4. Hubble's Law and Its Implications

Reading: If you haven't finished reading chapters 1-3, do it now. The new reading for this section is short: 5.1, 5.2, and 5.6. If you're feeling unsatisfied with your understanding of what the expansion of the universe means, you may want to jump ahead and read chapter 7, though we will get to that chapter later after covering some of the intervening subjects.

Hubble's Law

Hubble found that a galaxy's recession velocity is proportional to its distance.

This discovery can be summarized by an equation:

$$v = H_0 d.$$

v = velocity of galaxy away from the Milky Way [km/sec]

d = distance of the galaxy [Mpc]

H_0 = Hubble's constant [km/s/Mpc]

In principle, one can use any set of consistent units in this equation (e.g., v in miles/hour, d in furlongs, and H_0 in miles/hour/furlong).

However, it is conventional to give galaxy distances in Mpc (megaparsecs) and velocities in km/sec, which yields reasonable numbers.

In this case, the units of H_0 must be km/sec/Mpc so that the units of the equation work out.

Measuring H_0 is hard, for reasons we will come to shortly. but we now think we know it pretty well.

A good current estimate is

$$H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}},$$

and throughout the course you should assume this value of H_0 except when told otherwise.

The current uncertainty in the value of H_0 is about 3%, and astronomers are working hard to get this uncertainty down to 1%.

The Center is Everywhere

At first glance, Hubble's discovery seems to imply that we are at a special place in the universe: every galaxy is rushing away from *us*.

However, with a diagram and a small bit of calculation (Homework 1, Part III), you can show that if Hubble's law holds then observers in *every* galaxy see other galaxies rushing away, also following Hubble's law.

Hubble's law arises because the expansion of the universe is stretching the distances between galaxies, and the further away the galaxy is, the bigger the effect.

However, the universe is not expanding *into* anything, and there is no unique *center* to the expansion.

Alternatively, since one sees the same expansion law from any place in the universe, we could say that "the center is everywhere."

Two analogies are useful, though each has shortcomings.

Analogy 1: An infinite universe with 3-dimensional space. The universe is an infinite raisin cake in an infinite oven, and the raisins represent the galaxies. As the cake rises, the expansion of space drags the galaxies away from each other.

Analogy 2: A finite, *two-dimensional* universe confined to the surface of a spherical balloon. As the balloon inflates, the expansion of space drags the galaxies away from each other.

One is tempted to view the center of the balloon as a center of expansion, but every place on the *surface* of the balloon, and thus in the universe, is equivalent.

Analogy 2 makes sense because Einstein's theory of gravity allows *three-dimensional* space to be curved, and in fact to curve back on itself like a sphere does.

The rubber band example in §7.3 of the book is a 1-dimensional version of this analogy.

Peculiar Velocities

Galaxies do not follow Hubble's law exactly.

In addition to the expansion of the universe, galaxy motions are affected by the gravity of specific, nearby structures, such as the pull of the Milky Way and Andromeda galaxies on each other.

Each galaxy therefore has a *peculiar velocity*, where peculiar is used in the sense of "individual," or "specific to itself."

Thus, the recession velocity of a galaxy is really

$$v = H_0 d + v_{\rm pec},$$

where $v_{\rm pec}$ is the peculiar velocity of the galaxy along the line of sight.

If peculiar velocities could have any value, then this would make Hubble's law useless.

However, peculiar velocities are typically only about 300 km/sec, and they very rarely exceed 1000 km/sec.

Hubble's law therefore becomes accurate for galaxies that are far away, when $H_0 d$ is much larger than 1000 km/sec.

Furthermore, we can often estimate what a galaxy's peculiar velocity will be by looking at the nearby structures that will be pulling on it.

Mapping the Distribution of Galaxies

Hubble's law is a tremendous asset when it comes to mapping the 3-dimensional distribution of galaxies, because it means that we can estimate a galaxy's distance from its redshift.

As we have seen, measuring distances directly is hard, and the more distant the galaxy the harder it gets.

But measuring the redshift is fairly straightforward: measure the galaxy's spectrum and look for the shifts of its absorption or emission lines.

Then use $d = v/H_0$ to get the distance.

Peculiar velocities will "fuzz out" the map and distort it slightly, but they will not wreck it completely.

Before the 1990s, astronomers usually measured the redshifts of galaxies one at a time, and the largest 3-d maps had about 10,000 galaxies.

The *Sloan Digital Sky Survey*, which started operating around 2000 (after a decade of construction) has revolutionized this process by using drilled aluminum plates plugged with optical fibers to measure the spectra of 640 objects at one time. (In 2009 this was upgraded to 1000 objects at a time.)

