4% at $k \simeq 0.2h\,\text{Mpc}^{-1}$.

In Figure 2.7b, we consider the variations of the spectral index $n_s$. The power spectrum recovery is independent of the assumed value of $n_s$ on large scales. However, small differences arise at $k \geq 0.1h\,\text{Mpc}^{-1}$ given the $w_p(r_p)$ constraints on small scales. For comparison, we also show the difference in the shape of $P_{\text{lin}}(k)$ with different $n_s$, normalized at $k = 0.05h\,\text{Mpc}^{-1}$ as in practice. Note that in principle we can compute $b^2(k)$ for each $n_s$ value and check the self-consistency of the slope of $P_{\text{rec}}(k)$ at low $k$, instead of using $b^2(k)$ of the fiducial model. Figure 2.7c plots the changes from the Hubble constant $h$ variations. Faster expansion gives rise to a smaller horizon size at the matter-radiation equality epoch, leading to a turn-over in
$P_{\text{lin}}(k)$ at higher $k$. The difference at $k \geq 0.1h\text{Mpc}^{-1}$ is larger than in Figure 2.7b, because the fixed normalization $\sigma_8$ accompanies the vertical shift of the $P_{\text{lin}}(k)$ shape.

We have so far assumed that satellite galaxies in halos trace the dark matter distribution characterized by a NFW profile with the same concentration parameter $c_{\text{dm}} = c_{\text{gal}}$. Though we do not plot in Figure 2.7, we find that less than 1% uncertainties arise at $k \leq 0.5h\text{Mpc}^{-1}$ when we consider lower concentration parameters for satellite galaxies as suggested in numerical simulations (e.g., Nagai & Kravtsov 2005). While we only show $P_{\text{rec}}(k)$ using $P_R(k)$, the overall trend remains unchanged for the other observable power spectra $P_{Z-R}(k)$ and $P_0(k)$, with varying levels of FoG compression, over $0.01h\text{Mpc}^{-1} \leq k \leq 0.5h\text{Mpc}^{-1}$.

In Figure 2.8, we show the galaxy power spectra of the same model sequence in Figure 2.7 and their ratio to the fiducial model prediction of $P_R(k)$ is shown in the bottom panel, normalized at $k = 0.05h\text{Mpc}^{-1}$ to account for large statistical uncertainties present in power spectrum measurements at lower $k$. The uncertainty in the assumed value of $\sigma_8$ has small impact on the prediction of $P_R(k)$ or equivalently power spectrum recovery; the long dashed line is nearly identical to the fiducial model (solid) within the current measurement uncertainty. The variations of $n_s$ and $h$ show little change in the power spectrum shape when normalized at $k = 0.05h\text{Mpc}^{-1}$, while the difference in the amplitude is at the 10% level at $k \lesssim 0.05h\text{Mpc}^{-1}$, where measurement uncertainty is largest.
In summary, our method provides $P_{\text{obs}}(k)$ or $b^2(k)$ uniquely, once cosmological parameters are specified. However, with the present level of power spectrum measurement uncertainties, the parameters $\sigma_8$, $n_s$, and $h$ can be considered as nuisance parameters in the fiducial model when computing $b^2(k)$, since their impact on the power spectrum recovery is relatively small compared to the statistical uncertainty.

2.5. **Bias Function for Luminous Red Galaxies**

While we have focused on the SDSS main galaxy samples so far, the most precise measurements in galaxy power spectrum come from the luminous red galaxy (LRG) sample (Eisenstein et al. 2001) because of the large effective volume that LRGs span. However, there are two additional complications in considering this sample. First, fewer simulations are available to test our analytic model over a large volume with mass resolution high enough to resolve group or cluster halos that LRGs reside in. Second, constraints on HOD parameters of LRGs are not yet thoroughly investigated. Here we simply adopt HOD parameters from Zheng et al. (2007) and ignore the impact of uncertainties in these parameters for the moment. It will allow us to understand how the scale-dependence of LRG bias may differ from that of the Sloan main galaxies, while complete investigation of HOD parameters and their uncertainties will be required before applying our method to real LRG data.
Zheng et al. (2007) obtain the HOD parameters of the LRG samples with absolute-magnitude limit $-23.2 \leq M_g \leq -21.2$ and $-23.2 \leq M_g \leq -21.8$ by matching the projected correlation function $w_p(r_p)$ and mean space density $\bar{n}_g = 9.7 \times 10^{-5} (h^{-1}\text{Mpc})^{-3}$ and $2.4 \times 10^{-5} (h^{-1}\text{Mpc})^{-3}$, and by accounting for error covariance matrix taken from Zehavi et al. (2005a) given the best-fit $\Lambda$CDM cosmological parameter of Tegmark et al. (2006) (see Table 1 of Zheng et al. 2007 for the HOD parameters). LRGs are predominantly central elliptical galaxies of group or cluster sized halos $M \gtrsim 10^{13.5} h^{-1} M_\odot$, and their mean redshift is $\bar{z} = 0.3$. Note that our analytic model is tested against $N$-body simulations in a regime adequate for galaxy samples at $z = 0$ residing in lower mass halos. With only a handful of large-volume $N$-body simulations just available for the analysis of LRG clustering, it is necessary to test our analytic model against those large-volume $N$-body simulations at earlier redshifts for further investigation. Here we use a new large $N$-body simulation (Warren et al. 2006) to test our analytic model predictions in a regime adequate for LRGs. The simulation is performed with the Hashed Oct-Tree code (Warren & Salmon 1993), evolving $1024^3$ particles in a volume of comoving $1086 h^{-1}\text{Mpc}$ on a side from $z = 34$ to the present, using a $\Lambda$CDM cosmology ($\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, $\Omega_b = 0.046$, $h = 0.7$, $n_s = 1$, and $\sigma_8 = 0.9$). Dark matter halos are identified using the friends-of-friends algorithm with $b = 0.2$. Since we only have the simulation output at $z = 0$, we cannot directly compute quantities of interest at
the mean redshift of LRGs ($\bar{z} = 0.3$), but we can test whether our analytic model is accurate.

Figure 2.9 plots the real-space and redshift-space monopole correlation functions measured from the $N$-simulation at $z = 0$, populated by using the HOD parameters of the LRG sample with higher $n_g$, obtained by Zheng et al. (2007). The error bars are computed by Jackknife resampling of the eight octants of the cube. Our analytic predictions are in good agreement with the populated $N$-body simulation. The marginally significant discrepancy at $r = 4h^{-1}\text{Mpc}$ could represent that our halo exclusion is inaccurate at the 10% level, while it is difficult to assess the statistical significance of the discrepancy with only one simulation. Figure 2.10 shows the analytic predictions for $P_R(k)$ and $P_0(k)$, where the Poisson errors are computed from the measured number of Fourier modes in the simulation. The analytic model provides good approximations to $P_R(k)$ and $P_0(k)$ at $k < 0.3h\text{Mpc}^{-1}$. The shot-noise power spectrum dominates the measurements of $P_R(k)$ and $P_0(k)$ at $k > 0.3h\text{Mpc}^{-1}$, and the discrepancy at $k \approx 0.3h\text{Mpc}^{-1}$ may indicate that our shot-noise subtraction scheme is imperfect. Nevertheless, more simulations are necessary to better quantify the statistical significance of the deviation at $k \approx 0.3h\text{Mpc}^{-1}$. The $N$-body test at $z = 0$ shows that our analytic model predicts the correlation functions and power spectra in the LRG regime with reasonable accuracy, and we suspect that these predictions would remain accurate at $z = 0.3$. 

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Figure 2.11 plots the scale-dependence of LRG bias at $z = 0.3$, predicted by using the analytic model. Note that we now use the cosmological model, consistent with the WMAP3 results (Zheng et al. 2007 adopt to obtain the best-fit HOD parameters). Noticeably, the bias shapes are in marked contrast to those for the $M_r \leq -21$ and $M_r \leq -20$ samples that closely follow the nonlinear matter power spectrum (circles). This is mainly because the fraction of satellite galaxies is small $\approx 5\%$ in the LRG samples, and because their host halos are massive ($\gg 10^{13} h^{-1} M_\odot$) and hence highly clustered. Moreover, LRG pairs in a single halo have significant impact on the power spectra even at $k \approx 0.2 h \text{Mpc}^{-1}$ due to the large virial radii of halos that host LRGs, leading to the larger departure from $P_{nl}(k)$. The low fraction of satellite galaxies also suppresses the redshift-space multipoles arising from virial motions of satellite galaxies in halos, and the $P_{Z-R}(k)$ and $P_0(k)$ shapes have little difference compared to the $P_R(k)$ shape. In Figure 2.11c and 2.11d, relatively small changes in the bias shapes arise between two thresholds on FoG compression, because both cases basically suppress all the halos that host two or more LRGs.

The dotted curves plot the $Q$-model prescription with $Q = 30.3$, used in Tegmark et al. (2006). The good fit of the $Q$-model prescription in Tegmark et al. (2006) is somewhat surprising, because the quantity they measure is somewhere between our $P_{Z-R}(k)$ and $P_0(k)$ with FoG compression threshold $\sigma_h = 750 \text{ km s}^{-1}$. Note that Tegmark et al. (2006) put more weight on $P_0(k)$ than $P_2(k)$ or $P_4(k)$ when computing their $P_{2g}(k)$ due to the larger uncertainties in measurements of $P_2(k)$.
and $P_t(k)$ (see, the modeling approach described in § 2.3), and hence $P_{gg}(k)$ is not exactly the same as our $P_{Z-R}(k)$, while we suspect it is close to our $P_{Z-R}(k)$ and $P_0(k)$. However, the difference in the scale-dependent bias shape could perhaps be explained by differences in how FoG effects are compressed \textit{provided} that the Zheng et al. (2007) HOD is approximately correct. Though we need to fully quantify the uncertainties in our adopted HOD parameters for the LRG samples, our result shows that it would favor a model with more small scale power than found in Tegmark et al. (2006), if the uncertainties were indeed small. Nevertheless, the $Q$-model prescription with high value of $Q$ approximately captures the scale-dependence of LRG bias at $k < 0.08h\text{Mpc}^{-1}$, while the discrepancy at $k > 0.08h\text{Mpc}^{-1}$ is worrisome. Two dashed curves in Figure 2.11c show the $Q$-model prescription with $Q = 20.7$ and 17.3 that best fit our calculations of $P_{Z-R}^{750}(k)$ and $P_0^{750}(k)$. While the lower $Q$-values are favored to minimize the deviations at $k > 0.1h\text{Mpc}^{-1}$, it is less likely that the $Q$-model provides a correct description of LRG bias for any value of $Q$.

Figure 2.12 plots the scale-dependent bias of the brighter LRG sample, in the same format as Figure 2.11. This LRG sample is more biased by 15%, compared to the previous LRG sample and yet shows less scale-dependence at $k \lesssim 0.1h\text{Mpc}^{-1}$: the linear bias approximation is accurate to 5%. The best-fit $Q$-model prescription with $Q = 14.9$ and 12.9 can provide a reasonably ($\sim 5\%$) good description of the LRG bias, while it also favors a model with more small scale power than found
in Tegmark et al. (2006). This conclusion remains unchanged for power spectrum measurement of the combined sample of the LRGs with the two different thresholds, since the combined sample is close to the sample with higher number density.

2.6. Summary

We have developed an analytic model to predict observable galaxy power spectra $P_{\text{obs}}(k)$ for specified cosmological and galaxy HOD parameters, and verified its applicability to power spectrum analysis using $N$-body simulations. As $P_{\text{obs}}(k)$, we have considered the real-space $P_R(k)$, redshift-space monopole $P_0(k)$ and pseudo real-space $P_{Z-R}(k)$, with varying levels of FoG compression for the latter two. We have used the analytic model to extend recovery of the primordial matter power spectrum by calculating scale-dependent bias functions $b^2(k) = P_{\text{obs}}(k)/P_{\text{lin}}(k)$ when HOD parameters are determined to fit the number density $\bar{n}_g$ and projected correlation function $w_p(r_p)$ of the observed SDSS galaxy samples given a specified cosmological model. Our main findings are as follows:

1. Our analytic model for calculating $w_p(r_p)$ follows the method described in Tinker et al. (2006), with the improved treatment of the scale-dependent halo bias and ellipsoidal halo exclusion corrections. Drawing on the Tinker (2007) model for redshift-space distortion, the analytic model is extended to incorporate calculating real-space and redshift-space power spectra. We have tested its predictions for
$w_p(r_p)$ and $P_{\text{obs}}(k)$ against populated $N$-body simulations, spanning cosmological parameter range $\Omega_m = 0.1 - 0.63$ and $\sigma_8 = 0.6 - 0.95$ with HOD parameters matched to represent two SDSS galaxy samples with absolute-flux limits $M_r \leq -20$ and $M_r \leq -21$ (Zehavi et al. 2005b). The analytic model reproduces the numerical results of $w_p(r_p)$ to 5% or better, and the predictions of $P_{\text{obs}}(k)$ are consistent with the numerical results to 2% at $k = 0.1 - 1h\text{Mpc}^{-1}$ and to 10% at $k = 0.25 - 0.1h\text{Mpc}^{-1}$, though the finite box size of the simulations makes it difficult to assess the statistical significance at low $k \leq 0.1h\text{Mpc}^{-1}$.

2. For the $M_r \leq -20$ galaxy sample, $P_{Z\rightarrow R}(k)$ recovers $P_R(k)$ to 2% at $k \leq 0.2h\text{Mpc}^{-1}$, while the deviation is already 10% at $k = 0.1h\text{Mpc}^{-1}$ if the scaled $P_0(k)$ is used to recover $P_R(k)$. $P_{Z\rightarrow R}(k)$ is more robust to nonlinearity than $P_0(k)$, since the redshift-space multipoles deviate from the linear theory predictions on nonlinear scales, but their departures are partially canceled out in the linear combination, reducing the deviation of $P_{Z\rightarrow R}(k)$. However, it becomes substantial at $k \geq 0.3h\text{Mpc}^{-1}$, resulting from the contributions of higher-order multipoles and changes in amplitude of $P_0(k)$, $P_2(k)$, and $P_4(k)$. This can be partly remedied by FoG compression, removing the higher-order multipoles mainly from the random motions of satellite galaxies in a halo; $P_{Z\rightarrow R}^{750}(k)$ can recover $P_R(k)$ to 5% at $k \leq 0.45h\text{Mpc}^{-1}$, and at higher $k$ for $P_{Z\rightarrow R}^{400}(k)$, though $P_0^{400}(k)$ can only achieve 10% accuracy at $k \leq 0.3h\text{Mpc}^{-1}$.
3. The nonlinear matter power spectrum accurately describes the real-space power spectra to 1% at $k \leq 0.2 h \text{Mpc}^{-1}$ for the $M_r \leq -20$ and $M_r \leq -21$ galaxy samples. The bias shape for $P_{Z\rightarrow R}(k)$ is qualitatively similar to $P_R(k)$ at $k \leq 0.3 h \text{Mpc}^{-1}$, whereas the bias shape for $P_0(k)$ is completely different over the entire range we consider here. FoG compression brings about little change in the bias shape for $P_{Z\rightarrow R}(k)$, but substantial change at $k \geq 0.2 h \text{Mpc}^{-1}$ arises in the bias shape for $P_0(k)$ when combined with FoG compression, though the difference persists in the bias shapes for $P_{Z\rightarrow R}(k)$ and $P_0(k)$. The $Q$-model prescription traces our calculation of $P_R(k)$ well at $k \geq 0.1 h \text{Mpc}^{-1}$ and differs at a few percent level at low $k$, while it poorly describes the calculations of $P_{Z\rightarrow R}(k)$ and $P_0(k)$.

4. Systematic uncertainties in recovering $P_{\text{lin}}(k)$ arise in our method from the adopted fiducial model of HOD and cosmology. We have tested the former by adopting a flexible HOD parametrization with more freedom to explore all the plausible halo occupation functions. For the $M_r \leq -20$ sample, the uncertainty is 2% at $k = 0.2 h \text{Mpc}^{-1}$, progressively becomes smaller at lower $k$, and climbs up to 4% at $k = 0.5 h \text{Mpc}^{-1}$. The uncertainty is a factor of two smaller for the $M_r \leq -21$ sample, roughly equivalent to the ratio of the fractional measurement errors of the two samples. We have investigated the other systematic uncertainty by assuming the range of current parameter uncertainties. Cosmological parameters that are less constrained in power spectrum measurement give rise to uncertainty of $\sim 5%$ at $k = 0.3 h \text{Mpc}^{-1}$ in recovering the $P_{\text{lin}}(k)$ shape, and smaller uncertainty at lower
These two types of systematic uncertainty can be accurately computed and marginalized over in deriving parameter estimates.

5. We have used one $N$-body simulation at $z = 0$ (Warren et al. 2006) with large volume and high mass resolution adequate for the LRG clustering analysis, and we have tested our analytic model predictions against the numerical results, obtained by populating the simulation with the HOD parameters (Zheng et al. 2007) to represent the LRG sample. The analytic model provides predictions for $\xi_0(r)$ and $\xi_R(r)$ accurate to 5% or better over the range of $0.1 h^{-1}\text{Mpc} \leq r \leq 20 h^{-1}\text{Mpc}$, and the predictions for $P_0(k)$ and $P_R(k)$ with the same accuracy over the range of $0.35 h\text{Mpc}^{-1} \leq k \leq 0.2 h\text{Mpc}^{-1}$.

6. For the LRG sample with absolute-magnitude limit $-23.2 \leq M_g \leq -21.8$, the linear bias approximation is accurate to 2% or better at $k \leq 0.08 h\text{Mpc}^{-1}$ and its accuracy rises to 5% at $k = 0.1 h\text{Mpc}^{-1}$. The linear bias is less accurate for the LRG sample with higher number density and absolute-magnitude limit $-23.2 \leq M_g \leq -21.2$, to 2% at $k \leq 0.05 h\text{Mpc}^{-1}$ and to 10% at $k = 0.1 h\text{Mpc}^{-1}$. $P_{Z\rightarrow R}(k)$ and $P_0(k)$ have shapes similar to $P_R(k)$ over the range of $0.01 h\text{Mpc}^{-1} \leq k \leq 0.3 h\text{Mpc}^{-1}$, since LRGs are mainly central galaxies in massive halos, suppressing the redshift-space multipoles due to the random motions of satellite galaxies. Compared to the LRG sample with higher $\bar{n}_g$, the brighter LRG sample is less biased to $P_{\text{lin}}(k)$ at $k \geq 0.1 h\text{Mpc}^{-1}$ due to the lower fraction of satellite galaxies. The $Q$-model prescription poorly describes all of the LRG power
spectra, since its functional form is too restricted for the linear bias over the range of $0.01h\text{Mpc}^{-1} \leq k \leq 0.1h\text{Mpc}^{-1}$. Especially, the discrepancy is maximum at $k = 0.04 - 0.2h\text{Mpc}^{-1}$, where the scale-dependent bias correction is most important.

