2. Measuring distances

Reading: As introduction to the course (going with §1 of the notes), you should read Chapter 1 of the textbook. If you need to remind yourself how scientific notation works, pay special attention to p. 14.

In Sections 2 and 3 of the course we will cover material that is in Chapters 2 and 3 of the textbook. We won't go in quite the same order as the text, but probably it is easiest to read the text in order rather than match to the order in lecture.

Bottom line: you should read chapters 1, 2, and 3 by Wednesday, September 5.

Bonus points: The crosswords at the end of each chapter are a good way to check that you have picked up the basic points, at least at the level of terminology. If you turn in the *crossword for Chapter 1* (on p. 22) by *Wednesday, September 5* (beginning of class), then I will add 4 bonus points to your score on homework assignment 1. You can either photocopy and fill in the crossword or turn in a list of the words for each of the numbered across and down entries (but it has to be clear what your answers are for each entry).

The homework assignments are graded on a standard 100-point scale, so 4 points is enough to, say, raise a B+ to an A-. Each homework assignment is worth 10% of the total course grade. You won't get bonus points for the Chapter 2 or 3 crosswords, but they are still a good way of solidifying what you have read for yourself.

Determining distance

Determining the distance to a celestial object is the fundamental problem of observational astronomy.

We see something in the sky; how can we tell how far away it is, if we can't go there?

What strategies do we have in everyday life?

Binocular vision. Compare from two different points of view.

Judge angular size relative to true size. More distant objects "look smaller."

At night: judge distance to a car based on apparent brightness of its headlights. More distant lights look fainter.

Parallax

Demonstrate with finger and closed eyes.

How can we see a celestial object from two significantly different points of view?

Look 6 months apart, baseline is twice the earth's distance from the sun.

Need a reference frame. Nearby stars should shift position relative to distant stars.

Most (but not all) ancient Greeks thought that the sun revolved around the earth, instead of vice versa.

One of the main arguments: if the earth revolved around the sun, we should see shifts of the stars because of parallax.

Seems like a pretty good argument. What's wrong with it?

Stars are so far away that the parallax is too small to detect with the naked eye.

The distance to stars: another route

Make a guess: suppose that stars are things like the sun.

Use their apparent brightness, relative to the sun, to infer their distance, relative to the earth-sun distance.

First done by Christian Huygens in the mid-1600s, using pinholes and glass beads to reduce the brightness of the sun until it matched that of the bright star Sirius.

Yielded the first approximately correct answer to the distance to stars.

To make this idea work quantitatively, we need to be more precise about what we mean by brightness.

We'll also need an equation relating brightness to distance.

Luminosity and apparent flux

The luminosity of a star (or any other kind of object, for that matter) is the amount of energy that it radiates into space every second.

Luminosity is *intrinsic*, doesn't depend on distance.

Luminosity has *units* of energy/time. Energy and time can themselves be measured in different units (e.g., seconds or hours or years for time).

We will generally use the standard metric unit for luminosity, which is a *watt*.

The watt is itself a composite unit: one watt is equal to one joule per second. A joule is the standard metric unit of energy, equal to twice the energy of a 1 kilogram mass moving at 1 m/sec.

The wattage of a bulb tells you how intrinsically bright it is. The apparent brightness of the light bulb depends on how far away it is.

(Technically, we define the wattage of a light bulb to be the rate at which it *consumes* energy. The light that it *produces* depends partly on how efficient it is.)

The luminosity of the sun is 3.9×10^{26} watts. (To determine this fact, you need to first determine the distance to the sun, in meters.)

The *apparent brightness* (a.k.a. *apparent flux*, or simply *flux*) of a star depends on how far away it is.

We measure apparent flux by measuring how much energy we receive in a given amount of time *in a given collecting area*.

Our standard unit of flux will therefore be watts/m^2 (watts per square meter).

When the sun is directly overhead, its flux at the earth's surface is about 1,400 watts/m^2.

Flux, luminosity, and distance

If we move an object twice as far away, its apparent flux gets *four* times fainter.

The light spreads out from the object in two dimensions, so if it has gone twice as far it is spread over an area that is four times larger.

More generally,

$$f = \frac{L}{4\pi d^2}.$$

L = intrinsic luminosity of the source [watts]

d = distance of the source [meters]

f = apparent brightness (flux) of source [watts/m^2]

In words: The apparent brightness of a source is equal to the intrinsic luminosity divided by 4π times the square of the distance.

We can interpret this equation by thinking of the energy carried by the light from the source being "spread out" over a sphere whose area is $4\pi d^2$.

The distance to α-Centauri

The star α-Centauri is (we now know) one of the closest stars to the sun, and it is quite similar to the sun in its properties.
Let's apply the Huygens argument to α-Centauri.
With careful measurements, one can show that the flux of the Sun is 4.8×10^{10} (48 billion) times that of α-Centauri.

