Astronomy 1143: Assignment 4

This assignment is due at the beginning of class on *Friday*, November 16.

You may consult with others in the class when you are working on the homework, but you should make a first attempt at everything on your own before talking to others, and you must write up your eventual answers independently.

You are *welcome* to come to my office hours or Dan Stevens's office hours for advice. You should get as far as you can on the assignment *before* you come to office hours, so that you know what points you are stuck on.

The office hour times and office numbers are listed on the syllabus. Wednesday afternoon office hours will start out in the classroom (right after class), and we'll stay there unless/until we need to vacate the room for another class. If you are unable to attend the scheduled office hours because of unavoidable conflicts, you can e-mail Dan or me to schedule an appointment.

Homework assignments turned in to my mailbox after class but before 5:30 pm on Friday will be accepted but marked down 10 points for lateness. Homework assignments turned in after that but before 5:30 pm on Monday will be accepted but marked down 20 points. Assignments will not be accepted after 5:30 pm on Monday.

Please staple or paper clip all sheets together. Be sure that your name is on your assignment. It's your responsibility to write clearly enough that we can grade your answers.

Part I: Some Short Questions (35 points)

Parts (a)-(d) are worth 5 points each, part (e) is worth 10 points.

(a) If I took a lump of coal at room temperature (300° K) out into interstellar space, it would cool off by emitting photons that carried away its heat energy. Initially those photons would have a characteristic wavelength of about 0.01 mm.

- As the lump of coal cools, what is changing about the molecules that make it up?
- As the lump of coal cools, does the characteristic wavelength of the photons that it emits get longer or shorter?

(b) Suppose that you found a star that resembled the sun but whose chemical composition was 90% hydrogen, and 10% helium (by mass). Assuming that your measurements of the composition were convincing, why would the existence of such a star pose a challenge to the big bang theory?

(c) Deuterium (heavy hydrogen, with one proton and one neutron in the nucleus) can be destroyed in stars because it gets fused into helium. However, based on everything we know about stars, it is impossible for them to produce additional deuterium because as soon as they make it they convert it into helium. Despite this problem, there is roughly one deuterium atom in the universe for every 100,000 "normal" hydrogen atoms.

Why does the existence of deuterium provide evidence for the hot early universe predicted by the big bang theory?

(d) When the universe was 350,000 years old, its temperature was higher than 3000° K, and it was opaque to visible light. When the universe was 450,000 years old, its temperature was lower than 3000° K, and it was transparent to visible light. How did the universe go from opaque to transparent in such a (relatively) short time?

(e) Two of the remarkable things that we have learned in section 10 of the course are

- Most of the atoms in your body — i.e., all of them except hydrogen and helium — were once inside a star.
- If you go outside, you are bathed every second by trillions of cosmic microwave background photons that are the faded glow of the hot early universe.

Write a one-paragraph essay giving your reaction to these surprising facts.

Part II: CMB Spots and the Geometry of Space (25 points)

Each part is worth 5 points.

Background: In addition to the homework assignment, you should have a separate handout with two figures (it is also available on the course web page). The upper one illustrates the general idea of this part of the assignment: we are going to use the size of hot spots on the cosmic microwave background (CMB) to figure out whether the universe is closed (positively curved, spherical geometry), flat, or open (negatively curved, potato chip geometry). The lower one is an actual map of the CMB over a portion of the sky, made in 1999 by the Boomerang balloon experiment. Color indicates temperature, with red being hot and blue being cold.

Although the colored blobs on this map have a variety of shapes and sizes, there is a characteristic scale that you can pick out by eye. This characteristic scale is imprinted by pressure waves that travel in the early universe before recombination, when the universe is still opaque and photons are able to push matter around. These pressure waves travel at close to the speed of light, so they can go a distance of about 400,000 light years before recombination occurs and the universe becomes transparent. The universe has expanded by a factor of 1000 since recombination, stretching that 400,000 light years to 400 million light years. A more accurate calculation gives a size of about 220 million light years for the typical diameter of spots on the CMB, where the difference acounts for the speed of pressure waves being somewhat slower than the speed of light and the universe expanding while the pressure waves are moving. However, we don't directly see the physical size of the spots, we see their *angular* size.