With the release of SDSS DR5, the measurement precision has increased substantially, compared to the data we analyzed here. Therefore, $b^2(k)$ or $P_{\text{obs}}(k)$ can be more accurately computed by providing tighter constraints on HOD parameters used in our method. However, LRG sample has emerged as the most powerful probe in galaxy power spectrum measurements due to its largest effective volume. Adopting the HOD parameters for LRGs from Zheng et al. (2007) and ignoring the uncertainties associated with these parameters, we have demonstrated the applicability of our method and investigated $b^2(k)$ of the LRG sample. Though we suspect that these predictions would remain similar for a full analysis accounting for uncertainties in the adopted HOD parameters, careful investigation of the LRG HOD parameters is necessary to apply our method to the SDSS-II LRG sample, taking full advantage of precise measurements on quasi-linear scales that are marginalized over by adopting the $Q$-model prescription in previous analysis. Given the growth of current and future galaxy surveys in depth and redshift, it is also desirable to refine our method to provide accurate estimates of the broad band shape of power spectrum at $k \simeq 0.2h\text{Mpc}^{-1}$, considering the recent attention to the baryonic acoustic oscillations in galaxy power spectrum as a standard ruler. Precise
measurement of the primordial matter power spectrum will play a crucial role in constraining cosmological parameters and testing dark energy models.
Fig. 2.1.— Projected correlation functions $w_p(r_p)$ for SDSS galaxy samples with absolute-magnitude limits $M_r \leq -21$ (upper points) and $M_r \leq -20$ (lower points). Solid lines represent the best-fit analytic model predictions of $w_p(r_p)$ for five different combinations of cosmological parameters, with increasing $\sigma_8$ and decreasing $\Omega_m$ from lowest to highest curves (see Table 2.6). Shaded regions show the statistical uncertainty of the central cosmological model only, computed from the error on the mean of five independent $N$-body simulations. For comparison, we plot the $w_p(r_p)$ measurements of the galaxy samples from Zehavi et al. (2005b), and the error bars are the diagonal elements of the covariance matrix.
Fig. 2.2.— Dimensionless real-space and redshift-space multipole power spectra $\Delta^2(k)$ for the SDSS galaxy sample with $M_r \leq -20$. Various curves represent the analytic predictions of the central model for the corresponding galaxy power spectra indicated in the legend, and shaded regions are the statistical uncertainty on the real-space galaxy power spectrum, computed from the five $N$-body simulations. The light, short dashed curve, labeled $P_{Z,R}(k)$ (see, eq. [2.19]), is a linear combination of redshift-space multipoles that reduces to $P_R(k)$ in linear regime; it is largely obscured by the solid curve. Thick dashed curves represent the linear (long) and nonlinear (short) matter power spectra. Note that the quadrupole $P_2(k)$ crosses zero at $k = 0.3h\text{Mpc}^{-1}$, and $-P_2(k)$ is used at larger $k$. 
The upper panel plots pseudo real-space galaxy power spectra and redshift-space monopoles with and without Finger-of-God (FoG) compression relative to the analytic model prediction for $P_R(k)$, assuming HOD parameters that fit the SDSS galaxy sample with $M_r \leq -20$. Thresholds for the FoG compression are 400 km s$^{-1}$ and 750 km s$^{-1}$, as indicated in the legend. The redshift-space monopoles are scaled by a constant factor predicted by linear theory to match $P_R(k)$ at large scales. The bottom panel shows the fractional difference between the analytic model calculations and simulation results on the corresponding galaxy power spectra. Shaded regions represent fractional statistical uncertainty on $P_R(k)$ from the N-body results. The other power spectra have larger statistical uncertainties due to the finite simulation volume.
Fig. 2.4.— Bias shapes $P_R(k)$, $P_{Z\rightarrow R}(k)$, and $P_0(k)$ of the galaxy sample with $M_r \leq -20$. Individual panels assume the different thresholds of FoG compression. Different curves represent bias shapes for the power spectra indicated in the legend and dotted curves are bias shapes of the $Q$-model that best fit to our predictions (see text). For comparison, we plot the nonlinear matter power spectrum relative to the linear matter power spectrum as circles.
Fig. 2.5.— Bias shapes $P_R(k)$, $P_{Z-R}(k)$, and $P_0(k)$ of the galaxy sample with $M_r \leq -21$, in the same format as Fig. 2.4.
Fig. 2.6.— Effects of systematic errors in our adopted HOD parametrization and uncertainties in HOD parameters on recovering the linear matter power spectrum from the galaxy samples with $M_r \leq -20$ and $M_r \leq -21$. Solid curves show 10 HOD models with $\chi^2 \leq 1$ (relative to the best-fit HOD model) from a Markov Chain Monte Carlo obtained by fitting the $w_p(r_p)$ data from Zehavi et al. (2005b) with a flexible parametrization of HOD. These models represent 1 − $\sigma$ errors in our parametrization and parameters. Left panels: mean halo occupation functions of total and satellite galaxies. Right panels: ratios of recovered power spectrum to the true linear matter power spectrum for the ten HOD models.
Fig. 2.7.— Impact of uncertainties in the fiducial cosmological model on the power spectrum recovery. The true cosmological model is assumed to have the value of the parameter in the legend, different from the fiducial model \((\sigma_8 = 0.9, n_s = 1.0, h = 0.7)\), and \(P_{\text{rec}}(k) = P_{\text{obs}}(k)/b^2(k)\) is obtained by using \(b^2(k)\) of the fiducial model. Here we only show the case of \(P_{\text{obs}}(k) = P_R(k)\). The circles in the panel (b) and (c) show the ratio of the true \(P_{\text{lin}}(k)\) to \(P_{\text{lin}}(k)\) of the fiducial model.
Fig. 2.8.— Differences in the galaxy power spectra of the cosmological models with the varying parameter listed in the legend. The bottom panel shows the ratio in $P_R(k)$ relative to the fiducial model, normalized at $k = 0.05h$ Mpc$^{-1}$. 
Fig. 2.9.— Test of analytic model predictions for real-space $\xi_R(r)$ and redshift-space monopole $\xi_0(r)$. Here we take the HOD parameters for the luminous red galaxy (LRG) sample from Zheng et al. (2007) as an input, and populate one large volume $N$-body simulation at $z = 0$. Symbols represent measurements of $\xi_R(r)$ and $\xi_0(r)$ from the simulation and curves represent analytic model predictions. The attached bottom panel shows the fractional differences between the analytic model calculations and the simulation results. Statistical uncertainties are computed by Jackknife resampling of the eight octants of the simulation box and shown as error bars for $\xi_R(r)$ and shaded regions for $\xi_0(r)$. 
Fig. 2.10.— Test of analytic model predictions for $P_R(k)$ and $P_0(k)$, in the same format as in Fig. 2.9. The statistical uncertainties are computed by assuming a Poisson distribution of Fourier modes measured from the simulation. The thin dashed curves are bias shapes of the $Q$-model that best fit to the simulation results.
Fig. 2.11.— Bias shapes $P_R(k)$, $P_{Z-R}(k)$, and $P_0(k)$ of the luminous red galaxy (LRG) sample with $-23.2 \leq M_g \leq -21.2$, in the same format as Fig. 2.4. Adopting the Zheng et al. (2007) HOD parameters, we use the analytic model to predict the scale-dependence of LRG bias at $z = 0.3$. The dotted curve in each panel plots the $Q$-model prescription with $Q = 30.3$ used in Tegmark et al. (2006), and two short dashed curves in panel (c) show the $Q$-model prescription with $Q = 20.7$ and 17.3 that best fit our calculations of $P_{Z-R}^{750}(k)$ and $P_0^{750}(k)$, which approximate the decorrelated measurements of galaxy power spectrum in Tegmark et al. (2006).
Fig. 2.12.— Bias shapes $P_R(k)$, $P_{Z\rightarrow R}(k)$, and $P_0(k)$ of the luminous red galaxy (LRG) sample with $-23.2 \leq M_g \leq -21.8$, in the same format as Fig. 2.11. The best-fit $Q$-values in panel (c) are 14.9 and 12.9 for $P_{Z\rightarrow R}^{750}(k)$ and $P_0^{750}(k)$, respectively.
Note. — The HOD parameters of the five \( N \)-body models are determined to reproduce the same clustering \( w_p(r_p) \) of the SDSS galaxy samples with \( M_r \leq -20 \) and \( M_r \leq -21 \), and to match the number densities \( \bar{n}_g = 5.74 \times 10^{-3} \ (h^{-1}\text{Mpc})^{-3} \) for the \( M_r \leq -20 \) sample and \( \bar{n}_g = 1.17 \times 10^{-3} \ (h^{-1}\text{Mpc})^{-3} \) for the \( M_r \leq -21 \) sample, respectively.

Table 2.1. HOD Parameters of the five \( N \)-body models

<table>
<thead>
<tr>
<th>Model</th>
<th>( \Omega_m )</th>
<th>( \sigma_8 )</th>
<th>( M_{\text{min}}(h^{-1}M_\odot) )</th>
<th>( M_1(h^{-1}M_\odot) )</th>
<th>( M_{\text{cut}}(h^{-1}M_\odot) )</th>
<th>sample</th>
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<td>( 2.95 \times 10^{11} )</td>
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<td>( M_r \leq -20 )</td>
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<tr>
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<td>( 1.38 \times 10^{13} )</td>
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Chapter 3

From Galaxy-Galaxy Lensing to Cosmological Parameters

3.1. Introduction

In the current paradigm of structure formation, galaxies form by the dissipative collapse of baryons in halos of cold dark matter (CDM). Understanding the relation between the galaxy and dark matter distributions is the key challenge in interpreting the observed clustering of galaxies. Large area imaging surveys have provided a new tool for untangling this relationship, galaxy-galaxy weak lensing, which uses the subtle distortion of background galaxy shapes to measure average mass profiles around samples of foreground galaxies. The last few years have seen rapid growth in this field, with the first tentative detections (Brainerd et al. 1996) giving way to high signal-to-noise ratio measurements over a substantial dynamic range (e.g., Fischer et al. 2000; McKay et al. 2001; Hoekstra et al. 2002; Sheldon et al. 2004; Mandelbaum et al. 2006).
In a cosmological context, the strength of the galaxy-galaxy lensing signal for a given galaxy sample should depend mainly on the mean matter density $\Omega_m$ and the amplitude of dark matter fluctuations $\sigma_8$, since increasing either parameter enhances the average amount of dark matter around galaxies and thereby amplifies the lensing signal.\footnote{Here $\sigma_8$ is the rms linear theory matter fluctuation in spheres of radius $8 \ h^{-1}\text{Mpc}$, with $h \equiv H_0/100 \ \text{km} \ \text{s}^{-1}\text{Mpc}^{-1}$.} In this paper, we develop tools for constraining $\Omega_m$ and $\sigma_8$ with galaxy-galaxy lensing and galaxy clustering measurements, using halo occupation models of galaxy bias that are applicable from the linear regime into the fully non-linear regime. Our approach extends and complements earlier work by Seljak (2000), Guzik & Seljak (2001, 2002), Tasitsiomi et al. (2004), and Mandelbaum et al. (2005).

Galaxy-galaxy lensing measures the profiles of mean tangential shear around galaxies. With knowledge of source and lens redshift distributions, this tangential shear can be converted to excess surface density,

$$\Delta \Sigma(r) \equiv \bar{\Sigma}(<r) - \bar{\Sigma}(r),$$

(3.1)

where $\bar{\Sigma}(<r)$ is the mean surface density interior to the disk of projected radius $r$ and $\bar{\Sigma}(r)$ is the averaged surface density in a thin annulus of the same radius.
The excess surface density profile is itself related to the galaxy-matter cross-correlation function \( \xi_{gm} \) by

\[
\Delta \Sigma (r) = \rho_c \Omega_m \left[ \frac{2}{r^2} \int_0^r \int_{-\infty}^{\infty} r' \xi_{gm} \left( \sqrt{r'^2 + z^2} \right) \, dz \, dr' - \int_{-\infty}^{\infty} \xi_{gm}(r, z) \, dz \right],
\]

(3.2)

where \( \rho_c \) is the critical density of the universe. Johnston et al. (2007) discuss and test methods of inverting \( \Delta \Sigma (r) \) to obtain the three-dimensional \( \xi_{gm}(r) \). Here we treat \( \Delta \Sigma (r) \) as the primary observable and concentrate on predicting it directly.

On large scales, where matter fluctuations are linear, the relation between the matter auto-correlation function \( \xi_{mm} \), the galaxy-matter cross-correlation function \( \xi_{gm} \), and the galaxy auto-correlation function \( \xi_{gg} \) may be adequately described by the linear bias model,

\[
\xi_{gg} = b^2 \xi_{mm},
\]

(3.3)

\[
\xi_{gm} = b \xi_{mm},
\]

(3.4)

where the linear bias factor \( b \) is the same in both equations (Kaiser 1984). Thus, measurements of \( \xi_{gg} \) and \( \Delta \Sigma \propto \xi_{gm} \Omega_m \) can be combined to yield \( \Omega_m \). Since the amplitude of galaxy fluctuations is \( \sigma_{8g} = b \sigma_8 \) in the linear bias model, this method in turn constrains the product \( \sigma_8 \Omega_m \). Redshift-space distortions of the

\footnote{We interchangeably use \( r \) to refer to a projected (two-dimensional) or a three-dimensional radius.}
galaxy power spectrum and the abundance of rich galaxy clusters as a function of mass both depend on a parameter combination that is approximately $\sigma_8 \Omega_{m}^{0.6}$ (Kaiser 1987; White et al. 1993), so the combination of galaxy-galaxy lensing with either of these measurements can break the degeneracy between $\sigma_8$ and $\Omega_m$. However, the linear bias approximation may break down on the scales $r \lesssim$ several $h^{-1}$Mpc where $\Delta \Sigma(r)$ is measured with high precision. Moreover, if the relation between galaxy and matter density contrasts is linear but stochastic, then the linear bias factor $b$ in equation (3.4) should be replaced by $br_{gm}$, where $r_{gm}$ is the galaxy-matter cross-correlation coefficient (Pen 1998; Dekel & Lahav 1999), and the constrained combination becomes $\sigma_8 \Omega_m r_{gm}$ even in the linear regime. The addition of $r_{gm}$ as a free parameter reduces the cosmological constraining power of the $\Delta \Sigma$ and $\xi_{gg}$ combination, and restoring it requires an independent determination of $\xi_{mm}$. Cosmic shear measurements can provide such a determination (Blandford et al. 1991; Miralda-Escudé 1991b; Kaiser 1992), but these measurements are challenging and suffer larger systematic errors than galaxy-galaxy lensing.

Hoekstra et al. (2001, 2002) used imaging and photometric redshift data from the Red-Sequence Cluster Survey (Gladders & Yee 2001) to measure aperture fluctuations proportional to $\xi_{gg}$, $\xi_{gm}$, and $\xi_{mm}$. They provided tentative evidence that $b$ and $r$ are each individually scale-dependent but that the ratio $b/r_{gm}$ is approximately constant, with $b/r_{gm} \approx 1$ for $\Omega_m = 0.3$. Using the Sloan Digital Sky Survey (SDSS, York et al. 2000), which provides spectroscopic redshifts of lens
galaxies and image shapes and photometric redshifts of source galaxies. Sheldon et al. (2004) detected galaxy-galaxy lensing and measured the galaxy-matter correlation function from 0.025 to 10 $h^{-1}\text{Mpc}$ (see Fischer et al. 2000 and McKay et al. 2001 for earlier SDSS measurements, and Mandelbaum et al. 2006 for more recent measurements at $r \leq 2 \ h^{-1}\text{Mpc}$). They found that the galaxy-galaxy correlation function and the galaxy-matter correlation function agree in shape, with an amplitude ratio that implies $b/r_{gm} = (1.3 \pm 0.2)(\Omega_m/0.27)$ for galaxy samples of mean luminosity $\langle L \rangle \sim L_*$.

To circumvent the limitations of the linear bias approximation, we model galaxy-galaxy and galaxy-matter correlations using halo occupation methods, following the lead of Seljak (2000), Berlind & Weinberg (2002), and Guzik & Seljak (2002). The halo occupation distribution (HOD) provides a fully non-linear description of the relation between galaxies and mass by specifying the probability $P(N|M)$ that a halo of virial mass $M$ contains $N$ galaxies of a particular class, along with any spatial or velocity biases within individual halos. Numerous authors have used this approach to compute analytic approximations for galaxy and dark matter clustering statistics (e.g., Ma & Fry 2000; Seljak 2000; Scoccimarro et al. 2001; Seljak 2001; Sheth et al. 2001a; White 2001; see review by Cooray & Sheth 2002).

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3Throughout this paper, we use the term “halo” to refer to a dark matter structure of overdensity $\rho/\rho_m \simeq 200$, in approximate dynamical equilibrium, which may contain a single bright galaxy or a group or a cluster of galaxies.
and to model observed galaxy clustering (e.g., Jing et al. 1998, 2002; Peacock & Smith 2000; Kochanek & White 2001; Bullock et al. 2002; Magliocchetti & Porciani 2003; Yang et al. 2003; Porciani et al. 2004; Zehavi et al. 2004, 2005a; Zheng 2004; Abazajian et al. 2005; Collister & Lahav 2005; Lee et al. 2006; Tinker et al. 2006).

Theoretical predictions for the HOD of different galaxy types have been calculated using semi-analytic models, hydrodynamic simulations, and high resolution \(N\)-body calculations that identify “galaxies” with dark matter substructures (Kauffmann et al. 1997; Benson et al. 2000; White et al. 2001; Yoshikawa et al. 2001; Berlind et al. 2003; Kravtsov et al. 2004; Zentner et al. 2005; Zheng et al. 2005).

Our basic approach to modeling galaxy-galaxy lensing in the HOD framework is similar to that adopted by Tinker et al. (2005, 2006) for modeling mass-to-light ratios and redshift-space distortions. Since measurements of the galaxy power spectrum, cosmic microwave background anisotropies, and the \(\text{Ly}\alpha\) forest yield tight constraints on the shape of the linear matter power spectrum \(P_{\text{lin}}(k)\) (see, e.g., Spergel et al. 2003; Tegmark et al. 2004; Cole et al. 2005; Seljak et al. 2005), we take this shape to be fixed and investigate the parameter space spanned by \(\Omega_m\) and \(\sigma_8\). For a given choice of \(\Omega_m\) and \(\sigma_8\), we first choose HOD parameters to match observations of the projected galaxy correlation function \(w_p(r_p)\) (see, e.g., Zehavi et al. 2004, 2005a).

We then predict the excess surface density profile \(\Delta \Sigma(r)\) for this combination of \(\Omega_m\), \(\sigma_8\), and HOD. Comparison to galaxy-galaxy lensing measurements then determines the acceptable combinations of \(\Omega_m\) and \(\sigma_8\). We impose the observed galaxy-galaxy lensing measurements to compare with models.
correlation function as a constraint on the HOD, instead of taking ratios as in the
linear bias analysis. There is no need for an unknown cross-correlation coefficient $r_{gm}$
because any “stochasticity” between galaxy and mass density fields is automatically
incorporated in the HOD calculation. Our strategy complements that of Guzik
& Seljak (2001, 2002) and Mandelbaum et al. (2005), who focus on constraining
halo masses, halo profiles, and satellite fractions using $\Delta \Sigma(r)$ alone, rather than
constraining $\Omega_m$ and $\sigma_8$ from the combination of $\Delta \Sigma(r)$ and $\xi_{gg}(r)$.

Our eventual conclusions about the cosmological constraining power of
galaxy-galaxy lensing measurements rest on an analytic model for computing $\Delta \Sigma(r)$
given $P_{\text{lin}}(k)$, $\Omega_m$, $\sigma_8$, and the galaxy HOD. This model is similar in spirit to that
of Guzik & Seljak (2002), but it differs in many details, in part because we define
the calculational problem in different terms. We test the analytic model against
numerical calculations, in which we use a specified HOD to populate the halos of
$N$-body simulations, placing “central” galaxies at the halo potential minimum and
“satellite” galaxies at the locations of randomly selected dark matter particles.
Both our analytic model and our method of populating $N$-body halos ignore the
impact of dark matter subhalos around the individual satellite galaxies orbiting
in a larger halo. To begin, therefore, we test the validity of the “populated halo”
approach itself, by comparing $\Delta \Sigma(r)$ for the galaxy population of a smoothed
particle hydrodynamics (SPH) simulation to that found by populating the dark
matter halos of this simulation with “galaxies” placed on randomly selected dark
matter particles. This test shows that satellite subhalos have minimal impact on $\Delta \Sigma(r)$ and that the populated halo approach is acceptable for our purpose. We also show that any environmental dependence of halo galaxy content at fixed halo mass (Gao et al. 2005; Harker et al. 2006) has little discernible impact on the galaxy-galaxy or galaxy-matter correlation functions in our SPH simulations. More generally, our SPH and $N$-body tests indicate that the analytic model should be accurate at the $5-10\%$ level on scales $r \gtrsim 0.1 \, h^{-1}\text{Mpc}$. This level of accuracy is acceptable for present purposes, since the current measurement errors are typically $\gtrsim 25\%$ per radial bin (e.g., Sheldon et al. 2004), but still higher accuracy will be needed in the long term.