Let's introduce some notation:
$f_\odot$ = flux of the Sun
$L_\odot$ = luminosity of the Sun
$d_\odot$ = distance from earth to the Sun
f_α = flux of α-Centauri
L_α = luminosity of α-Centauri
d_α = distance to α-Centauri

Our flux-luminosity-distance relation must apply to both the Sun and α-Centauri. Thus

$$f_\odot = \frac{L_\odot}{4\pi d_\odot^2} \qquad ; \qquad f_\alpha = \frac{L_\alpha}{4\pi d_\alpha^2}.$$

We also know from measurements that $f_\odot / f_\alpha = 4.8 \times 10^{10}$.

To find the distance to α-Centauri, divide the left side of the first equation by the left side of the second equation and the right side of the first equation by the right side of the second equation.

$$4.8 \times 10^{10} = \frac{f_\odot}{f_\alpha} = \frac{L_\odot / 4\pi d_\odot^2}{L_\alpha / 4\pi d_\alpha^2} = \frac{L_\odot}{L_\alpha} \frac{d_\alpha^2}{d_\odot^2}.$$

Now we'll assume that L_α is equal to $L_\odot$, so their ratio is just one.
Then take the square-root of both sides of the equation to get

$$\frac{d_\alpha}{d_\odot} = \sqrt{\frac{f_\odot}{f_\alpha}} = \sqrt{4.8 \times 10^{10}} = 2.19 \times 10^5 = 219,000.$$

This combination of measurements (of $f_\odot$ and f_α), assumption (that $L_\odot = L_\alpha$), and calculation (using $f = L/4\pi d^2$, which is a general equation for how light behaves) tells us that the distance to α-Centauri is 219,000 times larger than the distance from the earth to the sun.
In fact, α-Centauri is about 50% more luminous than the sun ($L_\alpha = 1.5 L_\odot$), so this calculation underestimates its true distance by about 25%.

To get the distance to α-Centauri in meters (for example), we also need to know $d_\odot$, the distance from the earth to the sun, in meters.
However, this result is already enough to tell us how big the parallax to α-Centauri is: draw a triangle where one side is $d_\odot$ (the change in the earth's position as it goes 1/4 of the way around the sun) and the other is $219,000 d_\odot$.
The apparent position of α-Centauri changes by 1/219,000 of a radian, or 0.00026 degrees.
This is much too small a change to see with the naked eye, and α-Centauri is the second *closest* star to the earth.

Angles, arc-seconds, and resolution

The most familiar unit of angle is the *degree.* There are 90 degrees in a right angle, 360 degrees in a full circle.

To roughly gauge angles on the sky, it is useful to know that your closed fist at arms length covers about 10 degrees, and the tip of your index finger at arms length covers about 1 degree.

In mathematical equations, it is often convenient to use *radians* as the unit of angle. There are 2π radians in a full circle, so one radian is $360/2\pi = 57.3$ degrees.

Astronomical objects outside our solar system are extremely far away, and their angular sizes are usually much, much smaller than a degree.

Rather than write awkwardly small numbers all the time, the unit of angle that we will typically use is the *arc-second.*

There are 3600 arc-seconds in 1 degree (analogous to 3600 seconds in one hour).

There are 206,265 arc-seconds in 1 radian.

If we can see that an object is extended (not just a point), we say it is *resolved.*

With the naked eye, you can resolve objects about 60 arc-seconds across (1 arc-minute).

Because of blurring by the atmosphere, the sharpest images made by a good ground-based telescope are about 1 arc-second across, making this a natural unit of angle for astronomy.

1 arc-second is the angle subtended by a quarter at a distance of about 3.1 miles.

Hubble Space Telescope, above the earth's atmosphere, makes images with resolution of about 0.1 arc-seconds (limited by the size of its mirror).

The sun and the moon are both about 1/2 degree across on the sky (1800 arc-seconds).

The diameter of the sun is 400 times the diameter of the moon, but it is also 400 times further away.

Since other stars are at least 200,000 times further away than the sun, their angular sizes are much less than one arc-second.

They are *unresolved*, appearing as points of light even in a very good telescope.

Other galaxies are very large (many thousands of light years across), so they are resolved, extended sources even when they are very far away (many millions of light years).

Giant clouds of gas in our own galaxy (the Milky Way) are resolved, extended sources in telescopes because they are big (a few light years across) and not *so* far away (a few thousand light years).

The Astronomical Unit (AU) and the parsec

The distance from the earth to the sun is 1.5×10^{11} m. (Strictly speaking, this is the average distance, since the earth's orbit is slightly elliptical. Also, it's really 1.496×10^{11} m.)

This distance plays a fundamental role in the determination of other astronomical distances, so it has a name, the *Astronomical Unit*, abbreviated AU.

$$1\,\mathrm{AU} = 1.5 \times 10^{11}\,\mathrm{m}.$$

For future reference, note that the speed of light is 3×10^8 m/sec, implying that 1 AU = 500 light-seconds.

