Introduction to Gravity and Orbits

Isaac Newton
• Born in England in 1642
• Invented calculus in early twenties
• Finally published work in gravity in 1687 – The Principia

Newton’s Laws of Motion
• 1: An object in motion will remain in motion unless there is a force acting on it
• 2: The size of an object’s acceleration is proportional to the force applied and inversely proportional to the mass of the object.
• 3: For every force applied to a object, there is an equal and opposite force in response (i.e. for every action there is an equal and opposite reaction)
Newton’s First Law a.k.a Law of Inertia

- Objects in motion (or at rest) remain in motion (or at rest) unless acted upon by an outside force
- Example: Riding in a car
  - Braking a car, you continue forward
  - Accelerating a car, you remain at rest

Newton’s Second Law

- The size of an object’s acceleration is proportional to the force applied and inversely proportional to the mass of the object.
- Force = mass x acceleration
- To accelerate a more massive object requires more force
- An object accelerates in the same direction as the force

Newton’s Second Law of Motion

- Which would you rather push?
Newton’s Second Law of Motion

- Who would you rather do the pushing?

Newton’s Third Law of Motion

- For every force applied to a object, there is an equal and opposite force in response
- Apple pushes down on the table
- Table exerts force upwards on apple

Law of Universal Gravitation

- Gravity is a FUNDAMENTAL force
- Gravity is an ATTRACTIVE force
  - Pulls objects together
- Gravity is a UNIVERSAL force
  - Affects material everywhere in the Universe
  - Works over large scales
- Gravity is a MUTUAL force
  - Operates between pairs of objects
The force of gravity keeps planets in orbit, causes objects to fall, or holds objects to the surface of the Earth.

Force of Gravity

- Depends on the masses of the objects
  - More massive, stronger gravitational force
- Depends on the distance between the objects
  - More distance, weaker gravitational force
- DOES NOT DEPEND ON ANYTHING ELSE!

\[ F = \frac{G M_1 M_2}{d^2} \]

Dropping an Apple: Mutual Attraction

- Gravitational Force between the Earth (E) and the Apple (a):
  \[ F_{E \text{ on } a} = F_{a \text{ on } E} = \frac{G M_E M_a}{R_E^2} \]

- Acceleration of the Apple:
  \[ a_a = \frac{G M_E}{R_E^2} = 9.8 \text{ m/s}^2 \]  
  gravitational acceleration: \( g \)

- Acceleration of the Earth:
  \[ a_E = \frac{G M_a}{R_E^2} = 10^{-25} \text{ m/s}^2 \]

- Earth much more massive than apple.
- A force of the same size will accelerate the Earth much less than for the apple
- We see the apple fall. We don’t see the Earth move up to the apple.
Gravitation Depends on Mass

- More Mass, more Gravitational force
- Weight is actually a measure of the Earth’s gravitational force on our body mass:
  \[ W = F_{grav} = M_{body} \times g_{earth} \] (where \( g = 9.8 \, \text{m/s}^2 \))
- Mass always remains the same

Newton on Earth: 150 lbs
If Earth were half its mass: 75 lbs
If Earth were twice its mass: 300 lbs
If Earth were 50 times its mass: 7500 lbs

Dependence of Force on Mass is Linear

Gravitation Depends on Distance

- Objects closer together, force is stronger
- Objects farther apart, force is weaker
- \( F \) proportional to \(1/\text{distance}^2\)

Newton on Earth: 150 lbs
In the Space Station: 136 lbs
200 miles above Earth
At the Moon’s distance: 0.04 lbs
238,850 miles above Earth

Force of gravity decreases as distance between objects increases according to the Inverse Square Law

Changes in Mass

\[ F = \frac{GM_1 M_2}{d^2} \]
Changes in Distance

\[ F = \frac{GM_1M_2}{d^2} \]

- On Earth: 150 lbs
- On the Moon: 24.9 lbs
- Moon's mass: 0.01 M_E
- Moon's radius: 0.27 R_E
- Moon's grav. accel: \( g_m = 1.34 \text{ m/s}^2 \)
- On Mercury: 56.7 lbs
- Mercury's mass: 0.06 M_E
- Mercury's radius: 0.4 R_E
- Mercury's grav. accel: \( g_m = 3.68 \text{ m/s}^2 \)
- On Jupiter: 354.6 lbs
- Jupiter's mass: 318 M_E
- Jupiter's radius: 11 R_E
- Jupiter's grav. accel: \( g_j = 25.8 \text{ m/s}^2 \)
- On the Moon: 24.9 lbs
- Moon's mass: 0.01 M_E
- Moon's radius: 0.27 R_E
- Moon's grav. accel: \( g_m = 1.34 \text{ m/s}^2 \)

\[ \text{Weight} = \text{Mass} \times g \]

\[ = \text{Mass} \times \left( \frac{G M_{\text{planet}}}{R_{\text{planet}}^2} \right) \]

Consequence of Gravity: Orbits

- The Force of Gravity keeps objects in Orbit
- Without Gravity: Moon would move in straight line at constant speed (at 1000 m/s)
  - Newton’s First Law
  - It would move away from the Earth
- Gravity of Earth pulls Moon into a curved path
  - Causes Moon to orbit, constantly “falling” to Earth
Kepler’s Laws of Planetary Motion

- 1: Planets orbit on ellipses with the Sun at one focus
- 2: Planets sweep out equal areas in equal times
- 3: The orbital period squared is proportional to the size of the semi-major axis cubed

- Newton generalized these laws using his Law of Universal Gravitation to apply to any two objects

Understanding Orbits


First Law of Orbital Motion - Kepler

- Planets orbit on ellipses with the Sun at one focus
- A circle is just a special case of an ellipse with no eccentricity
- Eccentricity: how “squashed”

\[
e = 0.5
\]

\[
e = 0.9
\]
First Law of Orbital Motion - Newton

• Two types of Orbits:
  – Closed curves (Ellipses, Circles)
  – Open curves
• Speed dictates type of orbit

First Law of Orbital Motion - Newton

• Escape Speed: minimum speed needed to escape the force of gravity of the central body pulling object into a closed orbit
• Trajectory follows an open curve
• If v>escape speed, does NOT ORBIT

Second Law of Orbital Motion

• Move faster when closer to the central object.
• Move slower when farther away from the central object.
• Kepler: Planets sweep out equal areas in equal times

http://astro.unl.edu/naap/postAnimations/kepler.html
Third Law of Orbital Motion

- The time it takes a planet to complete its orbit depends on the size of the orbit (i.e., the distance from the sun)

- Kepler:
  - The orbital period squared is proportional to the size of the semi-major axis cubed: \( P^2 = a^3 \) (\( P \) in years, \( a \) in AU)

- Newton:
  - Generalized the relationship beyond Sun-Earth
  - There is also a dependency on the masses of the objects

- Kepler’s form was only an approximation when one object was much larger than the other (i.e. planets orbiting the Sun).

Most Important Take-away Points

- Gravity:
  - Gravity is a MUTUAL, ATTRACTIVE force
  - Force of gravity is proportional to mass
  - Force of gravity is inversely proportional to the square of the distance (Inverse Square Law)
  - Force = mass x acceleration; \( F_{\text{grav}} = GM_1M_2/d^2 \)

- Orbits:
  - The speed of the object dictates the shape of its orbit
  - Orbiting objects speed up close in to the central object and slow down at a distance