In our $N$-body and analytical calculations, we use HOD parameters for SDSS galaxy samples with absolute-magnitude limits $M_r \leq -20$ and $M_r \leq -21$ (Zehavi et al. 2005b) for purposes of illustration. The results presented in § 3.5 therefore yield predictions of the weak lensing signal for these galaxy samples as a function of $\Omega_m$ and $\sigma_8$. The analytic model can be used to make predictions for other galaxy samples given measurements of the projected correlation function as input.

4 Throughout the paper, we quote absolute magnitudes for $h = 1$; more generally, these thresholds correspond to $M_r - 5 \log h$. 

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3.2. SPH Galaxies versus Populated Halos

To test the validity of our \( N \)-body method for calculating galaxy-galaxy lensing predictions (see § 3.3), we first examine an SPH simulation of a \( \Lambda \)CDM (inflationary cold dark matter with a cosmological constant) universe. This simulation is described in detail by Weinberg et al. (2004), who, among other things, present predicted galaxy-matter correlations and compare them to recent observations. Here we want to know whether the individual dark matter subhalos retained by baryonic galaxies in groups and clusters make an important contribution to the galaxy-galaxy lensing signal.

In brief, the simulation uses Parallel TreeSPH (Hernquist & Katz 1989; Katz, Weinberg, & Hernquist 1996; Davé et al. 1997) to model a 50 \( h^{-1}\) Mpc comoving cube with \( 144^3 \) dark matter particles and \( 144^3 \) gas particles. The cosmological parameters are \( \Omega_m = 0.4 \), \( \Omega_\Lambda = 0.6 \), \( h = 0.65 \), \( n = 0.95 \), \( \Omega_b h^2 = 0.02 \), and \( \sigma_8 = 0.80 \). The gravitational forces are softened with a 10 \( h^{-1}\) kpc (comoving) spline kernel. Radiative cooling leads to the formation of dense baryonic clumps (Katz et al. 1992; Evrard et al. 1994), which form stars according to the algorithm described by Katz, Weinberg, & Hernquist (1996). Galaxies are identified by applying the SKID (Spline-Kernel Interpolated DENMAX; see Katz, Weinberg, & Hernquist...
Tests with simulations of varying resolution show that the simulated galaxy population is complete above a baryonic mass (stars plus cold, dense gas) of $\sim 64 m_{\text{SPh}}$, corresponding to $5.4 \times 10^{10} M_\odot$ ($3.5 \times 10^{10} h^{-1} M_\odot$) for this simulation. The space density of galaxies above this mass threshold is $\bar{n}_g = 0.02 (h^{-1}\text{Mpc})^{-3}$, corresponding to that of observed galaxies with $M_r \leq -18.6 (L > 0.18 L_\odot$; Blanton et al. 2003). We use this mass-thresholded galaxy sample for the tests below.

We identify dark matter halos by applying the friends-of-friends algorithm (FOF; Davis et al. 1985) to the dark matter particle distribution, with a linking length of 0.2 times the mean interparticle separation, or $70 h^{-1}\text{kpc}$. We associate each SPH galaxy with the halo containing the dark matter particle closest to its center of mass. To create “populated halo” galaxy catalogs, we replace the SPH galaxies in each halo with an equal number of artificial galaxies positioned on dark matter particles. The first “central” galaxy of each occupied halo is placed at the location of the dark matter particle with lowest potential energy (computed using only halo members). Any additional, “satellite” galaxies are placed on randomly selected dark matter particles. Satellites therefore follow the radial profile of dark matter within each halo, while any detailed association between satellites and the centers of dark matter subhalos is erased.

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5We use the implementation of SKID by J. Stadel & T. Quinn, available at http://www-hpcc.astro.washington.edu/tools/skid.html
The left panels of Figure 3.1 compare the galaxy-galaxy correlation functions, galaxy-matter correlation functions, and excess surface density profiles of the SPH galaxies and the populated halo galaxy catalogs. Results for the populated halos are an average over 10 realizations of the galaxy locations, and error bars show the dispersion among the ten realizations. (Note that these do not represent the uncertainty on the mean, which would be a factor of three smaller, and they do not include the uncertainty owing to the finite simulation volume, since we are comparing galaxy catalogs in the same volume). The galaxy-galaxy and galaxy-matter correlations of the two catalogs are very similar at $r > 0.5 \, h^{-1}\text{Mpc}$, while at smaller separations the populated halo catalog has correlations that are stronger by up to 20%. The excess surface density profile is calculated by directly counting galaxy-dark matter particle pairs in projection to compute $\Sigma(< r)$ and $\Sigma(r)$, not by integrating the three-dimensional $\xi_{gm}(r)$. We count all projected pairs through the $50 \, h^{-1}\text{Mpc}$ box and average results from the three orthogonal projections of the box; noise from uncorrelated foreground and background particles cancels out because we have many galaxy targets. Relative to the SPH galaxy catalog, $\Delta \Sigma(r)$ for the populated halo catalog starts about 10% low, rises to 10% above at $r \sim 0.5 \, h^{-1}\text{Mpc}$, then agrees closely beyond $r \sim 1 \, h^{-1}\text{Mpc}$.

The modest deviations between the SPH galaxy and populated halo results could reflect either the impact of satellite subhalos or differences between the radial profiles of SPH satellites and dark matter. To separate the two effects, we adopt
a different method of populating halos that ensures identical radial profiles, by
placing each satellite at the radial distance of the corresponding SPH satellite but
choosing a random orientation for the radius vector. Results are shown on the
right panels of Figure 3.1. The differences in $\xi_{gg}(r)$, $\xi_{gm}(r)$, and $\Delta \Sigma(r)$ are greatly
reduced, demonstrating that they arise mainly from the different radial profiles
of SPH satellite galaxies and dark matter; specifically, the SPH satellites are less
concentrated towards the halo center than the dark matter. With matched radial
profiles, the populated halos still have slightly larger $\xi_{gm}(r)$ at $r \lesssim 0.1 \, h^{-1}\text{Mpc}$, in
part because there are usually offsets of this magnitude between the location of
the SPH central galaxy and the position of the most bound dark matter particle.
However, the differences in $\Delta \Sigma(r)$ are now smaller than 10% at all $r$.

We conclude that it is safe to ignore the subhalos of individual satellite galaxies
when computing $\Delta \Sigma(r)$ for a full galaxy sample. Indeed, the remaining residuals in
Figure 3.1, a consequence of the central galaxy offsets mentioned above, are opposite
in sign to those expected from satellite subhalos. Satellites in the SPH simulation
do reside in individual dark matter subhalos (Weinberg et al. 2006), but these are
tidally truncated, and at small separations $\Delta \Sigma(r)$ is dominated by the contribution
of the more numerous, central galaxies (see § 3.4.2 below). The small impact of
satellite subhalos on $\Delta \Sigma(r)$ is therefore unsurprising, and was anticipated by earlier
analytic modeling (Guzik & Seljak 2002; Mandelbaum et al. 2005).
Nonetheless, one might worry that the absence of any subhalo signal in Figure 3.1 is an artifact of our simulation’s finite mass resolution, leading to an artificially high degree of tidal truncation. To test this possibility, we compare results from this simulation to those of a simulation of the same cosmological model with a factor of eight higher mass resolution but smaller volume. This simulation uses $128^3$ dark matter particles and $128^3$ gas particles in a volume $22.222 \ h^{-1}\text{Mpc}$ on a side. In each simulation, we select all halos in the mass range $6 \times 10^{12} \ h^{-1}\text{M}_\odot \lesssim M \lesssim 2 \times 10^{13} \ h^{-1}\text{M}_\odot$ and measure the galaxy-matter correlation function for satellites in these halos above the baryonic mass threshold of the larger volume, lower resolution simulation. The $50 \ h^{-1}\text{Mpc}$ box contains 78 halos and 188 satellite galaxies satisfying these cuts, while the $22.222 \ h^{-1}\text{Mpc}$ box contains 12 halos and 33 satellites. As shown in Figure 3.2, the satellite galaxy-matter correlations are equal in the two simulations to within the statistical errors, which are estimated by bootstrap resampling of the galaxies in the smaller simulation. The average mass profiles around satellites are therefore robust over a factor of eight in mass resolution.

Standard HOD calculations assume that the halo occupation function $P(N|M)$ has no direct dependence on a halo’s larger scale environment. This assumption is motivated by the excursion set derivation of the Extended Press-Schechter formalism (Bond et al. 1991), which, in its simplest form, predicts that a halo’s formation history is uncorrelated with its environment at fixed mass (White 1996). The
correlation of galaxy properties with large scale environment emerges indirectly from the correlation with halo mass because high mass halos are more common in dense environments. Blanton et al. (2006) showed that the observed correlation of red galaxy fraction with overdensity measured at 6 \( h^{-1}\)Mpc is entirely accounted for by the correlation with overdensity measured at the 1 \( h^{-1}\)Mpc scale characteristic of individual large halos. However, while early \( N\)-body studies showed at most weak correlations between halo formation time and environment at fixed mass for halos with \( M \gtrsim 10^{13}h^{-1}M_\odot \) (Lemson & Kauffmann 1999; Sheth & Tormen 2004), Gao et al. (2005) have recently shown that there is a much stronger correlation for lower mass halos, with the older halos being more strongly clustered (see also Sheth & Tormen 2002; Harker et al. 2006 discuss the potential origin of environmental correlations in the excursion set formalism). Berlind et al. (2003), examining the same SPH simulation and galaxy sample that we have used here, showed that the mean number of galaxies as a function of halo mass, \( \langle N \rangle_M \), is independent of halo environment within the statistical uncertainties imposed by the finite simulation volume. However, in light of Gao et al.’s (2005) result, we have carried out an experiment to explicitly examine the possible impact of environmental dependence of \( P(N|M) \) on galaxy-galaxy and galaxy-matter correlations.

In the populated halo calculations shown in Figure 3.1, the number of galaxies assigned to each halo is equal to the number of SPH galaxies, so any environmental dependence predicted by the SPH simulation is also built into the populated halo
distribution. We eliminate the environmental dependence by shuffling the galaxy populations among halos of similar mass. Specifically, we sort the halos by mass and replace the number of galaxies \( N_i \) in the halo of rank \( i \) with the number \( N_{i+1} \) in halo \( i + 1 \), then recalculate \( \xi_{gg}(r) \), \( \xi_{gm}(r) \), and \( \Delta \Sigma(r) \), averaging over ten realizations of galaxy positions within halos. We repeat the exercise with the substitutions \( N_{i+1} \rightarrow N_{i+2} \), \( N_{i-1} \), and \( N_{i-2} \) so that we can average over four different halo shufflings and compute the statistical error on the mean. The sampling of the halo mass function is sparse for the highest mass halos in the simulation, so at high masses we cannot exchange the galaxy contents of halos without significantly changing \( P(N|M) \) itself. We therefore keep the galaxy populations of the \( N_{\text{fix}} \) most massive halos fixed, with \( N_{\text{fix}} = 5 \) or 20. For \( N_{\text{fix}} = 5 \), we are shuffling the contents of all halos with \( M < 9.0 \times 10^{13} \, h^{-1} M_\odot \), and for \( N_{\text{fix}} = 20 \) we are shuffling the contents of all halos with \( M < 4.6 \times 10^{13} \, h^{-1} M_\odot \).

Figure 3.3 plots the fractional difference in \( \xi_{gg}(r) \), \( \xi_{gm}(r) \), and \( \Delta \Sigma(r) \) between the shuffled halo realizations and the original populated halos. We use the populated halos as the comparison standard rather than the SPH galaxies so that we can isolate the impact of environmental dependence of \( P(N|M) \). Error bars show the error on the mean from the four shufflings, but recall that we have only one realization of the original populated halos. For \( N_{\text{fix}} = 5 \), there is a 5% increase on \( \xi_{gg}(r) \) at \( r \lesssim 0.5 \, h^{-1} \text{Mpc} \). However, these scales lie in the 1-halo regime where environmental variation of \( P(N|M) \) should have no impact at all, so the increase is probably a
statistical fluctuation that reflects the particular sizes and concentrations of the halos present in the simulation. It is only slightly larger than the 1σ error bars, and the errors from point to point are highly correlated. For $N_{\text{fix}} = 20$, the changes in $\xi_{gg}(r)$ are less than 3% over the range $0.08 \ h^{-1}\text{Mpc} \lesssim r \lesssim 3 \ h^{-1}\text{Mpc}$. The three points at $r \gtrsim 5 \ h^{-1}\text{Mpc}$ are depressed by $\sim 5\%$ on average, which suggests that shuffling may slightly lower the large scale galaxy bias factor, but the statistical significance of this depression is difficult to assess with a single $50 \ h^{-1}\text{Mpc}$ simulation.

Shuffling changes $\xi_{gm}(r)$ by less than 5%, usually much less, except for the largest scale point with $N_{\text{fix}} = 5$. Most significantly for our present purposes, the changes in $\Delta \Sigma(r)$ are at most $\sim 2\%$ for $N_{\text{fix}} = 20$ at all scales, and only slightly larger for $N_{\text{fix}} = 5$. We conclude that ignoring any possible environmental dependence of $P(N|M)$ has minimal impact on the calculation of galaxy-galaxy lensing observables for a given cosmology and HOD, for a galaxy sample defined by a threshold in baryonic mass. There could be a few percent effect on the large scale bias of the galaxy-galaxy correlation function, which might lead to small errors in inferring the HOD from observations of $\xi_{gg}(r)$. Assessing the importance of this effect will require larger simulations. Croton, Gao & White (2007) have carried out a similar shuffling experiment for semi-analytic galaxy populations in the 500 $h^{-1}\text{Mpc}$ Millennium Run simulation (Croton et al. 2006), and they find few percent changes in large scale bias for galaxy samples defined by thresholds in mass or absolute magnitude (it was the hearing about their shuffling experiment that inspired us to carry out our own).
3.3. *N-Body Simulations*

To help us develop and test our analytic model, we have carried out five *N*-body simulations of a ΛCDM universe using *gadget* ([Springel et al. 2001]). Each simulation begins at expansion factor $a = 0.01$ with a scale-invariant ($n = 1$) fluctuation spectrum modulated by the transfer function of Efstathiou et al. (1992) with shape parameter $\Gamma = 0.2$. Our analytic model calculations in §3.5 use the *CMBFAST* transfer function ([Seljak & Zaldarriaga 1996]), which represents cosmological predictions more accurately, but the Efstathiou et al. (1992) representation should be adequate for calibrating and testing the analytic model itself. The simulations end at $a = 1.0$, when $\Omega_m = 0.1$, $\Omega_\Lambda = 0.9$, and the linear theory normalization of the power spectrum is $\sigma_8 = 0.95$. We use earlier outputs from the same simulations to represent models with the cosmological parameter combinations listed in Table 3.6: $(\Omega_m, \sigma_8) = (0.16, 0.90), (0.30, 0.80), (0.48, 0.69)$, and $(0.63, 0.60)$. Since we are adopting a fixed, observationally motivated form of the power spectrum instead of changing its shape with $\Omega_m$, this procedure is exact. We would obtain the same results if we ran a separate simulation for each model but started it at expansion factor $a = 0.01/a_{\text{out}}$, where $a_{\text{out}} = 0.84, 0.64, 0.49, 0.40$ for the four $(\Omega_m, \sigma_8)$ combinations. We refer the reader to Tinker et al. (2005, 2006) for the simulation details.
Our simulations use $360^3$ particles to model a volume $253 \, h^{-1} \text{Mpc}$ (comoving) on a side. The dark matter particle mass is $9.6 \times 10^{10} \Omega_m h^{-1} M_{\odot}$. We choose the mass resolution so that the lowest mass halos that host galaxies with $M_r \leq -20$, according to our HOD fits (see below), contain at least 32 particles. The gravitational force resolution is $\epsilon = 70 \, h^{-1} \text{kpc}$ (this is the approximate Plummer-equivalent value). The five simulations are identical except for the random number seed used to generate the initial conditions. We identify dark matter halos using FOF with a linking length equal to 0.2 times the mean interparticle separation, or $140 \, h^{-1} \text{kpc}$, and set the halo mass equal to the total mass of the linked particles.

We populate the $N$-body halos with galaxies using HOD parameters that are designed to reproduce the mean space density and projected correlation function of SDSS galaxies with $M_r \leq -20$, as measured by Zehavi et al. (2005b). The adopted form of the HOD is motivated by the results of Kravtsov et al. (2004) and Zheng et al. (2005). Halos below some minimum mass $M_{\text{min}}$ are devoid of galaxies. All halos above $M_{\text{min}}$ have a central galaxy, which is placed at the position of the dark matter particle with the lowest potential energy in each halo. The number of satellite galaxies is drawn from a Poisson distribution with mean $(M/M_1)^{\alpha_{\text{sat}}}$. Each satellite galaxy is placed on a randomly selected dark matter particle from the halo. Table 3.6 lists the values of $M_{\text{min}}$, $M_1$, and $\alpha_{\text{sat}}$ for our five $(\Omega_m, \sigma_8)$ combinations. Further details of the fitting procedure are given by Tinker et al. (2006). The specifics of the parametrization and details of the fitting method are not important.
to our purposes here, since we will test the analytic model predictions using the
same HOD parameters applied to the simulations. However, these parameter choices
ensure a galaxy population with realistic clustering properties.

Figure 3.4 shows $\xi_{gg}(r)$, $\xi_{gm}(r)$, and $\Delta \Sigma(r)$ for the five $N$-body models. The
five galaxy-galaxy correlation functions are nearly identical by construction, though
with the HOD parameters at our disposal it is not possible to exactly match the
observed correlation function over our full range of $\sigma_8$. The galaxy-matter correlation
function is higher for the more strongly clustered, high $\sigma_8$ models, as expected.
However, since $\Delta \Sigma(r)$ scales (approximately) with $\Omega_m \sigma_8$, and $\Omega_m$ falls faster than $\sigma_8$
grows in our simulation outputs, the order of models is reversed on the $\Delta \Sigma(r)$ panel.
We discuss the comparison between the $N$-body and analytic model results in the
following section.

3.4. Analytic Modeling of Galaxy-Matter Clustering

3.4.1. Formulation and Tests

Our analytic method of calculating $\xi_{gm}(r)$ for a given cosmology and HOD is
based on the methods that Zheng (2004) and Tinker et al. (2005a, see Appendix B)
used to calculate the galaxy-galaxy correlation function. These methods are based,
in turn, on ideas introduced by Scherrer & Bertschinger (1991), Ma & Fry (2000),
Seljak (2000), Peacock & Smith (2000) and Scoccimarro et al. (2001). We present a full technical description of our $\xi_{gm}$ calculation here but refer the reader to these earlier works for more general discussion. Our galaxy-galaxy correlation calculations follow Tinker et al. (2005), with ellipsoidal halo exclusion.