If we measure a star's parallax, we can determine its distance from geometry alone.

A common unit of distance in astronomy is the *parsec*, at which annual parallax would be 1 arc-second.

$$\begin{aligned} 1\ \mathrm{pc} &= 206,265\ \mathrm{AU} = 3.26\ \text{light years} \\ &= 3.1 \times 10^{13}\ \mathrm{km} = 1.9 \times 10^{13}\ \text{miles}. \end{aligned}$$

Also: 1 kpc = 1 kiloparsec = 1000 parsecs; 1 Mpc = 1 megaparsec = 1 million parsecs; 1 Gpc = 1 gigaparsec = 1 billion parsecs

If a star has parallax p arc-seconds, then its distance in parsecs is $d = 1/p$. Smaller parallax, larger distance.

Nearest stars are more than 1 parsec away $\longrightarrow$ ancients couldn't measure stellar parallaxes.

With modern techniques, can measure parallaxes as small as 0.01" from the ground, distances up to 100 pc. Satellite measurements can go to 0.001", 1000 pc. Future satellites will do better still.

Nonetheless, can only measure parallax for stars in our region of the Milky Way.

For more distant objects, must estimate distances indirectly.

The Standard Candle Method

The most common approach to measuring distances on cosmological scales (i.e., beyond the Milky Way) is the *standard candle* method.

Find some object of known luminosity L.

Measure its apparent flux f.

Find its distance from $d = \sqrt{L/4\pi f}$ (which is a rearrangement of the equation $f = L/4\pi d^2$.

The challenge: how do you know L in the first place?

Need to "calibrate" the luminosity by finding objects in systems of known distance, e.g., from parallax.

Cepheid Variables

Most stars are steady, staying nearly constant in brightness.

Some stars vary, getting brighter and fainter periodically.

Cepheid variables (named for the first known example, a star in the constellation Cepheus) have periods that range from a couple of days up to a couple of hundred days.

In the early 1900s, Henrietta Leavitt made a catalog of hundreds of variable stars in the Magellanic Clouds, large concentrations of stars visible in the southern hemisphere.

She noticed that Cepheids with longer periods had brighter apparent fluxes. Assuming that stars in the Magellanic Clouds are all at approximately the same distance, she concluded (correctly) that Cepheids with longer periods are more luminous.

We now have a fairly good understanding of Cepheid variables: bigger stars are brighter, and they also take longer to expand and contract.

Cepheids make excellent standard candles:

- they are very luminous, hundreds or thousands of times more luminous than the sun, so they can be seen to great distances
- their variability makes them relatively easy to discover
- once you measure the period by monitoring the star for a long time, you know the luminosity.

To calibrate these standard candles, either:

a) determine the distance to the Magellanic Clouds by other means (using other kinds of stars, whose luminosities were calibrated with parallax measurements)

b) measure the distances to Cepheids in the Milky Way using parallax.

Method b is preferable, but because Cepheids are rare even the closest ones are hundreds of parsecs away.

It has only recently become feasible to measure highly accurate parallaxes to them using space telescopes.

Edwin Hubble and the Andromeda Nebula

In the early 20th century, the nature of "spiral nebulae" was one of the hottest topics in astronomy.

Were they gas clouds within the Milky Way, or were they much more distant systems, themselves composed of enormous numbers of stars?

In 1921, Edwin Hubble used the new 100" telescope (100 inches being the diameter of the mirror) at Mount Wilson Observatory to settle the question.

For the first time, he discovered Cepheid variable stars in the Andromeda Nebula.

They were very faint (small apparent flux), and by using Leavitt's period-luminosity relation he determined that Andromeda was at very large distance.

Modern value: 700 kpc, about 2.1 million light years.

Implications:

Andromeda was much further than any stars in the Milky Way, and was itself a system of billions of stars like our own Galaxy.

Other spiral nebulae, much fainter and smaller on the sky, must be much more distant still.

The universe is filled with galaxies, and it's big.

Andromeda is the nearest large galaxy to the Milky Way, and is roughly similar in size and shape to the Milky Way.

There are some much smaller, irregular or spheroidal galaxies that are closer, including the Magellanic Clouds.

Standard Rulers

If we observe an object of known length, we can use it as a *standard ruler*.

Instead of $f = L/4\pi d^2$, the guiding formula is

$$\theta = l/d.$$

l = length of the source [meters]

d = distance of the source [meters]

θ = angular size of source, in radians (multiply by 206,265 to get angle in arc-seconds)

This formula is only accurate when l is much smaller than d, so that the angle θ is small, but this is nearly always the case for astronomical observations.

Unfortunately, there are fewer examples of good standard rulers than good standard candles.

We more often use this formula to get the intrinsic size of an object whose distance d and angular size θ have been measured.

For example, the Andromeda galaxy is about 2 degrees across on the sky. If it is 2.1 million light-years away, what is its diameter in light years?