Astronomy 1143: Assignment 3

This assignment is due at the beginning of class on *Friday*, November 2.

You may consult with others in the class when you are working on the homework, but you should make a first attempt at everything on your own before talking to others, and you must write up your eventual answers independently.

You are *welcome* to come to my office hours or Dan Stevens's office hours for advice. You should get as far as you can on the assignment *before* you come to office hours, so that you know what points you are stuck on.

The office hour times and office numbers are listed on the syllabus. Wednesday afternoon office hours will start out in the classroom (right after class), and we'll stay there unless/until we need to vacate the room for another class. If you are unable to attend the scheduled office hours because of unavoidable conflicts, you can e-mail Dan or me to schedule an appointment.

Homework assignments turned in to my mailbox after class but before 5:30 pm on Friday will be accepted but marked down 10 points for lateness. Homework assignments turned in after that but before 5:30 pm on Monday will be accepted but marked down 20 points. Assignments will not be accepted after 5:30 pm on Monday.

Please staple or paper clip all sheets together. Be sure that your name is on your assignment. It's your responsibility to write clearly enough that we can grade your answers.

Make sure that you read and answer all parts of each question. For questions that require calculations, you should first work out your answers on scratch paper, but the solution you turn in should show enough of your work that we can tell how you got to your answer. For full credit, your answers should include units as well as numbers in the calculations. (For example, $v = Hr = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \times 100\,\mathrm{Mpc} = 7000\,\mathrm{km\,s^{-1}}$, not just $70 \times 100 = 7000\,\mathrm{km\,s^{-1}}$.)

Part I: Some Short Questions (35 points)

Parts (a)-(c) are worth 5 points each, parts (d) and (e) are 10 points each.

(a) My mass is about 80 kg, and on earth I weigh about 175 lbs. If you look up numbers for the mass and radius of the earth and the moon, you will find that

$$\frac{GM_{\rm moon}}{R_{\rm moon}^2} = \frac{1}{6}\frac{GM_{\rm earth}}{R_{\rm earth}^2}.$$

If I went to the moon, what would my mass be? About how much would I weigh?

(b) Consider a planet with two moons, each of the same mass. The second moon is twice as far from the planet as the first moon. How much weaker is the gravitational force on the second moon? (To be precise, what is the ratio F_2/F_1 of the force F_2 on the second moon to the force on the first moon.)

(c) I drop a bowling ball and a golf ball, from a height of 1 meter. Using the language and ideas of Newton's theory of motion and of gravity, explain why both of them fall to the ground in the same amount of time, despite having different masses. Then explain the same fact using the language and ideas of Einstein's theory of gravity.

(d) Our discussion in the course suggests two possible fates for the universe. (1) If the density of matter is high enough, gravity could halt the expansion of the universe and make it recollapse, ending in a "big crunch" at some finite (but long) time in the future. (2) If the density of matter isn't high enough for gravity to reverse the expansion, then the universe will expand forever, and galaxies will drift further and further apart. Eventually (a long, long time from now) all stars will run out of nuclear fuel and burn out, and the universe will get darker and colder.

Which of these scenarios, a dramatic crunch or an infinitely drawn out cool-down, do you find more appealing? Why?

[You don't have to pick the one that you think is correct, just the one you would prefer to be correct. Of course, the universe has no duty to be appealing to us, and our preferences have no influence on which of these cosmic fates actually lies in store for our universe.]

(e) Newton's theory of gravity was highly successful at explaining the motions of planets and other phenomena, for more than two centuries. However, improved measurements, such as the small anomaly in the orbit of Mercury discussed in class, eventually showed that Newton's theory is imperfect, and that Einstein's theory of gravity is more accurate.

Write a paragraph about an example from your own experience where you had an idea that you thought was correct for some time but eventually realized was wrong based on some new evidence. Once you realized that your original idea was wrong, did you modify it slightly, replace it with something quite different, or abandon it despite having no replacement?

Your example needn't have anything to do with science — it could have to do with yourself, or other people, or politics, or sports, or art, or whatever you choose.