Contributions to $\xi_{gm}$ can come from galaxy-matter pairs\textsuperscript{6} residing in a single halo or in two distinct halos. We separate these two contributions as

$$1 + \xi_{gm}(r) = \left[1 + \xi_{gm}^{1h}(r)\right] + \left[1 + \xi_{gm}^{2h}(r)\right], \quad (3.5)$$

noting that it is pair counts (proportional to $1 + \xi_{gm}$) that add rather than the correlations $\xi_{gm}$ themselves. The one-halo contribution is

$$1 + \xi_{gm}^{1h}(r) = \frac{1}{4\pi r^2 n_g} \int_{M_{\text{min}}}^{\infty} dM \frac{dn}{dM} \langle N \rangle_M \frac{M}{\bar{\rho}_m} \frac{1}{2R_{\text{vir}}} F' \left(\frac{r}{2R_{\text{vir}}}\right), \quad (3.6)$$

where $dn/dM$ is the halo mass function (Press & Schechter 1974; Sheth & Tormen 1999; Jenkins et al. 2001), $\langle N \rangle_M$ is the mean number of galaxies in halos of mass $M$, $\bar{\rho}_m$ is the mean mass density, and $F(r/2R_{\text{vir}})$ is the average fraction of galaxy-matter pairs in halos of mass $M$ and virial radius $R_{\text{vir}}$ that have separation less than $r$ (Berlind & Weinberg 2002; $F'(x)$ is simply the derivative of $F(x)$ with respect to its argument). We define $R_{\text{vir}}$ such that the mean density within $R_{\text{vir}}$ is $\Delta_{\text{vir}} \bar{\rho}_m$, and unless otherwise stated we assume $\Delta_{\text{vir}} = 200$. We further split the one-halo term

\textsuperscript{6}By which we mean pairs of galaxies and dark matter particles.
by discriminating central and satellite galaxies (see, e.g., Berlind & Weinberg 2002; Yang et al. 2003; Zheng 2004),

\[ \langle N \rangle_M F'(x) = \langle N_{cen} \rangle_M F'_{cen}(x) + \langle N_{sat} \rangle_M F'_{sat}(x). \]  

(3.7)

Pairs involving a central galaxy simply follow the radial mass profile \( \rho(r) \), so

\[ F'_{cen}(x) \propto \rho(r)r^2. \]

The distribution \( F'_{sat}(x) \) of satellite galaxy-matter pairs is the convolution of the galaxy and matter profiles. We assume a spherical NFW profile (Navarro, Frenk & White 1997), truncated at \( R_{vir} \), for both dark matter and satellite galaxies. We compute the dark matter concentration parameter \( c_{dm} \) using the relation of Bullock et al. (2001). We allow the galaxy concentration to be different, \( c_{gal} = \alpha_c c_{dm} \), but adopt \( \alpha_c = 1 \) as our standard assumption.

On scales much larger than the virial diameter of the largest halo, the galaxy-matter correlation function is equal to \( \xi_{mm}(r) \) multiplied by a galaxy bias factor

\[ b_{gal} = \frac{1}{n_g} \int_0^\infty \frac{dM}{dM} \langle N \rangle_M b_h(M), \]  

(3.8)

where \( b_h(M) \) is the bias factor of halos of mass \( M \). However, an accurate calculation on intermediate scales must account for the finite extent of halos, for the scale dependence of \( b_h(M) \), and for halo exclusion — two spherical halos cannot be separated by less than the sum of their virial radii. It is convenient to do the calculation in Fourier space, where the convolutions of halo profiles become
multiplications of their Fourier transforms. Our complete series of expressions for the two-halo contribution to $\xi_{gm}(r)$ is

$$1 + \xi_{gm}^{2h}(r) = \left( \frac{n_g'}{n_g} \right) \left[ 1 + \xi_{gm}^{2h'}(r) \right],$$

(3.9)

where

$$\xi_{gm}^{2h'}(r) = \frac{1}{2\pi^2} \int_0^\infty dk k^2 P_{gm}^{2h'}(k|r) \frac{\sin(kr)}{kr},$$

(3.10)

is the Fourier transform of

$$P_{gm}^{2h'}(k|r) = P_m(k) \frac{1}{n_g'} \int_0^\infty dM_1 \frac{dn}{dM_1} \langle N \rangle_{M_1} b_h(M_1|r) y_g(k, M_1)$$

$$\times \int_0^\infty dM_2 \frac{dn}{dM_2} \frac{M_2}{\rho_m} b_h(M_2|r) y_m(k, M_2)p_{no}(x|M_1, M_2),$$

(3.11)

where $y_g(k, M)$ and $y_m(k, M)$ are the normalized Fourier counterparts of the galaxy and the matter profiles, and

$$n_g'^2 = \int_0^\infty dM_1 \frac{dn}{dM_1} \langle N \rangle_{M_1} \int_0^\infty dM_2 \frac{dn}{dM_2} \langle N \rangle_{M_2} p_{no}(x|M_1, M_2).$$

(3.12)

In these expressions, $p_{no}(x|M_1, M_2)$ represents the probability that two halos of mass $M_1$ and $M_2$ with scaled separation $x \equiv r/(R_{\text{vir},1} + R_{\text{vir},2})$ do not overlap. For spherical halos, $p_{no}(x)$ would be a step function at $x = 1$, but Tinker et al. (2005) found that an accurate separation of the 1-halo and 2-halo contributions to $\xi_{gg}(r)$ requires accounting for the non-spherical shapes of halos identified by the FOF algorithm. We
adopt their expression, based on a fit to Monte Carlo realizations of ellipsoidal halo pairs with a reasonable distribution of axis ratios: \( p_{n_0}(x) = 0 \) for \( x < 0.8 \), \( p_{n_0}(x) = 1 \) for \( x > 1.09 \), and \( p_{n_0}(x) = (3y^2 - 2y^3) \) with \( y = (x - 0.8)/0.29 \) for \( 0.8 \leq x \leq 1.09 \).

The restricted number density \( n_g' \) is the mean space density of galaxies residing in allowed (i.e., non-overlapping) halo pairs at separation \( r \). Since \( p_{n_0}(x \mid M_1, M_2) \) and the halo bias factors \( b_h(M \mid r) \) depend on \( r \), one must evaluate equations (3.9)–(3.12) separately for each value of \( r \) where one wants to know \( \xi_{gm}(r) \). The double integrals in equations (3.11) and (3.12) are non-separable because of the \( M_1 \)-dependence of \( p_{n_0}(x) \). In principle, one should separately compute equation (3.11) for central and satellite galaxies and sum the results, since \( g_g(k, M) \) is different in the two cases, but we have tested and found that ignoring this subtlety has negligible effect.

For scale-dependent halo bias factors, we adopt the expression

\[
b_h^2(M \mid r) = b_{\text{asym}}^2(M) \times \frac{[1 + 1.17 \xi_{gm}(r)]^{1.49}}{[1 + 0.69 \xi_{gm}(r)]^{2.09}},
\]

from Tinker et al. (2005). We also use Tinker et al.’s (2005a; Appendix A) expressions for the asymptotic bias factors \( b_{\text{asym}}(M) \). These follow the formulation of Sheth et al. (2001b), but with different parameter values that yield a substantially better fit to the simulations. For “concordance” cosmological parameters, these bias factors are similar to those of Seljak & Warren (2004), but they are more accurate for models with different matter power spectra. We use Smith et al.’s (2003) approximation for the non-linear power spectrum \( P_{gm}(k) \) and correlation function
\( \xi_{\text{mm}}(r) \) in equations (3.11) and (3.13). One could in principle use \( P_{\text{lin}}(k) \) instead of \( P_{\text{mm}}(k) \); this would require a different (though still scale-dependent) expression for \( b_h(M|r) \) with a separate \( N \)-body calibration.

The calculation as we have described it is a straightforward generalization of the \( \xi_{\text{gg}}(r) \) calculation presented by Tinker et al. (2005), who tested its accuracy over the range \( \sigma_8 = 0.6 - 0.95 \) for both the \( \Gamma = 0.2 \) simulations used here and for a similar set with \( \Gamma = 0.12 \). However, a significant technical, and to some degree conceptual issue arises with the evaluation of the second integral in equation (3.11). Since we assign all galaxy-matter pairs to either the 1-halo or 2-halo terms, we implicitly assume that all dark matter is in halos of some mass, and thus

\[
\int_{0}^{\infty} dM \frac{dn}{dM} M = \bar{\rho}_m. \tag{3.14}
\]

More importantly for present purposes, the distribution of matter is by definition unbiased with respect to itself, and therefore

\[
\int_{0}^{\infty} dM \frac{dn}{dM} \frac{M}{\bar{\rho}_m} b_h(M|r) = 1. \tag{3.15}
\]

The Jenkins et al. (2001) mass function and Tinker et al. (2005) halo bias factors used here are fits to simulations over a finite range of halo masses, and they do not satisfy either of these constraints. To impose the constraint (3.15) explicitly, we break the second integral of equation (3.11) at a halo mass \( M_{\text{brk}} = 10^8 h^{-1}M_\odot \) and
evaluate it as

\[ \int_0^\infty dM_2 \frac{dn}{dM_2} \frac{M_2}{\bar{\rho}_m} b_h(M_2|r) \ y_m(k, M_2) p_{no}(x|M_1, M_2) \]
\[ \approx \int_{M_{\text{brk}}}^\infty dM_2 \frac{dn}{dM_2} \frac{M_2}{\bar{\rho}_m} b_h(M_2|r) \ y_m(k, M_2) p_{no}(x|M_1, M_2) \]
\[ + \left[ 1 - \int_{M_{\text{brk}}}^\infty \frac{dM'}{dM'} \frac{M'}{\bar{\rho}_m} b_h(M'|r) \right]. \]  

(3.16)

For the term in brackets on the right hand side, we make the (good) approximation that \( p_{no}(x) = y(k, M) = 1 \) for halos with \( M < M_{\text{brk}} \) at all radii of interest for our calculation, then apply equation (3.15). We find that this procedure is necessary to obtain accurate results. The same problem does not arise for integrals involving \( \langle N \rangle_M \) because the mean occupation itself goes to zero at low halo masses.

We must make one further adjustment to the analytic model before testing it against the populated N-body halos described in § 3.3. These halos are identified by the FOF algorithm, which, roughly speaking, selects particles within an isodensity surface. The mean overdensity \( \Delta_{\text{vir}} \) within this surface depends on the halo profiles. To compute the effective value of \( \Delta_{\text{vir}} \) for our simulations, we calculate the mean density within spheres centered on the most-bound particles of the FOF halos that enclose the halo’s FOF mass. The canonical value of \( \Delta_{\text{vir}} = 200 \) is accurate for our central model with \( \Omega_m = 0.3 \) and \( \sigma_8 = 0.8 \). However, the \( \Delta_{\text{vir}} \) values for other models, listed in Table 3.6, deviate by up to 20%. The trend is as expected: halos in low \( \Omega_m \) models are more concentrated because they form earlier, and they have higher values of \( \Delta_{\text{vir}} \). However, the \( \Omega_m \)-dependence of FOF halos selected with
constant linking parameter is much weaker than the variation of virial densities predicted by the spherical collapse model (e.g., Bryan & Norman 1998). When calculating virial radii as a function of halo mass, we use the $\Delta_{\text{vir}}$ values in Table 3.6.

The attached bottom frame in Figure 3.4a shows the fractional difference between the analytic model and $N$-body results for the galaxy-galaxy correlation function, $(\xi_{\text{analytic}} - \xi_{N\text{-body}})/\xi_{N\text{-body}}$, for the five cosmological/HOD models listed in Table 3.6. Error bars represent the statistical uncertainty in the mean value of $\xi_{N\text{-body}}$, computed from the dispersion among the five independent simulations; for clarity, we show these only for the central model with $\Omega_m = 0.3$. The differences between the analytic and numerical results are usually less than 5% at all $r > 0.1 \, h^{-1}\text{Mpc}$. The one deviation that is clearly statistically significant is the rapid turn-up of the residuals at $r \approx 0.1 \, h^{-1}\text{Mpc}$, which reflects the smoothing effect of the simulation’s gravitational force softening. The marginally significant, $\sim 5\%$ discrepancy at $r \approx 0.8 \, h^{-1}\text{Mpc}$ suggests that our ellipsoidal exclusion correction still underestimates the number of close halo pairs, even though it allows halos to be separated by less than the sum of their virial radii. Without this correction, the deviation would fall off the bottom of the plot (see Fig. 10 in Tinker et al. 2005).

The $\sim 8\%$ divergence of the models at $r \approx 8 \, h^{-1}\text{Mpc}$ results from the deviations of the Smith et al. (2003) non-linear matter power spectrum from our simulation results. This difference could be partly an artifact of our finite box size, but the systematic dependence on cosmological parameters and reconvergence of results at
$r = 20\ h^{-1}\text{Mpc}$ suggest that it is mostly a result of slight non-universality of the 
Smith et al. (2003) formula, though it is hard to reach a definitive conclusion because of the substantial statistical uncertainties at these scales.

The bottom frames in Figure 3.4b and 3.4c show equivalent fractional differences for $\xi_{gm}(r)$ and $\Delta \Sigma(r)$. The residuals for $\xi_{gm}(r)$ are similar to those for $\xi_{gg}(r)$, with the rise at $r \approx 0.1\ h^{-1}\text{Mpc}$, again reflecting the force softening in the numerical simulation. Since $\Delta \Sigma(r)$ depends on $\xi_{gm}$ at all separations less than $r$ (see eq.[3.2]), and the deviation between analytic and $N$-body $\xi_{gm}$ grows at smaller separations, a comparison between the pure analytic calculation and the measurement of $\Delta \Sigma(r)$ from the simulation shows a substantial offset at $r \lesssim 1\ h^{-1}\text{Mpc}$. However, this offset reflects the limited resolution of the $N$-body simulation, not the failure of the analytic model. (Similar deviations at small scale were found by Mandelbaum et al. 2005.) To remove this numerical artifact from the comparison, we set $\Delta \Sigma$ for the analytic model equal to the $N$-body value at $r = 0.1\ h^{-1}\text{Mpc}$, then use the analytic calculation of $\xi_{gm}(r)$ to obtain the surface density for $r > 0.1\ h^{-1}\text{Mpc}$. With this correction, the analytic model for $\Delta \Sigma(r)$ is accurate to 5% or better for all five cosmological models at all radii $0.1\ h^{-1}\text{Mpc} \leq r \leq 20\ h^{-1}\text{Mpc}$. 

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3.4.2. Dissection of Correlation Functions

It is interesting to examine the separate contributions of central and satellite galaxies to the galaxy-galaxy and galaxy-matter correlation functions. Figure 3.5a shows the familiar decomposition of $\xi_{gg}(r)$ into 1-halo and 2-halo contributions (see Berlind & Weinberg 2002 for extensive discussion). We adopt the fiducial model with $\Omega_m = 0.3$, $\sigma_8 = 0.8$, and the corresponding HOD parameters listed in Table 3.6, and we use the $N$-body simulation measurements. The 1-halo term dominates at small scales, but it drops rapidly towards larger $r$ as halos with large virial diameters become increasingly rare. At large scales, the 2-halo term is a linearly biased version of the matter correlation function, but at small scales it turns over and drops because of halo exclusion. We plot $1 + \xi(r)$ rather than $\xi(r)$ itself because pair counts are additive, so individual contributions sum to give the total $1 + \xi(r)$; this consideration becomes especially important for the central-satellite decompositions shown in subsequent panels. The transition between 1-halo and 2-halo dominance occurs at roughly the virial diameter of $M_*$ halos, where $M_*$ is the characteristic scale of the halo mass function.

Figure 3.5b separates $1 + \xi_{gg}(r)$ into central and satellite galaxy contributions. Central-satellite pairs dominate $\xi_{gg}(r)$ at $r \lesssim 0.4$ $h^{-1}$Mpc, and central-central pairs at $r \gtrsim 1$ $h^{-1}$Mpc, while satellite-satellite pairs dominate by a small factor in the intermediate regime. Figure 3.5c and 3.5d show the separate central and satellite
contributions to the 1-halo and 2-halo terms, revealing the origin of the behavior in Figure 3.5b. Central-satellite pairs dominate $\xi^{1h}(r)$ at small scales because in this regime most pairs come from the most common halos that are large enough to host a galaxy pair. These halos have $\langle N \rangle_M < 3$, and they therefore have more central-satellite galaxy pairs than pairs that involve only satellites. Conversely, satellite-satellite pairs dominate $\xi^{1h}(r)$ at large scales because the halos with virial diameters large enough to host pairs at these separations have $\langle N \rangle_M > 3$.

Finally, pairs involving at least one central galaxy dominate the 2-halo term by a large factor at all separations, because only halo pairs in which both halos have $\langle N \rangle_M > 2$ contribute, on average, more satellite-satellite pairs than central-central or central-satellite pairs. Such halos are much less common than those with $1 \leq \langle N \rangle_M \leq 2$. Thus, satellite-satellite pairs make a major contribution to the total $\xi_{gg}(r)$ only over the radial range in which 1-halo contributions from high mass halos are dominant.

Figure 3.6 shows the equivalent dissection of $\xi_{gm}(r)$. The overall behavior is very similar to that seen in Figure 3.5. The 2-halo term extends to somewhat smaller scales because halos with mass near $M_{\text{min}}$ can still form galaxy-matter pairs with lower mass halos that have smaller virial radii. The satellite contribution to the 2-halo term is analogous to the sum of central-satellite and satellite-satellite pairs for $\xi_{gg}(r)$ because there are no “central” dark matter particles. (The normalization is lower than in Figure 3.2 because we now calculate the expected number of
galaxy-matter pairs using all galaxies instead of satellites alone.) However, it is still a factor of 3–5 below the central 2-halo term at nearly all separations. From Figure [3.6] we can understand why the individual halos of satellite galaxies appear to have so little impact on the SPH results discussed in § 3.2. Satellite galaxies dominate the 1-halo term of $\xi_{gm}(r)$ only beyond $r \approx 0.25 \, h^{-1}\text{Mpc}$. However, the individual halos of satellites orbiting in larger groups are usually tidally truncated well inside this radius, on scales where the signal is swamped by the contribution from central galaxies. Note that the satellite fraction of galaxy samples is typically less than 30%, and hence the lensing signal of satellite galaxies is smaller by an order of magnitude than that of central galaxies. The dark halos of satellites in groups and clusters can be measured by galaxy-galaxy lensing (Natarajan et al. 2002), but only by first identifying satellites and measuring $\Delta \Sigma(r)$ for them specifically.

3.5. From Galaxy-Galaxy Lensing to Cosmological Parameters

Having established the accuracy of the analytic model, we can now use it to investigate the dependence of $\Delta \Sigma(r)$ on $\Omega_m$ and $\sigma_s$. We consider a well-defined sample of galaxies, choose HOD parameters for each ($\Omega_m$, $\sigma_s$) combination by fitting the mean space density and projected correlation function of this sample, then predict $\Delta \Sigma(r)$. In this section, we focus on the sample of SDSS galaxies with
$M_r \leq -21$, with the projected correlation function, error covariance matrix, and mean space density $\bar{n}_g = 1.17 \times 10^{-3} h^3 \text{Mpc}^{-3}$ taken from Zehavi et al. (2005b). We also present predictions for a fainter luminosity threshold, $M_r \leq -20$, again using Zehavi et al. (2005b)'s observational constraints.\[7\] At large scales, we expect to recover the linear theory, linear bias result, $\Delta \Sigma \propto \Omega_m/b \propto \sigma_8 \Omega_m$, but we can extend the predictions to intermediate and small scales using the full analytic model.