To date, the SDSS has mapped more than 2 million galaxies and 200,000 quasars.

Large scale maps of the universe show that galaxies are clumped into groups and clusters that are themselves arranged into giant filaments and walls, interleaved with empty bubbles and tunnels.

The largest coherent structures that we see are roughly 100 Mpc (300 million light years) long.

Towards the end of the course, we will learn that these structures form by the action of gravity on tiny fluctuations imprinted in the universe during the first billionth of a second of cosmic history.

Measuring H_0

The Hubble constant is an important number in astronomy because we use it to determine the distances to objects beyond the Milky Way.

We in turn use the distances to determine the sizes and luminosities of objects, which are crucial to figuring out what they are.

As we will see shortly, H_0 also sets the scale for the *age* of the universe.

If there were no peculiar velocities, then measuring H_0 would be fairly easy:

• Measure the distance to a nearby galaxy — using Cepheids, for example

• Measure the galaxy's redshift

• Compute $H_0 = v/d$.

But because of peculiar velocities, to get an accurate value of H_0 we need to measure the distances to galaxies that are far away, so that $H_0 d$ is much larger than $v_{\rm pec}$.

Such galaxies are too far away to measure Cepheid distances directly.

Instead, we have to use Cepheid distances to nearby galaxies to calibrate "secondary distance indicators," which are more luminous and can therefore be seen at greater distances.

A variety of such indicators have been used, such as the total luminosity of galaxies that have a given rotation speed.

The most powerful approach today uses a class of violent stellar explosions called *Type Ia supernovae*, which arises when a white dwarf star becomes too massive to support itself against gravity.

Roughly speaking, Type Ia supernovae have the same *peak* luminosity (luminosity at the maximum brightness of the explosion), so they can serve as standard candles.

Very roughly, there is one supernova per century per galaxy, so if you monitor 1000 galaxies you will discover 10 supernovae per year.

The recipe for measuring H_0 then becomes:

• Using Cepheids, measure the distances to nearby galaxies that have well observed Type Ia supernovae.

• Using $L = 4\pi d^2 \times f$, calibrate the peak luminosity of Type Ia supernovae.

• Discover and measure Type Ia supernovae in galaxies that are 50-200 Mpc away. Measure the distances of these galaxies by using the supernovae as standard candles, $d = \sqrt{L/4\pi f}$.

• Compute $H_0 = v/d$ from these galaxies, for which peculiar velocities are a small effect.

Hubble Space Telescope (HST) has played a crucial role in getting accurate determinations of H_0 because of its ability to make very sharp images and to do very accurate flux measurements.

Specifically, HST has made it possible to:

• Find and measure Cepheids in more distant galaxies, and thus to more galaxies that have Type Ia supernovae

• Accurately measure supernovae in distant, 50-200 Mpc galaxies, where the supernovae themselves are apparently faint (low f)

and, most recently,

• Measure direct parallax distances to more Cepheids in the Milky Way, thus avoiding the need for more complicated calibrations via the Magellanic Clouds.

The *James Webb Space Telescope* (JWST), the successor to Hubble with expected launch in 2018, will be another powerful step forward.

The Age of the Universe: A First Glimpse

Example: Car race in the desert.
Cars moving 20 miles/hour are 20 miles from the center.
Cars moving 40 miles/hour are 40 miles from the center.
Etc.
When did the race start?

Apply the same reasoning to the expanding universe.
In time t, a galaxy moving away from us at speed v reaches a distance $d = vt$.
Galaxies moving twice as fast should be twice as far away, as per Hubble's law.
Run the movie backwards.
Conclusion: all the galaxies were packed tightly together in the past, at a time $t = d/v$.
Applying Hubble's law we conclude that the age of the universe is

$$t_0 = d/v = d/(H_0 d) = 1/H_0.$$

(The subscript 0 implies "age of the universe *today.*")
Using 1 Mpc $= 3.1 \times 10^{19}$ km and 1 year $= 3.16 \times 10^7$ sec yields $1/H_0 = 14$ billion years (14 Gyr).
Hubble's law suggests that the expansion of the universe began 14 billion years ago: the universe has a *finite* (but large) age.
What are possible ways out of this conclusion?

The Speed Limit

For sufficiently large distances, Hubble's law gives a velocity larger than the speed of light ($v > c$).
We should suspect that something breaks down before then.
In fact there are several complications that come in, but the main one is that we will have to account for the change in the expansion of the universe over time.
We'll come back to this point later, but for now be aware that Hubble's law is most accurate for velocities in the range $5000\,\text{km/sec} < v < 60,000\,\text{km/sec}$.
It becomes less accurate at small v because of peculiar velocities and at large v because of expansion of the universe over the time the light has been traveling.