Part II: Dark Matter in the Galaxy NGC 3198 (40 points)

Parts (a)-(f) are worth 5 points each; part (g) is worth 10 points.

(a) I have previously given the value of Newton's constant G in metric units, but for studying dark matter in galaxies it is convenient to use units of $\mathrm{km\,s^{-1}}$, solar masses ($M_\odot$), and kpc (1 kpc = 1 kiloparsec = 1000 parsecs.) Using the fact that the earth orbits the sun at a speed of $30\,\mathrm{km\,s^{-1}}$ in a circle of radius $r = 1\,\mathrm{AU} = 1/(206,265,000)\,\mathrm{kpc}$, show that G can be written

$$G = 4.36 \times 10^{-6} \left(\mathrm{km\,s^{-1}}\right)^2 \mathrm{kpc}\, M_\odot^{-1}.$$

Hint: start with the formula $M = v_{\mathrm{circ}}^2 r/G$ and solve for G.

(b) The plot on the next page shows the observed rotation curve of the galaxy NGC 3198: the circular velocity V is plotted against the distance R from the center of the galaxy.

At $R = 4\,\mathrm{kpc}$, what is the circular speed v_{circ}?

What is the mass $M_{\mathrm{int}}(R)$ interior to $R = 4\,\mathrm{kpc}$, in $M_\odot$?

Hint: Use the form of G given above in part (a).

(c) Half of the light of NGC 3198 is inside a radius $R = 4\,\mathrm{kpc}$.

Suppose that, after measuring the distance to the galaxy with Cepheids and measuring its apparent flux, you determine that its total luminosity is $3.6 \times 10^{10} L_\odot$ (36 billion solar luminosities). The light within $R = 4\,\mathrm{kpc}$ is half that, $1.8 \times 10^{10} L_\odot$.

Based on your result from (b), could the mass in the inner 4 kpc of NGC 3198 come entirely (or almost entirely) from stars like the sun? Why or why not?

[I tried to include an image of NGC 3198, but by the time I had copied and rescaled it and converted file formats it looked lousy. If you google NGC 3198 you will find lots of images of the galaxy. The full extent of visible region in such images is about 24 kpc in *diameter*, or 12 kpc in radius.]

(d) A radius $R = 8\,\mathrm{kpc}$ encloses nearly all of the light from the galaxy ($3.6 \times 10^{10} L_\odot$).

What is the circular speed at $R = 8\,\mathrm{kpc}$?

What is the mass $M_{\mathrm{int}}(R)$ interior to $R = 8\,\mathrm{kpc}$?

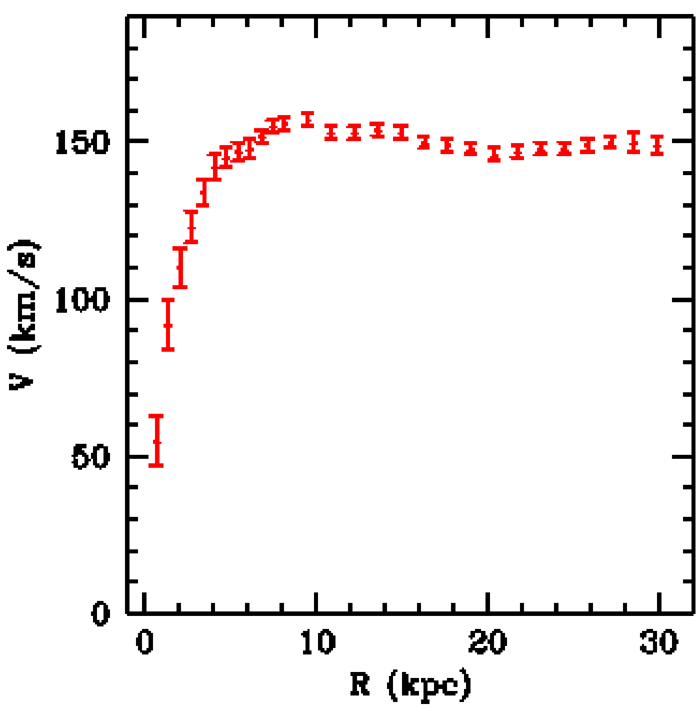
Could the mass in the inner 8 kpc of NGC 3198 come entirely (or almost entirely) from stars like the sun?