We make two significant changes in our application of the analytic model. First, we use a CMBFAST transfer function (Seljak & Zaldarriaga 1996), computed for $\Omega_m = 0.3$, $h = 0.7$, $\Omega_b = 0.04$, in place of the Efstathiou et al. (1992) parametrization adopted in our $N$-body simulations. This change to the transfer function has little effect on the HOD parameters inferred by fitting $w_p(r_p)$, but it has a noticeable effect on the $\chi^2$ values of these fits, and it affects the $\Delta \Sigma(r)$ predictions themselves. Note that we do not change the transfer function when changing $\Omega_m$ from our fiducial value of 0.3; because the power spectrum shape is empirically well constrained, we assume that any effect of changing $\Omega_m$ will be compensated by adjusting $h$, $\Omega_b$, or the inflationary index $n$ (which we set to one). Second, we define halo virial radii assuming $\Delta_{\text{vir}} = 200$ for all $\Omega_m$, instead of the varying $\Delta_{\text{vir}}$ values listed in Table 3.6 and used in § 3.4. This change simply amounts to a slight change in the halo definition; to identify these halos in $N$-body simulations, one would need to adjust

\[7\] Specifically, we use the Zehavi et al. (2005b) measurements for the $M_r \leq -20$ sample with limiting redshift $z = 0.06.$
the FOF linking length slightly with $\Omega_m$. The Bullock et al. (2001) concentration parameters are defined for different ($\Omega_m$-dependent) $\Delta_{\text{vir}}$ values, but we rescale them to our $\Delta_{\text{vir}}$ definition. We still adopt the Jenkins et al. (2001) halo mass function for all cosmological models, with no rescaling.

Figure 3.7 illustrates the results for a sequence of models with $\sigma_8$ ranging from 0.6 to 1.0, all for $\Omega_m = 0.3$. The HOD parameter values required to match the Zehavi et al. (2005b) $w_p(r_p)$ measurements are listed in Table 3.6, and the mean occupation functions $\langle N \rangle_M$ are shown in panel 3.7a. For lower $\sigma_8$, matching the observed clustering requires a larger fraction of galaxies in more massive, more biased halos, hence higher values of the $\langle N_{\text{sat}} \rangle_M$ slope $\alpha_{\text{sat}}$. The resulting galaxy correlation functions are very similar for all five values of $\sigma_8$, as shown in Figure 3.7d. Figure 3.7b plots $(M/\bar{\rho}_m) \, dn/d \ln M$, the fraction of mass contained in a (natural) logarithmic bin centered at mass $M$. For $\sigma_8 = 0.6$, this function peaks near $M \sim 10^{13} \, h^{-1} M_\odot$, while for $\sigma_8 = 1.0$ it peaks near $M \sim 10^{14} \, h^{-1} M_\odot$. Figure 3.7c plots the same function multiplied by $\langle N \rangle_M$, a product that is proportional to the number of 1-halo galaxy-matter pairs that arise in halos of mass $M$. This function peaks at $M \sim 3 \times 10^{14} \, h^{-1} M_\odot$ for $\sigma_8 = 0.6$ and $M \sim 10^{15} \, h^{-1} M_\odot$ for $\sigma_8 = 1.0$. The trend of $\alpha_{\text{sat}}$ with $\sigma_8$ partly compensates the trend of the mass distribution in Figure 3.7b, reducing the order-of-magnitude shift in the peak location to a factor of three. While high mass halos near the peak contribute a substantial fraction of all galaxy-matter pairs, these pairs are spread over a larger projected area, so the
contribution in a given \( r \sim r + dr \) bin is multiplied by an additional factor that scales roughly as \( R_{\text{vir}}^{-2} \sim M^{-2/3} \). For all values of \( \sigma_8 \), the fraction of 1-halo galaxy-matter pairs is tiny for \( M \geq 5 \times 10^{15} \ h^{-1} M_\odot \).

Figures 3.7e and 3.7f show the galaxy-matter correlation functions and excess surface density profiles, respectively, for this model sequence. At large scales, \( \xi_{\text{gm}}(r) \) and \( \Delta \Sigma(r) \) increase with \( \sigma_8 \propto 1/b \) as expected from linear theory. A similar increase appears on small scales because of the larger fraction of galaxy-matter pairs in more massive halos. In fact, the shapes of \( \xi_{\text{gm}}(r) \) and \( \Delta \Sigma(r) \) appear remarkably constant over the full range \( 0.1 \ h^{-1}\text{Mpc} \leq r \leq 20 \ h^{-1}\text{Mpc} \), a point we quantify below.

Figure 3.8 shows \( \Delta \Sigma(r) \) for three model sequences with different variations. In Figure 3.8a, we consider the same sequence of increasing \( \sigma_8 \), fixed \( \Omega_m \) shown in Figure 3.7, but we always keep halo concentrations fixed at the values predicted for \( \sigma_8 = 0.8 \). We adjust HOD parameters slightly from the values listed in Table 3.6 to obtain the minimum-\( \chi^2 \) fit to the projected correlation function with the new halo concentrations. In the large \( r \), 2-halo regime, \( \Delta \Sigma(r) \) is nearly identical to that shown in Figure 3.7f. However, fixing the concentrations to those of the central model has an important effect at small scales, causing the \( \Delta \Sigma(r) \) curves to converge. The constancy of shape in Figure 3.7f is thus partly a consequence of the changes in halo concentrations in different \( \sigma_8 \) models; higher \( \sigma_8 \) leads to earlier halo collapse and higher concentration, boosting \( \Delta \Sigma(r) \).
Figure 3.8b shows a sequence with fixed $\sigma_8 = 0.8$ and $\Omega_m$ varying from 0.2 to 0.4 in steps of 0.05. For this sequence, the HOD parameters $M_{\text{min}}$ and $M_1$ scale in proportion to $\Omega_m$, though we again make slight adjustments to fit $w_p(r_p)$. If halo concentrations were independent of $\Omega_m$, and we did not make those small adjustments, then $\xi_{\text{gg}}(r)$ and $\xi_{\text{gm}}(r)$ would be identical for all five models after this mass rescaling, since the halo mass function and halo bias factors are functions of $M/M_* \propto \Omega_m$ (see Zheng et al. 2002 for further discussion). In this case, $\Delta \Sigma(r)$ would have a constant shape and an amplitude proportional to $\Omega_m$. Figure 3.8b shows roughly this behavior, but the trend of higher concentration for lower $\Omega_m$ produces a weak convergence of models at small $r$.

We have so far assumed that satellite galaxies trace the dark matter in halos, with the same NFW radial profile. We now relax this assumption and allow the satellite profiles to have a lower concentration parameter, as suggested by some numerical studies (see §3.2 and Nagai & Kravtsov 2005). Figure 3.8c compares models with $\Omega_m = 0.3$, $\sigma_8 = 0.8$, and satellite concentration parameters $c_{\text{gal}} = \alpha_c c_{\text{dm}}$ with $\alpha_c = 0.3$, 0.5, 0.7, and 1.0. We again adjust HOD parameters to fit $w_p(r_p)$ after changing galaxy concentrations. These adjustments partly compensate for the changes in galaxy concentration, so the effect of a radial profile change is somewhat smaller here than in §3.2 (Fig. 3.1), where we kept other HOD parameters fixed. For $\alpha_c \geq 0.7$, the impact on $\Delta \Sigma(r)$ is under 3% at all $r$. For $\alpha_c = 0.3$, the effect rises to 7% at the scale $r \sim 0.5 - 1 \ Mpc$ where satellite galaxies make a dominant
contribution to \( \xi_{gm}(r) \) (see Fig. 3.6b). Small differences between galaxy and dark matter concentrations can thus be safely neglected, but large differences can have a small but measurable impact at \( r < 2 \, h^{-1}\text{Mpc} \).

Observational studies of galaxy-galaxy lensing often examine the ratio 
\[ \frac{\xi_{gm}(r)}{\xi_{gg}(r)} \],
which is equal to \( r_{gm}/b \) in the linear bias model (e.g., Hoekstra et al. 2001, 2002; Sheldon et al. 2004). Figure 3.9 plots this ratio for the four model sequences shown in Figures 3.7 and 3.8. Figure 3.9a shows the fixed-\( \Omega_m \), increasing \( \sigma_8 \) sequence of Figure 3.7. Since HOD parameters in each model are adjusted to match the projected correlation, model differences are driven almost entirely by changes in \( \xi_{gm}(r) \). At large scales, the ratio \( \frac{\xi_{gm}(r)}{\xi_{gg}(r)} \) is constant as predicted for linear bias, and \( \xi_{gm}(r) \) increases in proportion to \( \sigma_8 \propto 1/b \). Comparison to the bias factor defined by \( b^2 = \frac{\xi_{gg}(r)}{\xi_{mm}(r)} \) implies a cross-correlation coefficient \( r_{gm} \simeq 0.9 \) for all five models. However, in every case \( \frac{\xi_{gm}(r)}{\xi_{gg}(r)} \) rises sharply at a scale \( r \sim 1 \, h^{-1}\text{Mpc} \) near the 1-halo to 2-halo transition. For \( \xi_{gg}(r) \), this transition is fairly sharp, producing measurable deviations from a power-law (Berlind & Weinberg 2002; Zehavi et al. 2004). These deviations are smoothed out in \( \frac{\xi_{gm}(r)}{\xi_{gg}(r)} \) because the contribution of halos below \( M_{\text{min}} \) allows the 2-halo term to overlap more with the 1-halo term (compare Figs. 3.5a and 3.6a), and the ratio \( \frac{\xi_{gm}(r)}{\xi_{gg}(r)} \) therefore shows a sharp feature reflecting the break in \( \xi_{gg}(r) \). For higher \( \sigma_8 \), the 1-halo term extends to larger \( r \), and the jump in \( \frac{\xi_{gm}(r)}{\xi_{gg}(r)} \) is larger in amplitude but spread over a larger range of \( r \). The break in \( \frac{\xi_{gg}(r)}{\xi_{gg}(r)} \) is generally stronger for more strongly
clustered galaxy samples (Berlind & Weinberg 2002; Zehavi et al. 2005b), and we expect similar sample dependence for the $\xi_{gm}(r)/\xi_{gg}(r)$ jump.

At small scales, $\xi_{gm}(r)/\xi_{gg}(r)$ is again roughly flat, but at a level higher than the large scale ratio. As noted earlier, fixing halo concentrations causes the galaxy-matter correlation function of different $\sigma_8$ models to converge (Fig. 3.8a), so the $\xi_{gm}(r)/\xi_{gg}(r)$ ratios also converge in this case (Fig. 3.9b). Figures 3.9c and 3.9d show that the effects of $\Omega_m$ or $c_{gal}$ variations are much smaller than those of $\sigma_8$ variations. The $\sim 5\%$ model-to-model differences at small scales arise from their different halo concentrations, while the smaller differences at large scales reflect the slight changes in HOD parameters required to match $w_p(r_p)$. Present observations (Hoekstra et al. 2002; Sheldon et al. 2004) are consistent with $\xi_{gm}(r)/\xi_{gg}(r)$ that is approximately scale-independent, but the uncertainties are still fairly large, and testing for the feature predicted in Figure 3.9 will require more careful replication of observational procedures.

Figure 3.9 shows that the linear bias expectation of constant $\xi_{gm}(r)/\xi_{gg}(r)$ holds accurately for $r \geq 4 \, h^{-1}\text{Mpc}$ but fails at the 20–50$\%$ level in the non-linear regime. We can also ask how well the linear bias prediction $\Delta \Sigma \propto \Omega_m \sigma_8$ describes the scaling of $\Delta \Sigma(r)$ with cosmological parameters. To answer this question, and
to allow easy scaling of our predictions with cosmological parameters, we adopt the
more general formula

\[
\frac{\Delta \Sigma(r)}{\Delta \Sigma_{\text{FID}}(r)} = \left(\frac{\Omega_m}{0.3}\right)^\alpha \left(\frac{\sigma_8}{0.8}\right)^\beta
\]  

(3.17)

and determine best-fit values of \( \alpha \) and \( \beta \) at each separation \( r \). Here \( \Delta \Sigma_{\text{FID}}(r) \) is the
excess surface density prediction of the fiducial model with \( \Omega_m = 0.3 \) and \( \sigma_8 = 0.8 \),
and we fit \( \alpha \) and \( \beta \) using a full grid of models with \( \sigma_8 = 0.6, 0.7, 0.8, 0.9, 1.0 \) and
\( \Omega_m = 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45 \). We assume \( c_{\text{gal}} = c_{\text{dm}} \) in all cases.

Figures 3.10a and 3.10b plot the fitted values of \( \alpha \) and \( \beta \), respectively, as a
function of \( r \). Results for \( M_r \leq -20 \) galaxies and \( M_r \leq -21 \) galaxies are similar,
though the underlying \( \Delta \Sigma_{\text{FID}}(r) \) is different in the two cases. There are two notable
departures from the linear bias values \( \alpha = \beta = 1 \). At small scales, \( \alpha \) falls below
one, reflecting the weak convergence of \( \Delta \Sigma(r) \) curves seen in Figure 3.8b. This
convergence in turn reflects the \( \Omega_m \)-dependence of halo concentrations. At scales
\( r \sim 2 - 5 \ h^{-1}\text{Mpc} \), \( \beta \) rises above unity, corresponding to the slight divergence
of \( \Delta \Sigma(r) \) curves at these scales in Figure 3.7f. Figure 3.10c shows the rms and
maximum fractional errors between the \( \Omega_m \) or \( \sigma_8 \) dependences predicted by the full
analytic model and the scaling relation (3.17), calculated over our full model grid.
The rms errors range from \( \sim 1\% \) at large \( r \) to \( \sim 3\% \) at intermediate \( r \). The largest
errors arise for the \( \Omega_m = 0.45, \sigma_8 = 1.0 \) model, and they are roughly twice the rms
errors. Figure 3.10d shows the result of adopting the linear bias scaling \( \alpha = \beta = 1 \) in
equation (3.17). The linear bias predictions are accurate at the \( \sim 1\% \) (rms) to \( \sim 3\% \) (maximum error) level for \( r \geq 10 \, h^{-1}\text{Mpc} \), but the deviations become substantial at smaller \( r \), with errors of \( \sim 5 - 16\% \) at \( r = 3 \, h^{-1}\text{Mpc} \) and \( r = 0.1 \, h^{-1}\text{Mpc} \) for the \( \Omega_m = 0.45, \sigma_8 = 1.0 \) model. The errors of the linear bias scaling are typically a factor \( \sim 1.5 - 4.0 \) larger than those using our fitted values of \( \alpha(r) \) and \( \beta(r) \).

Table 3.3 lists \( \Delta_{\text{FID}}(r) \), the values of \( \Delta \Sigma(r) \) predicted by the analytic model for the \( M_r \leq -20 \) and \( M_r \leq -21 \) galaxy samples assuming \( \Omega_m = 0.3 \) and \( \sigma_8 = 0.8 \). It also lists the \( \alpha(r) \) and \( \beta(r) \) functions shown in Figure 3.10. Equation (3.17) can be used to scale these predictions to other values of \( \Omega_m \) and \( \sigma_8 \), and measurements of \( \Delta \Sigma(r) \) for these galaxy samples could then be used to obtain constraints in the \( \Omega_m - \sigma_8 \) plane. Our prediction of \( \Delta \Sigma_{\text{FID}}(r) \) is weakly dependent on the HOD parametrization that we adopt when fitting \( w_p(r_p) \). If we adopt the alternative parametrization used by Zehavi et al. (2005b), where \( \alpha_{\text{sat}} \) is fixed to one but the \( \langle N_{\text{sat}} \rangle_M \) cutoff is a fit parameter, then the \( \Delta \Sigma(r) \) predictions change by less than 5\% for all \( r > 0.1 \, h^{-1}\text{Mpc} \). We have not yet explored more general parametrizations, but we expect that the \( \Delta \Sigma(r) \) predictions would be robust at this level for all HOD models that fit the observed \( w_p(r_p) \).

Galaxy-galaxy lensing measurements are often made for flux-limited samples rather than absolute-magnitude limited samples to increase the signal-to-noise ratio, but such measurements are difficult to interpret quantitatively because they do not represent properties of a uniformly defined galaxy population. Because our
predictions apply on the non-linear scales where the measurement precision is higher, and because results from different radii can be combined, it should be possible to obtain precise constraints on $\sigma_8 \Omega_m$ from absolute magnitude-limited samples, and to use different samples to check for consistency. Since the values of $\alpha$ and $\beta$ vary with scale, it is possible in principle to break the degeneracy between $\Omega_m$ and $\sigma_8$. However, the deviations from linear scaling are not large, so while $\sigma_8 \Omega_m$ should be well constrained, individual parameter constraints are likely to be imprecise and sensitive to systematic uncertainties in the modeling.

Mandelbaum et al. (2006) have recently presented SDSS measurements for narrow bins of luminosity and of stellar mass, which are well suited to their goal of constraining halo virial masses and satellite fractions. For cosmological parameter constraints, we think it is better to use luminosity or mass-threshold samples, which provide higher signal-to-noise ratio, and which are easier to model robustly because there is no upper mass cutoff on $\langle N_{\text{cen}} \rangle_M$.

3.6. Summary

We have developed an analytic model to predict $\Delta \Sigma(r)$ for specified cosmological and galaxy HOD parameters and tested its validity using SPH and $N$-body simulations. We have used the analytic model to investigate the dependence of $\Delta \Sigma(r)$ on $\sigma_8$ and $\Omega_m$ when HOD parameters are chosen to reproduce the
observational space density and projected correlation function \( w_p(r_p) \) of the galaxy sample being measured. Our main findings are as follows:

1. In our SPH simulation, replacing the satellite galaxies of each halo with randomly selected dark matter particles has a 10–20\% effect on \( \xi_{gg}(r) \) and \( \xi_{gm}(r) \) at scales \( r \sim 0.5 \, h^{-1}\text{Mpc} \), and smaller impact at other scales. Most of this difference arises from the differing radial profiles of satellite galaxies and dark matter. If satellites are replaced in a way that preserves the radial profile but randomizes azimuthal positions, then changes to \( \xi_{gg}(r) \) and \( \xi_{gm}(r) \) are \( \lesssim 10\% \) at all radii, and changes to \( \Delta \Sigma(r) \) are \( < 5\% \). Dark matter subhalos around individual satellites orbiting in larger halos are present, but they have negligible impact on the global \( \Delta \Sigma(r) \).

2. If we randomly reassign the galaxy occupation number of each halo with \( M < 4.6 \times 10^{13} \, h^{-1}M_\odot \) to another halo of nearly equal mass, then changes to \( \xi_{gg}(r) \), \( \xi_{gm}(r) \), and \( \Delta \Sigma(r) \) are \( \lesssim 2\% \) at all \( r < 5 \, h^{-1}\text{Mpc} \). This result implies that any environmental dependence of the halo occupation function \( P(N|M) \) at fixed halo mass has minimal impact on these statistics for our simulated galaxy sample, which is defined by a baryonic mass threshold. For our largest scale point at 12 \( h^{-1}\text{Mpc} \), we find an effect of 10\% on \( \xi_{gg}(r) \), 5\% on \( \xi_{gm}(r) \), and 2\% on \( \Delta \Sigma(r) \), but the statistical uncertainties of our estimate are of comparable magnitude at this scale, so larger simulation volumes are needed to definitively establish the impact of any environmental dependence on the large scale bias factor. Taken together, results 1
and 2 show that the $\xi_{gg}(r)$ and $\Delta \Sigma(r)$ predictions of a full hydrodynamic simulation can be reproduced to 5% or better (usually much better) by populating the halos of a pure $N$-body simulation with the correct $P(N|M)$, provided that satellite galaxy populations have the correct radial profiles.

3. Our analytic model for $\Delta \Sigma(r)$ is based on the methods introduced by Seljak (2000) and Guzik & Seljak (2001), but it incorporates the scale-dependent halo bias and ellipsoidal halo exclusion corrections introduced by Zheng (2004) and Tinker et al. (2005) for $\xi_{gg}(r)$ calculations. We have tested the analytic model against numerical results from a grid of populated $N$-body simulations, which span the parameter range $\sigma_8 = 0.6 - 0.95$ and $\Omega_m = 0.1 - 0.63$, with HOD parameters chosen to match the space density and projected correlation function of SDSS galaxy with $M_r \leq -20$ (Zehavi et al. 2005b). The analytic model reproduces the numerical results to 5% or better over the range of $0.1 \, h^{-1}\text{Mpc} \leq r \leq 20 \, h^{-1}\text{Mpc}$. The residuals are consistent with the statistical errors of the numerical calculations, except for the innermost bin, where gravitational force softening in the $N$-body simulations artificially suppresses correlations.

4. For the $M_r \leq -20$ HOD parameters, pairs involving at least one central galaxy dominate the galaxy-galaxy correlation function at $r \lesssim 0.4 \, h^{-1}\text{Mpc}$ and $r \gtrsim 1 \, h^{-1}\text{Mpc}$. In the range $0.4 \, h^{-1}\text{Mpc} \lesssim r \lesssim 1 \, h^{-1}\text{Mpc}$, satellite-satellite pairs in large halos make the dominant contribution to $\xi_{gg}(r)$. In similar fashion, central galaxies dominate the galaxy-matter correlation function.
at small and large separations, while satellite galaxies dominate in the range $0.25 \ h^{-1}\text{Mpc} \lesssim r \lesssim 1.5 \ h^{-1}\text{Mpc}$. The halos of individual satellites make negligible contribution to $\Delta \Sigma (r)$ because they are usually tidally truncated below the scales at which the satellite contribution itself is important.

5. For samples with HOD parameters chosen to match the $M_r \leq -21$ SDSS sample of Zehavi et al. (2005b), the ratio $\xi_{gm}(r)/\xi_{gg}(r)$ is constant at $r \gtrsim 4 \ h^{-1}\text{Mpc}$, as predicted by the linear bias model, but it jumps by $20$--$50\%$ at scales $r \sim 1 \ h^{-1}\text{Mpc}$ near the transition from the 1-halo to 2-halo clustering regime, before settling to a new, higher value at small scales. The magnitude of the jump depends on $\sigma_8$, and it is likely to depend on the galaxy sample as well, being stronger for more highly clustered populations. In linear bias terms, the large scale values of $\xi_{gm}(r)/\xi_{gg}(r)$ correspond to a galaxy-matter cross-correlation coefficient $r_{gm} \simeq 0.9$, if we define the bias factor $b = [\xi_{gg}(r)/\xi_{mm}(r)]^{1/2}$.

6. We fit the dependence of $\Delta \Sigma (r)$ on cosmological parameters with a scaling formula $\Delta \Sigma (r) \propto \Omega_m^\alpha \sigma_8^\beta$, where $\alpha$ and $\beta$ are slowly varying functions of $r$. This scaling describes the results of our full analytic model with rms error $\lesssim 3\%$ over the parameter range $\Omega_m = 0.15 - 0.45, \sigma_8 = 0.6 - 1.0$. At large scales, $\alpha$ and $\beta$ approach the linear bias values $\alpha = \beta = 1$. However, forcing $\alpha = \beta = 1$ at all scales leads to errors that are larger by factors of $1.5 - 4.0$, relative to the scaling formula (3.17) with our fitted values of $\alpha$ and $\beta$. 

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Table 3.3 lists our predicted values of $\Delta \Sigma(r)$ for the $M_r \leq -20$ and $M_r \leq -21$ SDSS samples, assuming our fiducial cosmological model with $\Omega_m = 0.3$ and $\sigma_8 = 0.8$. Equation (3.17) allows scaling of these results to other values of $\sigma_8$ and $\Omega_m$, and measurements of $\Delta \Sigma(r)$ for these samples can be combined with these predictions to obtain cosmological constraints, which will be tightest on a parameter combination that is approximately $\sigma_8 \Omega_m$. Given the growth of the SDSS since the samples analyzed by Sheldon et al. (2004) and Zehavi et al. (2005b), it is probably preferable to extract $w_p(r_p)$ and $\Delta \Sigma(r)$ estimates for matched galaxy samples from the latest data sets. A full analysis should also investigate the effects of adding greater flexibility to the HOD parametrization itself, using, e.g., the 5-parameter formulation of Zheng et al. (2005). We have tested the effect of changing to a different 3-parameter description (see § 3.5), and we find changes of $\lesssim 5\%$ in the $\Delta \Sigma(r)$ predictions. We suspect that these predictions would remain similar for any choice of HOD parameters that reproduces the observed $w_p(r_p)$. Our SPH and $N$-body tests indicate that the analytic model predictions should be accurate to 5% or better given our HOD parametrization, though assumptions about galaxy profile concentrations have significant effect at $r < 2 \, h^{-1}\text{Mpc}$. This level of accuracy is adequate given the statistical errors expected for current samples, but refinement and testing on large simulations will be needed to take full advantage of future analyses of even larger, deeper surveys. Precise determinations of $\Omega_m$ and $\sigma_8$ can
play an important role in testing theories of dark energy and models of inflation, making galaxy-galaxy lensing an essential element of observational cosmology.
Fig. 3.1.— Galaxy-galaxy correlation functions (top panels), galaxy-matter correlation functions (middle panels), and $\Delta \Sigma (r)$ profiles (bottom panels) for the true galaxy population of an SPH simulation (solid lines) and the populated dark matter halos of this simulation (dotted lines, see text). Inset panels show the fractional difference between the SPH and populated halo results. In the left-hand panels, satellite galaxies in the populated halos are placed on randomly selected dark matter particles, while in the right-hand panels they are forced to follow the radial profile of satellite galaxies in the SPH simulation.
Fig. 3.2.— Comparison of galaxy-matter correlation functions for satellite galaxies of our standard SPH simulations (dashed lines), which uses $144^3$ particles in a $50\ h^{-1}\text{Mpc}$ box, to those of a higher resolution, smaller volume simulation ($128^3$ particles in a $22.222\ h^{-1}\text{Mpc}$ box, solid lines). In both simulations we select halos in the mass range $6 \times 10^{12}h^{-1}\text{M}_\odot \lesssim M \lesssim 2 \times 10^{13}h^{-1}\text{M}_\odot$ and satellite galaxies above the $3.5 \times 10^{10} h^{-1}\text{M}_\odot$ resolution limit of the larger volume run. Heavy lines show the full satellite-matter cross-correlation function, while light lines include only matter in the satellite’s parent halo (the one-halo term). Error bars are computed for the smaller simulation via bootstrap resampling.
Fig. 3.3.— Possible impact of environmental variation of the HOD on $\xi_{gg}(r)$, $\xi_{gm}(r)$, and $\Delta \Sigma(r)$. We shuffle the occupation numbers of halos of similar mass, leaving the populations of the five (filled circles) or 20 (open squares) most massive halos unchanged. Plots show the fractional difference between the shuffled halo results and the original results. Error bars in the points show the uncertainty in the mean calculated from four different shufflings (see text).
Fig. 3.4.— Large panels show $N$-body results for $\xi_{bg}(r)$, $\xi_{gm}(r)$, and $\Delta \Sigma(r)$ for the five cosmological parameter combinations indicated in the legend and detailed in Table 3.6. Attached bottom panels show the fractional difference between the analytic model calculations and the simulation results. Error bars represent fractional statistical uncertainty on the $N$-body results for the central model ($\Omega_m = 0.3$ and $\sigma_8 = 0.8$, solid lines), computed from the error on the mean of the five simulations.
Fig. 3.5.— Dissection of the galaxy-galaxy correlations for the fiducial model. (a) Contributions of one-halo (dashed) and two-halo (dotted) galaxy pairs to the full correlation function (solid). (b) Contributions of central-satellite (dashed), satellite-satellite (dotted), and central-central (dot-dashed) galaxy pairs. Panels (c) and (d) show the central/satellite decompositions of the one- and two-halo terms individually.
Fig. 3.6.— Dissection of the galaxy-matter correlations for the fiducial model, in the same format as Fig. 3.5. In panel (b)-(d), dashed and dotted lines show galaxy-matter pairs involving a central galaxy or a satellite galaxy, respectively.
Fig. 3.7.— Clustering contributions and clustering signals for the model sequence with fixed $\Omega_m$ and varying $\sigma_8$. (a) Mean halo occupation functions, determined by fitting the $w_p(r_p)$ data, with $\sigma_8$ increasing from top to bottom (see panel d legend). (b) Fraction of matter per logarithmic bin of halo mass. (c) Fraction of galaxy-matter pairs per logarithmic bin of halo mass. Note that pairs in higher mass halos are spread over a wider range of separations, diluting the contribution to any given $r \rightarrow r + dr$ bin. Panel (d), (e), and (f) show the galaxy-galaxy correlation function, galaxy-matter correlation function, and excess surface density, respectively.
Fig. 3.8.— Excess surface density profiles for other model sequence. We consider (a) the same model sequence as in Fig. 3.7, but with halo concentrations held fixed at the values for $\Omega_m = 0.3$ and $\sigma_8 = 0.8$, (b) a sequence of models with fixed $\sigma_8 = 0.8$ and $\Omega_m$ ranging from 0.2 (dotted) to 0.4 (long dashed) in steps of 0.05, and (c) models with $\Omega_m = 0.3$ and $\sigma_8 = 0.8$ in which galaxy profile concentrations satisfy $c_{gal} = \alpha_c c_{dm}$, with $\alpha_c = 0.3$ (dotted), 0.5 (long-dashed), 0.7 (dot-dashed), and 1.0 (solid). The inset panel shows fractional deviations from the $\alpha_c = 1.0$ model.
Fig. 3.9.— The ratio $\xi_{gm}(r)/\xi_{gg}(r)$, which is equal to $r_{gm}/b$ in the linear bias model. Panels (a)-(d) show the four model sequences illustrated in Figs. 3.7 and 3.8. Line types follow the same sequence as in those Figures, with $\sigma_8$ increasing from 0.6 (dotted) to 1.0 (long-dashed) in panels (a) and (b), $\Omega_m$ increasing from 0.2 (dotted) to 0.4 (long-dashed) in (c), and $\alpha_c$ increasing from 0.3 (dotted) to 1.0 (solid) in (d).
Fig. 3.10.— Parameters $\alpha(r)$ and $\beta(r)$ of the bias scaling relation $\Delta \Sigma(r) \propto \Omega_m^\alpha \sigma_8^\beta$ (eq.[3.17]) for samples matched to the SDSS $w_p(r_p)$ measurements of $M_r \leq -20$ (triangles) and $M_r \leq -21$ (circles) galaxies. Panel (c) shows the rms and maximum fractional errors of this scaling relation, relative to the full analytic calculation, over a model grid with $\Omega_m$ varying from 0.15 to 0.45 and $\sigma_8$ varying from $\sigma_8 = 0.6$ to 1.0. Panel (d) shows the same errors for linear bias scaling $\alpha = \beta = 1$. In all cases the maximum error arises for $\Omega_m = 0.45$, $\sigma_8 = 1.0$. 

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Note. — The HOD parameters of the fiducial model (Model 3) are chosen to reproduce the same clustering of the SDSS galaxy sample of $M_r \leq -20$ and to match the number density of galaxies $\bar{n}_g = 5.74 \times 10^{-3} \,(h^{-1}\text{Mpc})^{-3}$. HOD parameters of the other models are scaled with $\Omega_m$ from the HOD parameters of the fiducial model and adjusted to match $\xi_{gg}$ and $\bar{n}_g$.

Table 3.1. Parameters of GADGET Simulations and HOD Parameters
\[ M \frac{\Omega_m}{\sigma_8} \frac{M_{\text{min}}}{(h^{-1}M_\odot)} \frac{M_1}{(h^{-1}M_\odot)} \alpha_{\text{sat}} \]

<table>
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<tr>
<th>Model</th>
<th>( \Omega_m )</th>
<th>( \sigma_8 )</th>
<th>( M_{\text{min}}(h^{-1}M_\odot) )</th>
<th>( M_1(h^{-1}M_\odot) )</th>
<th>( \alpha_{\text{sat}} )</th>
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<tr>
<td>1</td>
<td>0.3</td>
<td>0.6</td>
<td>( 4.04 \times 10^{12} )</td>
<td>( 6.28 \times 10^{13} )</td>
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<td>2</td>
<td>0.3</td>
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<tr>
<td>3</td>
<td>0.3</td>
<td>0.8</td>
<td>( 4.71 \times 10^{12} )</td>
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<tr>
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</tr>
<tr>
<td>5</td>
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<td>1.0</td>
<td>( 4.95 \times 10^{12} )</td>
<td>( 1.23 \times 10^{14} )</td>
<td>1.19</td>
</tr>
</tbody>
</table>

Note. — The HOD parameters are chosen to reproduce the same clustering of the SDSS galaxy sample of \( M_r \leq -21 \) and to match the number density of galaxies \( \bar{n}_g = 1.17 \times 10^{-3} \ (h^{-1}\text{Mpc})^{-3} \).

Table 3.2. HOD Parameters for Different \( \sigma_8 \) Models
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<th>log $r$</th>
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<th>$M_r \leq -20$</th>
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<td>$\alpha$</td>
<td>$\beta$</td>
<td>$\log \Delta \Sigma (r)$</td>
<td>$\alpha$</td>
<td>$\beta$</td>
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<td>1.929</td>
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<td>1.13</td>
<td>1.582</td>
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<tr>
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<tr>
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<td>1.47</td>
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<td>1.04</td>
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<tr>
<td>0.666</td>
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<td>1.02</td>
<td>1.43</td>
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<tr>
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Table 3.3. The Weak Lensing Signal (cont’d)
<table>
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<th>$M_r \leq -20$</th>
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<td>$\log \Delta \Sigma (r)$</td>
<td>$\alpha$</td>
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<td>-0.145</td>
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<tr>
<td>1.205</td>
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<td>1.04</td>
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<tr>
<td>1.282</td>
<td>-0.292</td>
<td>1.04</td>
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Note. — The excess surface densities $\Delta \Sigma_{\text{FID}}$ of the fiducial model ($\Omega_m = 0.3$, $\sigma_8 = 0.8$) for SDSS galaxy samples of $M_r \leq -21$ and $M_r \leq -20$. The projected radius $r$ is in $h^{-1}\text{Mpc}$, and $\Delta \Sigma$ is in $hM_\odot\text{pc}^{-2}$. 

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Chapter 4

Probing Dark Energy with Cluster-Galaxy Weak Lensing

4.1. Introduction

Recent cosmological observations indicate that the universe is undergoing the second acceleration phase and the source of the accelerated expansion is an unknown energy component with negative pressure $P/\rho < -1/3$ that is prevalent over the entire universe and accounts for more than 70% of the matter-energy content of the universe, dubbed as dark energy. The nature of dark energy is one of the most outstanding problems in physical sciences and its understanding is believed to provide a quantum leap in our understanding of the fundamental physics beyond the standard model. However, in the absence of a persuasive theoretical explanation for dark energy, we must be guided by observations and the highest priority is therefore the verification (or nullification) of the simplest dark energy model of a “cosmological constant”. The defining property of dark energy, the repulsive gravity, accelerates the expansion of the universe and at the same time slows down the growth of structure driven by dark matter. Quantifying dark energy equation of
state $w = P/\rho$ is therefore the key goal of dark energy surveys that use these two different observable aspects of dark energy. Any discrepancy between these two types of measurements would alternatively indicate a tantalizing hint of possible failure of the general theory of relativity.

Cluster counting methods use the number of massive galaxy clusters in a given survey geometry to probe dark energy models. In particular, the number of the highest density peak at which massive clusters form is exponentially sensitive to the growth rate of structure, making its measurement one of the most promising tools in dark energy studies. However, the extreme sensitivity can only be fully utilized when individual cluster masses are measured at the level of accuracy better than that in the number of clusters required to separate different dark energy models. Especially since cluster masses are indirectly measured with mass-observables, such as optical or X-ray luminosity, and galaxy velocity dispersion, the scatter in the mass-observable relations can be misunderstood as different dark energy models when incorrectly accounted for.

Here we present a novel method to probe dark energy using cluster-galaxy weak lensing measurement of massive galaxy clusters. Cluster-galaxy weak lensing uses the weak distortion of background galaxy shapes to measure how matter is distributed around the sample of foreground lensing clusters, which provides a way to measure the statistical properties of lensing clusters without relying on the mass-observables. We rank-order clusters in a mass-observable and measure the statistical properties of
the stacked sample of rank-ordered clusters using cluster-galaxy weak lensing. Our method is also based on a mass-observable relation, but with minimal assumption: we require that the mass-observable is a monotonic function of true cluster mass on average, while no specific functional form is assumed. In §4.2, we elaborate how our method works and how we implement the scatter of mass-observable relations in our modeling. We develop an analytic model to compute cluster-galaxy weak lensing signals of the stacked sample of rank-ordered clusters in §4.3.1, investigate its sensitivity to cosmological parameters in §4.3.2, and present our main results on dark energy constraints obtainable from dark energy surveys in §4.3.3. Finally, we summarize our method and its prospects in §4.4.

4.2. Stacking Clusters of Galaxies

4.2.1. Idea

We describe a method to construct a sample of clusters for weak lensing measurements using a mass-observable, readily available from cluster surveys, such as optical luminosity, cluster richness, galaxy velocity dispersion, Sunayev-Zel’dovich effect, and X-ray luminosity. All of these mass-observables qualify to our minimal assumption that the mass-observable is a monotonic function of true cluster mass, only on average, while Sunayev-Zel’dovich effect and X-ray measurements are not
yet available for a large number of clusters. Here we characterize a mass-observable relation by an unknown function $F$, 

$$\log O_M = F(\log M), \quad (4.1)$$

with the monotonicity requirement,

$$\frac{\partial F}{\partial \log M} > 0, \quad (4.2)$$

where $O$ and $M$ indicate an observable and a true cluster mass. The overbar represents an ensemble average of clusters at a fixed $M$ and we implicitly assume that $O$ and $M$ are appropriately normalized to be dimensionless. For a given survey geometry, we rank-order observed clusters in a mass-observable and stack the top portion of the rank-ordered clusters for weak lensing measurements. In the absence of scatter in the mass-observable relation, the total number density of the observed clusters is

$$n_{\text{obs}}(> O_{\text{thr}}) = \int_{O_{\text{thr}}}^{\infty} dO \frac{dn}{dO} = \int_{M_{\text{thr}}}^{\infty} dM \frac{dn}{dM}, \quad (4.3)$$

where $O_{\text{thr}}$ and $M_{\text{thr}}$ are thresholds in the mass-observable and true cluster mass, and $\log O_{\text{thr}} = F(\log M_{\text{thr}})$. Given an assumed cosmological model, $M_{\text{thr}}$ can be obtained by matching $n_{\text{obs}}$ in equation (4.3) up to the uncertainty from a Poisson scatter of the observed clusters, even without any information on individual cluster masses or
the mass-observable relation $F$, and comparison to weak lensing measurements of
the stacked sample then determines the viability of the assumed cosmological model.

However, formation of massive galaxy clusters is hardly built upon simple
merger events of constituent galaxies and their observable properties cannot be
determined by a single parameter, cluster mass $M$. In other words, various detailed
microphysics, unknown in most cases, take part in the cluster formation, making it
appear as a stochastic process: Individual clusters can have a range of observable
properties at a fixed $M$, or scatter in mass-observable relations. In the presence
of scatter in mass-observable relations, our simple method should be modified
to account for the scatter distribution, $P(\log O | \log \hat{O}_M)$, and yet we illustrate in
Figure 4.1 that our method works for arbitrary mass-observable relations and it is
relatively robust to mass-observable scatter and bias.

Figure 4.1a shows a mass-observable relation, where points indicate individual
clusters and the thick line represents its mean relation $F$. Given an observational
threshold $O_{\text{thr}}$ (horizontal dotted line), we stack clusters with $O \geq O_{\text{thr}}$ (circles
filled with dark and gray), while $M_{\text{thr}}$ obtained from $n_{\text{obs}}(> O_{\text{thr}})$ yields a sample
of clusters that are correctly accounted for (dark-filled circles) and are missing in
the stacked sample (filled triangles). Though unwanted clusters are included in
the stacked sample (gray-filled circles), the majority of the constituent clusters
(dark-filled circles) are unaffected because only clusters with $M \simeq M_{\text{thr}}$ can scatter
into or out of the sample. Note that while the scatter distribution should be correctly
modeled for precise calculations, our method is only sensitive to the scatter around 
\( M \simeq M_{\text{thr}} \), as opposed to the cluster counting methods that require full knowledge 
of scatter distribution over the entire mass range \( M \geq M_{\text{thr}} \).