(e) Based on the rotation curve, what is the mass interior to radius $R = 20\,\mathrm{kpc}$? If all of the stars in NGC 3198 are similar to the sun, what fraction of the mass inside 20 kpc could they account for?

(f) Based on the rotation curve, what is the mass interior to radius $R = 30\,\mathrm{kpc}$? If all of the stars in NGC 3198 are similar to the sun, what fraction of the mass inside 30 kpc could they account for?

(g) Another astronomer discovers a new kind of star, which he calls a "D star," that has the same luminosity as the sun but is four times more massive ($L = L_\odot$, $M = 4M_\odot$). He proposes that NGC 3198 does not contain any mysterious "dark matter" but is simply made of D stars instead of sun-like stars.

Can this idea successfully explain your measurements? Why or why not?



Part III: Is the Universe Bound or Unbound? (25 points)

Each part (a)-(e) is worth 5 points.

When talking about distances within galaxies, it is convenient to use kpc, since the visible regions of galaxies are typically 10-50 kpc across, and their dark matter halos extend to distances of 100-300 kpc. When talking about larger scales in the universe, it is usually convenient to use Mpc (1 Mpc = 1 million parsecs = 1000 kpc), since the typical separations between galaxies are a few Mpc. For this Part, therefore, you will want to express G as

$$G = 4.36 \times 10^{-9} \left(\mathrm{km\,s^{-1}}\right)^2 \,\mathrm{Mpc}\, M_\odot^{-1}.$$

In Part II, we used the formula $M_{\rm int}(r) = v_{\rm circ}^2 r/G$. For this Part, we want to know whether galaxies at the edge of a spherical volume are moving faster or slower than the escape speed $v_{\rm esc}$. I told you in class that the escape speed is just $v_{\rm esc} = \sqrt{2} \times v_{\rm circ}$. Therefore, the corresponding expression for mass, which you will need in (b) below, is just

$$M_{\rm int}(r) = \frac{v_{\rm esc}^2 r}{2G}.$$

(a) Consider a spherical region of the universe with radius $r = 10\,\mathrm{Mpc}$, centered on the Milky Way. Using $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, compute the recession velocity v of galaxies at the edge of this spherical region (i.e., at a distance of $10\,\mathrm{Mpc}$). Express your answer in $\mathrm{km\,s^{-1}}$.

(b) If the average density of the universe is equal to the critical density, then these galaxies should be moving at exactly the escape speed. Using the mass formula above and your velocity from part (a), show that the mass inside the sphere is $M_{\rm int}(r) = 5.62 \times 10^{14} M_\odot$.

(c) Remember that the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$. Using your results from (a) and (b), argue that the critical density of a universe with $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ is $\rho_{\rm crit} = 1.34 \times 10^{11} M_\odot\,\mathrm{Mpc^{-3}}$. (Your answer should have the numbers that lead to this value and should briefly recap the reasoning.)

(d) Galaxies like the Milky Way are estimated to have a *total* mass, including the dark matter halo, of about $10^{12} M_\odot$ (a trillion solar masses). The average density of Milky Way-like galaxies in the universe is about $0.02\,\mathrm{galaxies/Mpc^3}$, i.e., two galaxies per hundred cubic megaparsecs. We should also account for the mass attached to less luminous but more numerous galaxies, which in total is about equal to the mass in Milky Way-like galaxies. Using these numbers, argue that the average density of matter in the universe is approximately $\bar{\rho} = 4 \times 10^{10} M_\odot\,\mathrm{Mpc^{-3}}$.

(e) What is the ratio $\bar{\rho}/\rho_{\rm crit}$ of the average density that you computed in part (d) to the critical density you computed in part (c)?

Based on this result, should the universe continue to expand forever, or will it eventually stop expanding and recollapse? Explain your answer.