In Figure 4.1b, we show the halo mass function (dashed line), normalized to the 
cumulative number density \( n_{\text{obs}}(>O_{\text{thr}}) \). Since there are always more low mass halos 
than high mass halos, the fraction of up-scattered halos is larger than the fraction 
of down-scattered halos. Figure 4.1c shows that this trend rapidly increases with 
\( M_{\text{thr}} \) and the fraction of scattered halos becomes a significant part of the total \( n_{\text{obs}} \) 
at high \( M_{\text{thr}} \) due to the rarity of the massive halo abundance.

The bottom panels of Figure 4.1 show another example of a mass-observable 
relation, illustrating that our method works for arbitrary mass-observable relations.
Notice that the mass-observable relation has a very complicated form in this 
case and a mass estimate \( M_{\text{obs}}(\bar{O}) \) based on this observable can have bias, 
\( \Delta \equiv M_{\text{obs}}(\bar{O}_M) - M \neq 0 \), even on average, arising from incorrect (or imperfect) 
assumptions about clusters, such as hydrodynamic equilibrium, isothermality, and 
so on. Especially, the bias is likely to be changing as a function of mass, \( \Delta = \Delta(M) \), 
further complicating the standard cluster counting methods. On the contrary, our 
method requires no knowledge on the specific functional form of mass-observable
relations $F$, nor bias $\Delta(M)$. Given an observational scatter $\sigma_{\log O}$ around the mean $\bar{O}_M$, the scatter in true cluster mass is

$$\sigma_{\log M} = \left| \frac{\partial F^{-1}}{\partial \log \bar{O}_M} \right| \sigma_{\log O} = \left| \frac{1}{F'(\log \bar{M})} \right| \sigma_{\log O} \quad (4.4)$$

and it is only the scatter $\sigma_{\log M}$ around $M_{\text{thr}}$ that matters in our calculation. Note that even with a constant observational scatter $\sigma_{\log O}$, the scatter $\sigma_{\log M}$ in cluster mass may not be a constant over mass because of the derivative of the mass-observable relations $F$. However, we only need two unknowns $M_{\text{thr}}$ and $\sigma_{\log M}(M_{\text{thr}})$ to characterize the stacked sample, assuming $\sigma_{\log M}$ is slowly varying around $M_{\text{thr}}$. Figure 4.1e shows that our method works better and the fraction of scattered halos is smaller than in the upper panels of Figure 4.1, given the same $\sigma_{\log O} = 0.3$ but different functional forms $F$. We notice that it is difficult to find the best $M_{\text{thr}}$ that minimizes the scattered halos shown in Figure 4.1f, unless the functional form of the mass-observable relation $F$ is a priori known. However, we emphasize that our method works relatively robustly with minimal assumptions on mass-observable relations.

Our method for stacking rank-ordered clusters has more advantages than the insensitivity of the scatter in mass-observable relations at $M \gg M_{\text{thr}}$: (1) Clusters provide high shear signals, making its measurements easier and more reliable than cosmic shear measurements. (2) Since clusters are stacked and averaged, the irregular shape of individual clusters are smoothed and the stacked sample can
be well approximated as a spherical dark matter halo. (3) Central cD galaxies in clusters also facilitate weak lensing measurements by providing apparent lens centers so that there is no need to rely on two point statistics, as there is in cosmic shear measurements, where foreground lensing matter is diffuse and invisible. (4) By stacking many clusters, we further increase the number of background source galaxies available for lensing measurements, which makes it possible to apply our method to high redshift clusters and thereby obtain tomographic information on the growth of structure. (5) Since only the top portion is used in rank-ordered clusters, the stacked sample is relatively free from the completeness issue, unless we use $O_{\text{thr}}$ close to the survey limit. Two unknowns of our method can be constrained by $n_{\text{obs}}$ and weak lensing measurements of the stacked sample over a range of angular separation, and further information can be extracted by using multiple $O_{\text{thr}}$ of rank-ordered clusters, though at the cost of increasing free parameters.

### 4.2.2. Modeling the Stacked Sample of Rank-Ordered Clusters

We model individual massive galaxy clusters as dark matter halos of overdensity 200 times the cosmic mean matter density $\rho_m$ in approximate dynamical equilibrium and typical mass $M \gg 10^{13} h^{-1} M_\odot$, and we associate the true mass $M$ of halos with the mass of galaxy clusters obtainable from a variety of the observational methods.
mentioned in § 4.1 with the identical definition of mass. These halos may contain a large number of bright galaxies and dark matter substructure that are orbiting around a cluster potential. However, since many clusters are stacked in our method, it is only statistically averaged properties of clusters that are relevant to modeling the stacked samples of clusters. Here we use the halo mass function obtained from a suite of large $N$-body simulations to model the mean comoving number density of dark matter halos (Jenkins et al. 2001).

For the purpose of illustration, we take the Dark Energy Survey (DES) as our fiducial cluster survey, which is an optical-near infrared survey covering an area of $\Omega_{\text{DES}} = 5000 \text{ deg}^2$ of the southern Galactic cap with the SDSS $griz$ filters on the Blanco 4-m telescope at Cerro Tololo Inter-American Observatory (CTIO). The multiple repeat imaging will achieve $\sim 24^{\text{th}}$ magnitude, detecting $\sim 200,000$ optical clusters up to $z \approx 1.5$. We also consider a fiducial cosmological model of a spatially flat $\Lambda$CDM universe with $\Omega_m = 0.24$, $\Omega_\Lambda = 1 - \Omega_m$, $n_s = 0.95$, and $\sigma_8 = 0.75$, consistent with the recent analysis of the Sloan luminous red galaxy power spectrum and the WMAP cosmic microwave background measurements (Tegmark et al. 2006). The linear matter fluctuation is computed using a CMBFAST transfer function with $h = 0.73$ and $\Omega_b = 0.04$ (Seljak & Zaldarriaga 1996).

In Figure 4.2, we show the expected number of clusters given the survey geometry and the effects of cosmological parameters on the stacked sample of
clusters. Figure 4.2a shows the number of clusters of mass greater than the threshold mass at each redshift \( z_{cl} \) shown in the legend,

\[
N_{\text{obs}}(> M_{\text{thr}}) = \Omega_{\text{DES}} \times \int_{z_{cl}-\Delta z/2}^{z_{cl}+\Delta z/2} \frac{d^2V}{d\Omega dz} n_{\text{obs}}(> M_{\text{thr}}) dz, \quad (4.5)
\]

where \( d^2V/d\Omega dz \) is the physical volume element in a unit solid angle and redshift bin, and the redshift depth is \( \Delta z = 0.1 \) for each slice. Note that \( d^2V/d\Omega dz, n_{\text{obs}}, \) and \( N_{\text{obs}} \) are all dependent on redshift, though we have not marked this explicitly in equation (4.5). Basically, more physical volume is available given the fixed survey area at higher redshift, providing more clusters at a fixed threshold mass. However, this trend is reversed at high mass, since the matter fluctuation amplitude is correspondingly smaller at higher redshift and the number of the highest density peaks at which massive clusters form declines exponentially faster than the physical volume grows.

In Figure 4.2b, we show the changes in the number of clusters for cosmological parameter variations. Two distinct factors affect the number of clusters: the expansion of the universe and the growth of structure. The former changes the physical survey volume given the survey area, while the latter changes the halo number density, or halo mass function. The thick (thin) dotted line shows the effect of higher (lower) value of \( n_s \) by 10%, compared to the fiducial model. The physical survey volume remains unchanged for the \( n_s \)-variation, while the halo mass function changes due to the increase in small-scale power of the matter fluctuation. This
change results in more low mass halos but reduces the abundance of high mass halos given the fixed amplitude of the fluctuation amplitude $\sigma_8$. The opposite effect can be easily understood when the sign of the variation is reversed. Similar effects arise for the $\sigma_8$-variation, where the expansion of the universe is unaffected. However, the increase in $\sigma_8$ gives rise to a nontrivial change, distorting the shape of the halo mass function by increasing the characteristic mass scale $M_\ast$, because more massive halos are present at the fixed halo abundance.

The $\Omega_m$-variation alters the two factors at the same time: a faster expansion in a universe with higher $\Omega_m$ reduces the physical survey volume but more mass accelerates the growth rate of structure, lowering the fluctuation amplitude at $z > 0$ with the fixed normalization amplitude $\sigma_8$ at present. However, the change in the fluctuation amplitude at $z > 0$ is too small $\sim 1\%$ to result in any discernible distortion in the shape of the halo mass function, while the smaller physical volume is compensated by the increase in the overall halo abundance. Therefore, the number of clusters increases evenly in the threshold mass. Finally, a higher value of the dark energy equation of state parameter $w$ accelerates the expansion of the universe due to the additional energy component, while slowing down the growth rate of structure given the fixed amount of matter. Therefore, the net effect is similar to the $\sigma_8$-variation with the overall reduction in observed halos due to the smaller physical volume. Notice that the fractional change is by far the smallest among the variations we consider in Figure 4.2b.
Similarly, we can estimate the fractional change in $M_{\text{thr}}$ obtained by matching the observed number of clusters (eqs. [4.3] & [4.5]) for cosmological parameter variations. For example, smaller physical volume at higher $\Omega_m$ requires higher $n_{\text{obs}}(> M_{\text{thr}})$ given the fixed number of observed clusters, while the higher overall abundance of halos counterbalances higher $n_{\text{obs}}(> M_{\text{thr}})$, resulting in higher $M_{\text{thr}}$. However, this can be readily read off from Figure 4.2b. At a fixed threshold mass, higher $\Omega_m$ increases $N_{\text{obs}}(> M_{\text{thr}})$ and hence higher $M_{\text{thr}}$ is required to match the fixed number of observed clusters at higher $\Omega_m$, and so can the other cosmological parameters in the same way.

### 4.2.3. Mass-Observable Distribution

In reality, observable properties of clusters at a fixed $M$ have distributions $P(\log O | \log \bar{O}_M)$, and our calculation for $n_{\text{obs}}(> O_{\text{thr}})$ in equation (4.3) should be modified to implement the scatter distribution of mass-observable relations,

$$n_{\text{obs}}(> O_{\text{thr}}) = \int_{O_{\text{thr}}}^{\infty} dO \int_0^{\infty} dM \frac{dn}{dM} P(\log O | \log \bar{O}_M),$$

(4.6)

and the observed number of clusters $N_{\text{obs}}(> O_{\text{thr}})$ can then be obtained by using equation (4.6) in place of $n_{\text{obs}}(> M_{\text{thr}})$ in equation (4.5). Here we model the scatter distribution of a mass-observable relation as a Gaussian distribution at a fixed $M$,

$$P(\log O | \log \bar{O}_M) = \frac{1}{\sqrt{2\pi}\sigma_{\log O}} \exp \left[-\frac{(\log O - \log \bar{O}_M)^2}{2\sigma_{\log O}^2}\right].$$

(4.7)
Notice that the scatter distribution has no bias because $\bar{O}_M$ is an average by definition, and this approximation is not as restrictive as it may seem, if $\sigma_{\log O}$ is allowed to be a function of $M$ and also redshift $z_{cl}$. In most of cluster dark energy studies, the scatter distribution is modeled in terms of mass estimate $M_{\text{obs}}(O)$ of an observable $O$ and hence another parameter needs to be modeled in this case: bias $\Delta(M)$ in the mass estimate. However, in our approach, we formulate the distribution in terms of observable $O$ with the unknown mean relation $F$ of a mass-observable, which lacks the need for bias. Note that this reduction of a free parameter is made possible by using the unknown function $F$, because our method is relatively insensitive to how a mean mass-observable $\bar{O}_M$ is related to true cluster mass $M$.

With the assumption of a Gaussian distribution, the total number density of observed clusters is

$$n_{\text{obs}}(>O_{\text{thr}}) = \int_{0}^{\infty} \frac{dM}{dM} \frac{1}{2\sqrt{2\pi}\sigma_{\log O}} \text{erfc} \left[ \frac{\log O_{\text{thr}} - \log \bar{O}_M}{\sqrt{2\sigma_{\log O}}} \right],$$

(4.8)

where erfc is the complementary error function obtained by integrating over $O$ at a fixed $M$ and hence the equation is valid for arbitrary $\sigma_{\log O}$ that varies with mass and redshift. The argument of the complementary error function can be expanded by

$$\frac{\log O_{\text{thr}} - \log \bar{O}_M}{\sqrt{2\sigma_{\log O}}} = \frac{\log M_{\text{thr}} - \log M}{\sqrt{2\sigma_{\log O}/F'(\log M_{\text{thr}})}}$$

$$\times \left( 1 + \frac{F''(\log M_{\text{thr}})}{F'(\log M_{\text{thr}})} \log \frac{M}{M_{\text{thr}}} + \cdots \right),$$

(4.9)

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further simplifying our method. Note that the complementary error function quickly becomes unity as \( M \gg M_{\text{thr}} \) given the monotonicity condition (eq.[4.2]), making our method insensitive to the unknown higher-order derivatives of \( F \) and the scatter \( \sigma_{\log \Omega} \) at \( M \gg M_{\text{thr}} \). In other words, the total number density of observed clusters can be well approximated as

\[
n_{\text{obs}}(> \Omega_{\text{thr}}) = \int_0^\infty dM \frac{dn}{dM} \frac{1}{\sqrt{2\pi} \sigma_{\log M}(M_{\text{thr}})} \left[ \frac{\log M_{\text{thr}} - \log M}{\sqrt{2\sigma_{\log M}(M_{\text{thr}})}} \right],
\]

which requires two unknowns to be specified, \( M_{\text{thr}} \) and \( \sigma_{\log M}(M_{\text{thr}}) \). This approximation is perfect for the example given in Figure 4.1a, where \( F^{(n)} = 0 \) with \( n \geq 2 \), and is accurate at the sub-percent level for the example in Figure 4.1d.

In the presence of scatter, the expected number of observed clusters in Figure 4.2 changes depending on the details of the scatter distribution, while we note that the qualitative picture remains valid, regarding the cosmological parameter dependence.

### 4.3. Cluster-Galaxy Weak Lensing

#### 4.3.1. Analytic Modeling of Cluster-Galaxy Weak Lensing

Cluster-galaxy weak lensing uses the subtle distortion of background source galaxy shapes to measure the profiles of mean tangential shear around a sample of
foreground lensing clusters. The tangential shear profile can be computed by the excess surface density of a sample of lensing clusters,

$$\Delta \Sigma(r_p) \equiv \bar{\Sigma}(< r_p) - \bar{\Sigma}(r_p),$$

(4.11)

where $\bar{\Sigma}(< r_p)$ is the mean surface density interior to the disk of projected radius $r_p$ and $\bar{\Sigma}(r_p)$ is the averaged surface density in a thin annulus of the same radius (Miralda-Escude 1991a; Sheldon et al. 2004), and the critical surface density given the redshifts of source $z_s$ and lens $z_l$,

$$\Sigma_c(z_l, z_s) \equiv \frac{c^2}{4\pi G} \frac{D_s(z_s)}{D_l(z_l) D_{sl}(z_l, z_s)},$$

(4.12)

where $D_l$, $D_s$, and $D_{sl}$ represent the angular diameter distances to lens, source, and between lens and source, respectively. For a given redshift distribution $dn_g/dz$ of background source galaxies, the mean tangential shear at observed angle $\theta$ from the center of a foreground lensing cluster is

$$\bar{\gamma}_T(\theta) = \frac{1}{n_g(> z_{cl})} \int_{z_{cl}}^{\infty} dz \frac{d\Sigma(D_l \theta)}{d\Sigma_c(z_{cl}, z)},$$

(4.13)

where $n_g(> z_{cl})$ is the number density of background galaxies with redshift greater than $z_{cl}$. 

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We model the mean surface density of a stacked sample of rank-ordered clusters with the cluster-matter cross-correlation function $\xi_{cm}(r)$,

$$\Sigma(r_p) = \bar{\rho}_m \int [1 + \xi_{cm}(r_p, \chi)] d\chi,$$

(4.14)

where $\chi$ is the line-of-sight distance, $r^2 = r_p^2 + \chi^2$. The cluster-matter cross-correlation function obtainable from tangential shear measurements provides the average mass profile $\rho(r)$ of the sample of clusters on small scales, such as a spherical NFW profile (Navarro, Frenk & White 1997). However, on large scales, beyond the virial radius, where the density profile is truncated, $\xi_{cm}$ encodes additional information, cluster-bias that can be used to constrain both the cluster sample and the underlying cosmology. Our method for calculating $\xi_{cm}(r)$ draws on the method developed in Yoo et al. (2006) to compute the galaxy-matter cross-correlation function $\xi_{gm}(r)$, with simple modification for clusters in place of galaxies: Clusters as a whole are “operationally” modeled in our analytic calculation as a central galaxy occupying a massive dark matter halo that hosts zero satellite galaxies, since our primary interest lies in clusters, not galaxies in a cluster.

The DES will measure $\sim 300$ million galaxies with photometric redshifts. However, only galaxies with the largest angular size will be used for weak lensing measurements, substantially reducing the shape noise to $\sigma_\gamma = 0.16$, at the cost of lowering the number density of background source galaxies. The effective source
galaxy density is therefore $n_{\text{eff}} = 12 \text{ arcmin}^{-2}$ and the redshift distribution of background source galaxies is modeled by

$$\frac{dn_g}{dz} \propto z^2 \exp \left[ -\frac{z}{z_0} \right],$$

with the maximum redshift cutoff $z_{\text{max}} = 2$ and the median redshift $z_{\text{med}} = 0.68$, where $z_0 = z_{\text{med}} / 2.68$. The DES will have a training sample of galaxies, a small subset of observed galaxies with spectroscopic redshift to calibrate the photometric redshift parameters, providing the redshift distribution $dn_g/dz$ of background source galaxies. Therefore, we ignore any uncertainties in photometric redshift measurements of source galaxies. This assumption is adequate for our method, because we only need to use photometric redshift measurements to separate galaxies at $z > z_{\text{cl}}$ from those at $z < z_{\text{cl}}$ given the overall redshift distribution from a training sample, whereas in weak lensing tomography, it is vital to have accurate photometric redshift estimates and its error distribution.\footnote{However, we do note that it is rather optimistic to assume that spectroscopic redshifts will be fully available up to $z_{\text{max}}$, and hence photometric redshift estimates are necessary to overcome the difficulty in spectroscopic redshift measurements at certain redshift.} Notably, it gains little information on the lensing matter distribution in our method to separate source galaxies into several tomographic bins, given the fixed redshift of lensing clusters, as opposed to cosmic shear measurements, where the redshift of lensing matter depends on the mean source redshift and thereby tomography makes a significant difference. Note that we also assume no uncertainties in photometric redshift measurements of lensing
clusters, since the photometric errors are known to be small ($\Delta_z \simeq 0.01$) for clusters, compared to spectroscopic redshift, where $\Delta_z$ is the dispersion in $z_{\text{spec}} - z_{\text{photo}}$ of a Gaussian distribution (Koester et al. 2007), arising from the special property of the brightest cluster galaxies (BCG) in color space. However, we note that the accuracy of photometric redshift measurement of clusters quickly deteriorates at $z_{\text{cl}} > 0.6$ due to the fall-out of the cluster 4000Å continuum break in $r$-band, and hence we restrict the use of clusters in our method for those at $z_{\text{cl}} \leq 0.6$.

In Figure 4.3, we show $\Delta \Sigma$ and $\bar{\gamma}_T$ of the stacked sample of 5000 clusters at $z_{\text{cl}} = 0.5$, of which the threshold mass is $M_{\text{thr}} = 9.0 \times 10^{13} h^{-1} M_\odot$ in the absence of scatter in mass-observable relations. Since $\Delta \Sigma(r_p) \propto \bar{\Sigma}(r_p)$ when $\bar{\Sigma}(r_p)$ is a power-law, $\Delta \Sigma(r_p)$ (solid) in Figure 4.3a basically represents the projected density profile of the stacked sample on small scales, and it extends smoothly beyond the virial radius $\sim 1 h^{-1} \text{Mpc}$, approaching an asymptotic value. On large scales, where linear theory is accurate, this asymptotic relation is $\Delta \Sigma \propto \Omega_m \xi_{\text{mm}} \propto \Omega_m b_{\text{cl}} \xi_{\text{mm}}$, where $\xi_{\text{mm}}$ is the matter auto-correlation function and a cluster bias factor is

$$b_{\text{cl}}(> O_{\text{thr}}) = \frac{1}{n_{\text{obs}}} \int_{O_{\text{thr}}}^{\infty} dO \int_0^\infty dM \frac{dn}{dM} b_h(M) P(\log O | \log O_M), \quad (4.16)$$

and $b_h(M)$ is the bias factor of halos of mass $M$ (e.g., Sheth et al. 2001b). Similarly, $\bar{\gamma}_T$ in Figure 4.3b reflects this trend as a function of observable angular separation $\theta$, where the scales of the two panels in Figure 4.3 are the same for the fiducial cosmological model. The conversion between the panels of Figure 4.3 is purely
geometrical, depending only on the angular diameter distances of source and lens, regardless of how we construct the stacked sample, shown in Figure 4.3.

4.3.2. Cosmological Parameter Sensitivity

In Figure 4.3, we also show the variations of the predicted lensing signals for cosmological models with different $\Omega_m$ and for different stacked samples of clusters. For a fixed threshold mass, higher $\Omega_m$ (dotted) in Figure 4.3a enhances the overall lensing signal of the sample, while the lower concentration of halos at a fixed mass produces a weak increase of $\Delta \Sigma$ on small scales. The stacked sample (long dashed) with the fixed number of clusters has higher $M_{\text{thr}}$ producing higher $\Delta \Sigma$ than the sample at the fixed $M_{\text{thr}}$ (dotted), since there exist more clusters at a fixed mass shown in Figure 4.2b. Smaller angular diameter distance for models with higher $\Omega_m$ gives rise to larger $\theta$ at a fixed $r_p$ in Figure 4.3b, while higher $\Sigma_c$ results in lower $\gamma_T$ at a fixed $\Delta \Sigma$. In total, a glancing shift is performed from $\Delta \Sigma(r_p)$ to $\gamma_T(\theta)$, reducing the difference at $\theta < 10''$ where the slope of $\Delta \Sigma(r_p)$ is rather flat. Therefore, it is better to measure $\gamma_T(\theta)$ at the outskirt of the stacked sample around the virial radius, where the fractional difference is larger, than near the center.

We also note that baryon physics becomes important at the cluster center and the model prediction may have systematic errors. However, one should also note that it becomes quickly challenging to measure $\gamma_T(\theta)$ outside the virial radius, where the signal is extremely small, for example, $\gamma_T(\theta)/\sigma_\gamma < 1\%$ at $\theta \geq 500''$. 

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We investigate the sensitivity of the weak lensing measurements to other cosmological parameters in Figure 4.4, where we show the differences in $\gamma_T$ from the fiducial model predictions, instead of fractional differences, because it is extremely difficult to measure $\gamma_T(\theta) < 10^{-4}$, at which the fractional difference may be the largest among different models. In Figure 4.4a, we consider the effect of spectral index $n_s$-variations that affects only the growth of structure, and hence the differences in $\gamma_T$ solely result from the differences in $\Delta \Sigma$. Higher $n_s$ (solid) increases the abundance of low mass halos, while reducing high mass halos, due to the change in the matter power spectrum. In terms of halo abundance, the net effect is small so that $M_{\text{thr}}$ remains similar as shown in Figure 4.2b. However, higher concentration of lower mass halos enhances $\Delta \Sigma$ and $\gamma_T$ on small scales, while the mean mass of the sample changes little. At $z_{\text{cl}} = 0.3$ (thin solid), the difference is larger than at $z_{\text{cl}} = 0.5$ (thick solid). Since $M_{\text{thr}}$ is lower and the change in the halo abundance is more pronounced.

The $\sigma_8$-variation in Figure 4.4b shows the similar trend, without changing $D_l$ and $\Sigma_c$. However, the change in this case is mainly driven by the shift of the mean mass of the sample, increasing $\Sigma$, and this is borne out by the substantial change in $M_{\text{thr}}$ in Figure 4.2b. In Figure 4.4c, we show the $\gamma_T$ difference of the $\Omega_m$-variation. Note that $\Omega_m$ is different from the fiducial model value by 10%, while 50% variation is used in Figure 4.3 to contrast the difference. As shown in Figure 4.3b, the model variation in $\gamma_T$ become smaller than in $\Delta \Sigma$, because of the
change in angular diameter distances of source and lens. Finally, in Figure 4.4d the dark energy equation of state $w$-variation changes $\Delta \Sigma$ similar to the $\sigma_s$-variation, only with smaller amplitude, while the change in angular diameter distances further reduces the difference in $\tilde{\gamma}_T$ among models with different $w$.

In Figure 4.5, we first investigate the sensitivity of our method to $\Omega_m$ and $\sigma_s$ at several angular separation $\theta$ of the stacked sample of 5000 clusters at $z_{cl} = 0.5$, then we repeat the experiment for other cosmological parameters in Figure 4.6. Figure 4.5 shows the parameter combination of $\Omega_m$ and $\sigma_s$ that can reproduce $\tilde{\gamma}_T$ of the fiducial model value at each $\theta$. First, we consider the geometrical effect: smaller $D_l$ in higher $\Omega_m$ maps out the observed (fixed) angle $\theta$ into smaller $r_p$, corresponding to higher $\Delta \Sigma(r_p)$, while higher $\Sigma_c$ results in smaller $\Delta \Sigma$ given the observed (fixed) $\tilde{\gamma}_T$, independent of $\theta$. However, the effect of the growth of structure works in the opposite way: higher $\Omega_m$ results in a stacked sample of more massive halos, enhancing the overall amplitude of $\Delta \Sigma$. Therefore, lower $\sigma_s$ value is required to compensate the change in $\Delta \Sigma$ for higher $\Omega_m$ given the observed $\tilde{\gamma}_T(\theta)$. The slope of the parameter combination becomes steeper as $\theta$ increases, since larger $\theta$ corresponds to larger $r_p$, at which $\Delta \Sigma(r_p)$ changes rapidly with $r_p$, and hence larger change in $\sigma_s$ is necessary to reproduce the observed $\tilde{\gamma}_T(\theta)$.

Figure 4.6 summarizes the sensitivity to cosmological parameters, where only two parameters in consideration vary in each panel with other cosmological parameters fixed to be the same as in our fiducial cosmological model. Large spread
in parameter space indicates good handle on the parameters, since changes in the parameter combination accompany comparable changes in $\tilde{\gamma}_T$. Basically, Figure 4.6 shows that our method is sensitive to $\Omega_m$, $\sigma_8$, and $n_s$, and $\tilde{\gamma}_T$-measurements at several $\theta$ help differentiate cosmological parameters, since different parameter combinations are required to reproduce observations at different $\theta$. The bottom panels show the similar behavior for the $n_s$-variation. However, note that the contours are vertical at large $\theta$, insensitive to the $n_s$-variation, since the dependence of $\xi_{mm}$ on $n_s$ is weak on quasi-linear scales. The top panels show that all the contours are virtually vertical, reflecting the difficulty in $w$-measurements. Note that the spread in the top left panel is somewhat misleading, since $\Delta \tilde{\gamma}_T$ at $\theta \geq 300''$ is smaller than $10^{-5}$ for the range of $w$. In other words, there exists difference and yet it is extremely difficult to measure observationally.

### 4.3.3. Cosmological Parameter Constraints

We present our main results on dark energy constraints obtainable from a DES type survey using cluster-galaxy weak lensing in Figure 4.7. We choose the number of clusters at each redshift such that $M_{\text{thr}}$ is close to $10^{14} h^{-1} M_\odot$ in our fiducial cosmology, since we have only few clusters at higher $M_{\text{thr}}$ and the scatter in mass-observable relations becomes overwhelming at lower $M_{\text{thr}}$. About 30,000 clusters are stacked in total up to $z_{cl} \leq 0.6$ with $\Delta z = 0.1$, and we use three subsamples at each redshift that are defined with two new observational
thresholds, $O_{\text{thr}} < O_{\text{thr},1} < O_{\text{thr},2}$ with fixed number of clusters in each subsample,

$$N_{\text{obs}}(> O_{\text{thr},2}) < N_{\text{obs}}(> O_{\text{thr},1}) < N_{\text{obs}}(> O_{\text{thr}}),$$

corresponding to three $M_{\text{thr}}$. This procedure mainly helps calibrate mass-observable relations and its scatter distribution, because samples with different $M_{\text{thr}}$ respond somewhat differently to the same scatter distribution $P(\log O | \log \mathcal{O}_M)$. However, we note that there exists a limit on the number of subsamples: each subsample should have many clusters enough for our statistical description to be valid. Here we assume a constant scatter $\sigma_{\log O}$ in a mass-observable relation over the redshift range and a flat universe without any priors on cosmological parameters from other measurements, such as cosmic microwave background radiation. The total number of free parameters is therefore $23$: $3$ $M_{\text{thr}}$ at each redshift bin, $\sigma_{\log O}$, $\Omega_m$, $\sigma_8$, $w$, and $n_s$. We marginalize all the parameters but four cosmological parameters $\Omega_m$, $\sigma_8$, $w$, and $n_s$.

The bottom panel of Figure 4.7 shows the confidence level contours on $\Omega_m$ and $\sigma_8$, superior to the current analysis of the SDSS galaxy sample and WMAP (Tegmark et al. 2006), since our method draws on the exponential sensitivity of high mass clusters and a large number of background source galaxies allow for precision measurements of weak lensing signals up to high redshift $z_{\text{cl}} \leq 0.6$. As illustrated in Figure 4.6, $\mathring{\gamma}_T$ measurements of the stacked samples at each $\theta$ break the degeneracy in general lensing measurements, tightening the constraints. However, notice that the error ellipse is still aligned along the parameter combination of $\sigma_8 \Omega_m^\alpha$, where $\alpha \simeq 0.4$ is somewhat different from $\alpha = 0.6$ in cluster abundance studies at $z = 0$. 

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(White et al. 1993), and $\alpha = 1.0$ in the galaxy-galaxy lensing study (Yoo et al. 2006), since clusters with different $M_{\text{thr}}$ at many redshift slices are used. Similarly, the constraint on $n_s$ shown in the middle panels is also strong, comparable to the current direct measurement from galaxy power spectrum (Tegmark et al. 2006). Our method constrains $n_s$ mostly through the effect on constituent clusters of the stacked samples, well separated from the effects of other cosmological parameters, showing little degeneracy with $\Omega_m$ or $\sigma_8$. If $\gamma_T$ were to be measured at larger $\theta$, it is the $n_s$ constraints that could shrink substantially among other cosmological parameters due to the direct sensitivity of $\zeta_{mm}$ on large scales. The top panels show the constraints on dark energy equation of state that are larger by at least factor of 2 than in the other panels, while strong enough to potentially rule out many dynamic dark energy models with $w > -0.9$. Joint analysis of our method and other measurements should yield powerful constraints on $w$, while our constraining power would weaken, once we account for the time evolution of dark energy equation of state.

4.4. Summary

We have developed a method for weak lensing measurement of stacked clusters to probe dark energy models and applied our method to a DES type survey to project the utility of our method and to forecast cosmological parameter constraints obtainable from future wide-field imaging surveys. Our method is based on a
minimal assumption on mass-observable relations, readily available from cluster surveys, requiring the monotonic condition of mass-observables and true cluster masses. Rank-ordered clusters are then stacked for weak lensing measurements, which makes our method insensitive to the scatter distribution in mass-observable relations. Our main findings are as follows:

1. For a mass-observable relation that is a monotonic function of true cluster mass, the observed number density of rank-ordered clusters can be used to infer the threshold mass without any mass estimates of individual clusters in the absence of scatter in the mass-observable relation. This process remains approximately correct, even in the presence of scatter in the mass-observable relation, since the top portion of rank-ordered clusters is stacked regardless of the presence of scatter. While modeling the stacked sample of clusters requires a description of the scatter distribution, our method for stacking clusters works for arbitrary mass-observable relations and scatter distributions with minimal number of free parameters, and is relatively insensitive to their detailed functional forms.

2. The number of observed clusters is determined by two distinct factors: the physical volume and the halo mass function of an assumed cosmology given a survey geometry. The change in \( n_s \) and \( \sigma_8 \) only affects the latter, increasing the number of observed clusters for higher values, while decreasing the observed clusters at high mass threshold \( \gtrsim 10^{14} h^{-1} M_\odot \) for the \( n_s \)-variation. The change in \( \Omega_m \) and \( w \) affects
both factors, while the effects are approximately independent of threshold mass and opposite to each other: higher $\Omega_m$, more clusters, but higher $w$, less clusters.

3. Given an observational threshold, the number of observed clusters is fixed. However, the observed lensing signals are determined by the underlying cosmology, since the constituent clusters of the sample are different and the observable angle and tangential shear measurement are mapped into different excess surface densities at different separations for each cosmological model. The change in $n_s$ and $\sigma_8$ produces larger deviation in observed $\tilde{\gamma}_T$ than the change in $\Omega_m$ and $w$, since the former only affects the stacked sample, while the latter also changes the mapping, reducing the difference in $\tilde{\gamma}_T$.

4. Observations of tangential shear at several angular separations provide leverage to overcome the difficulty in discriminating the small difference of model predictions, and to break the degeneracy in cosmological parameters. We find that our method is sensitive to $\Omega_m$ and $\sigma_8$ at all angular separations, and $n_s$ is relatively well constrained at angular separation smaller than that corresponding to cluster virial radius. Observations at large scale $\theta > 10'$ can provide valuable information on cosmological parameters, particularly $n_s$, while it is difficult to measure $\tilde{\gamma}_T$ at such scales. Dark energy equation of state produces by far the least change in $\tilde{\gamma}_T$, making its measurement even more challenging.
5. Weak lensing measurements of clusters at $z_{\text{cl}} \leq 0.6$ in a DES type survey can provide tight constraints on $\Omega_m$ and $\sigma_8$, with 1-$\sigma$ error of 0.005 for both, taking full advantage of the exponential sensitivity of high mass clusters, while avoiding the problem of getting accurate mass estimate for individual clusters. The constraint on $n_s$ is also strong, $\sigma(n_s) = 0.01$, comparable to the current direct measurement from galaxy power spectrum. Our method can rule out dynamical dark energy models with $w > -0.98$ at the 68% confidence level, assuming there is no time evolution of dark energy equation of state.

With the advent of large wide-field imaging surveys, ambitious goal to probe the nature of dark energy seems ever more promising and even reachable to some degree. Our method provides a way to put strong constraints on dark energy models, and to complement other proposals using measurements of type Ia supernovae, cosmic shear, baryonic acoustic oscillations, and number of clusters. However, there are several aspects of our method that we need to investigate and improve before we apply our method to the future surveys: (1) Uncertainties in redshift estimates for lensing clusters should be accounted for when applied to clusters at $z_{\text{cl}} > 0.6$. (2) Small but nonzero deviation of central cD galaxies from the cluster center can affect weak lensing measurements, while we suspect the effect should be minimal for measurements at $r_p \geq 100h^{-1}\text{kpc}$ given observational evidences for the deviation, a few kpc. (3) Large-scale structure along the line-of-sight can give rise to projection effects that plague lensing measurements and nearby clusters that happen to lie
along the line-of-sight and hence are identified as one massive clusters. None of them should constitute more than few percent level of systematic errors, while proper account should be given to achieve the level of precision that the future surveys can deliver. Precise measurement of dark energy equation of state would test the simplest dark energy model, a “cosmological constant,” shedding new light on the fundamental physics beyond the standard model.
Fig. 4.1.— Robustness of our method for stacking clusters. Assuming that the mass-observable is a monotonic function of true cluster mass on average, clusters are rank-ordered in the mass-observable and stacked given an observational threshold ($O_{\text{thr}}$). A simple theoretical calculation in eq. [4.3] gives a threshold mass in true cluster mass ($M_{\text{thr}}$) by matching the total number of clusters and assuming no scatter in the mass-observable relation. (a) Mass-observable relation: Points indicate individual clusters and the thick solid line is the mean relation of true cluster mass and mass-observable. The horizontal dotted line represents an observational threshold and the vertical dotted line corresponds to $M_{\text{thr}}$. The filled circles represent clusters that satisfy both the observational and theoretical threshold masses, while the triangles and circles filled with light gray are clusters that satisfy one threshold mass but fail to meet the other. (b) Fraction of the up-scattered and the down-scattered halos. The dashed line shows the halo mass function, normalized to the cumulative number density $n_{\text{obs}}(> O_{\text{thr}})$, and the vertical dotted line is $M_{\text{thr}}$ obtained from $n_{\text{obs}}(> O_{\text{thr}})$ in eq. [4.3]. (c) Fraction of the up-scattered and the down-scattered halos as a function of threshold mass. (d), (e), and (f) Another example of a mass-observable relation, illustrating that our method works for arbitrary mass-observable relations and it is relatively robust to mass-observable scatter and bias.
Fig. 4.2.— Fiducial survey projection and effects of cosmological parameters. (a) Expected number of clusters with mass greater than a threshold mass at each redshift. (b) Fractional changes in the expected number of clusters at $z_{cl} = 0.5$ when a cosmological parameter in the legend is higher (thick lines) or smaller (thin lines) by 10% than the assumed fiducial cosmological model.
Fig. 4.3.— Excess surface densities and tangential shear profiles of the stacked samples for cosmological models with different $\Omega_m$. The stacked sample in the fiducial model (solid) is obtained by stacking 5000 clusters at $z_{cl} = 0.5$ and the threshold mass is $9.0 \times 10^{13} h^{-1} M_\odot$. For each $\Omega_m$ variation, we consider two different stacked samples for which either the number of clusters or the threshold mass is fixed. The values of $\Omega_m$ for the model variations differ by 50% of the fiducial model value.
Fig. 4.4.— Effects of cosmological parameters on tangential shear profiles of the stacked samples obtained by the fixed number of clusters. The stacked samples in the fiducial model include 1000 and 5000 clusters at $z = 0.3$ and 0.5 with threshold masses $1.7 \times 10^{14} h^{-1} M_\odot$ and $9.0 \times 10^{13} h^{-1} M_\odot$, respectively. The cosmological parameter shown in each panel is different by 10% of the fiducial model parameter.
Fig. 4.5.—Cosmological parameter contour that reproduces the tangential shear profile of the fiducial model at a given observed angle. Other cosmological parameters are fixed to be the same as in our fiducial cosmological model. The stacked sample is obtained by stacking 5000 clusters at $z_{cl} = 0.5$ for each combination of $\Omega_m$ and $\sigma_8$. 

\[ \Omega_m \]

\[ \sigma_8 \]
Fig. 4.6.— Cosmological parameter contours that reproduce the tangential shear profile of the fiducial model at a given observed angle. The lines are the same as Fig. 4.5.
Constraints on dark energy models from a DES-type survey. We marginalize all the parameters in deriving the constraints on the four cosmological parameters, $\Omega_m$, $\sigma_8$, $w$, and $n_s$, assuming a flat universe without any priors on cosmological parameters from other measurements.


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