Caveat: I am fudging some of the numbers in this problem to make it mathematically simple. The basic ideas are correct, but a proper calculation is more difficult than the one you are doing here.

(a) Take the distance to the CMB to be $d = 14$ billion light years (since CMB photons have been traveling almost the entire history of the universe to get to us). If space is flat, obeying the laws of Euclidean geometry, then we can use the formula $\theta = l/d$ radians to compute the predicted angular size of CMB spots.

Using $l = 220$ million light years, what is the predicted angular size θ? Convert it from radians to degrees, remembering that one radian is 57.3 degrees.

(b) Choose six strong spots from the central area of the map (enclosed by the curved line) and measure their size in mm. Specifically, you should measure the size of the yellow region, going across the shortest dimension if the spot is elongated. Avoid the three small, circled spots, which are radio galaxies rather than CMB spots.

Estimate the size of each spot to the nearest 0.1 mm, and list each of your six measured values. What is the average size of the spots in mm?

(If you don't have a metric ruler, you can do this part and the next one in inches, but it will be harder to do it accurately.)

(c) Using the vertical axis on the left side of the plot, measure how many mm corresponds to 10 degrees. Based on this measurement and your result from (b), what is the average angular size of CMB spots in degrees?

(d) As you might imagine, this measurement can be done more precisely using a more sophisticated technique, and it yields a characteristic size of 0.9 degrees. For a flat universe, the predicted size is what you got in part (a) above. However, in Homework Assignment 3, Part III, you found that the average density of matter in the universe, $\bar{\rho}_{\rm matter}$, is only about 1/4 of the critical density required to make space flat.

For a negatively curved universe with $\bar{\rho}/\rho_{\rm crit} = 0.25$, the predicted angular size of CMB spots is smaller (see the top illustration), about 0.4 degrees. Is the observed characteristic size of 0.9 degrees closer to the size predicted for a flat universe or for a negatively curved universe with $\bar{\rho}/\rho_{\rm crit} = 0.25$?

(e) Recall (see course notes section 8, the last section) that the curvature of space actually depends on the *sum* of the density of matter and the density of exotic energy. From part (d), what can you conclude about exotic energy?

Part III: Supernovae and Cosmic Acceleration (35 points)

Each part is worth 5 points.

For this part I had originally hoped to have you use data from the Nobel-prize winning papers by Adam Riess and collaborators and Saul Perlmutter and collaborators. However, I concluded that it was going to be too complicated to guide you through some of the details of how to use those measurements, so here I have made up some artificial data, and I am putting things in a simplified form that is designed to capture the main point.

(a) The attached sheet has a graph of velocity (in $\rm km\,s^{-1}$) vs. distance (in Mpc). On this graph, draw a straight line that shows the relation $v = H_0 d$, with $H_0 = 70\,\rm km\,s^{-1}\,Mpc^{-1}$.

(b) Here is a table of distances and velocities for six relatively nearby supernovae. The velocities were measured from Doppler shifts, and the distances were estimated by the standard candle method $d = \sqrt{L/4\pi f}$. Plot them as points on the graph. The distance estimates are not perfect because supernovae are not exact standard candles, so the points won't lie exactly on a line.

Supernova	distance	velocity
A	90 Mpc	$6600\,\rm km\,s^{-1}$
B	140 Mpc	$12000\,\rm km\,s^{-1}$
C	150 Mpc	$11000\,\rm km\,s^{-1}$
D	180 Mpc	$13000\,\rm km\,s^{-1}$
E	200 Mpc	$14500\,\rm km\,s^{-1}$
F	260 Mpc	$19500\,\rm km\,s^{-1}$

(c) Here is a table of distances and velocities for eight much more distance supernovae. Plot them as points on the graph.

Supernova	distance	velocity
G	1070 Mpc	$68,000\,\rm km\,s^{-1}$
H	1300 Mpc	$77,000\,\rm km\,s^{-1}$
I	1460 Mpc	$81,000\,\rm km\,s^{-1}$
J	1420 Mpc	$84,000\,\rm km\,s^{-1}$
K	1430 Mpc	$90,000\,\rm km\,s^{-1}$
L	1610 Mpc	$96,000\,\rm km\,s^{-1}$
M	1640 Mpc	$105,000\,\rm km\,s^{-1}$
N	1760 Mpc	$105,000\,\rm km\,s^{-1}$

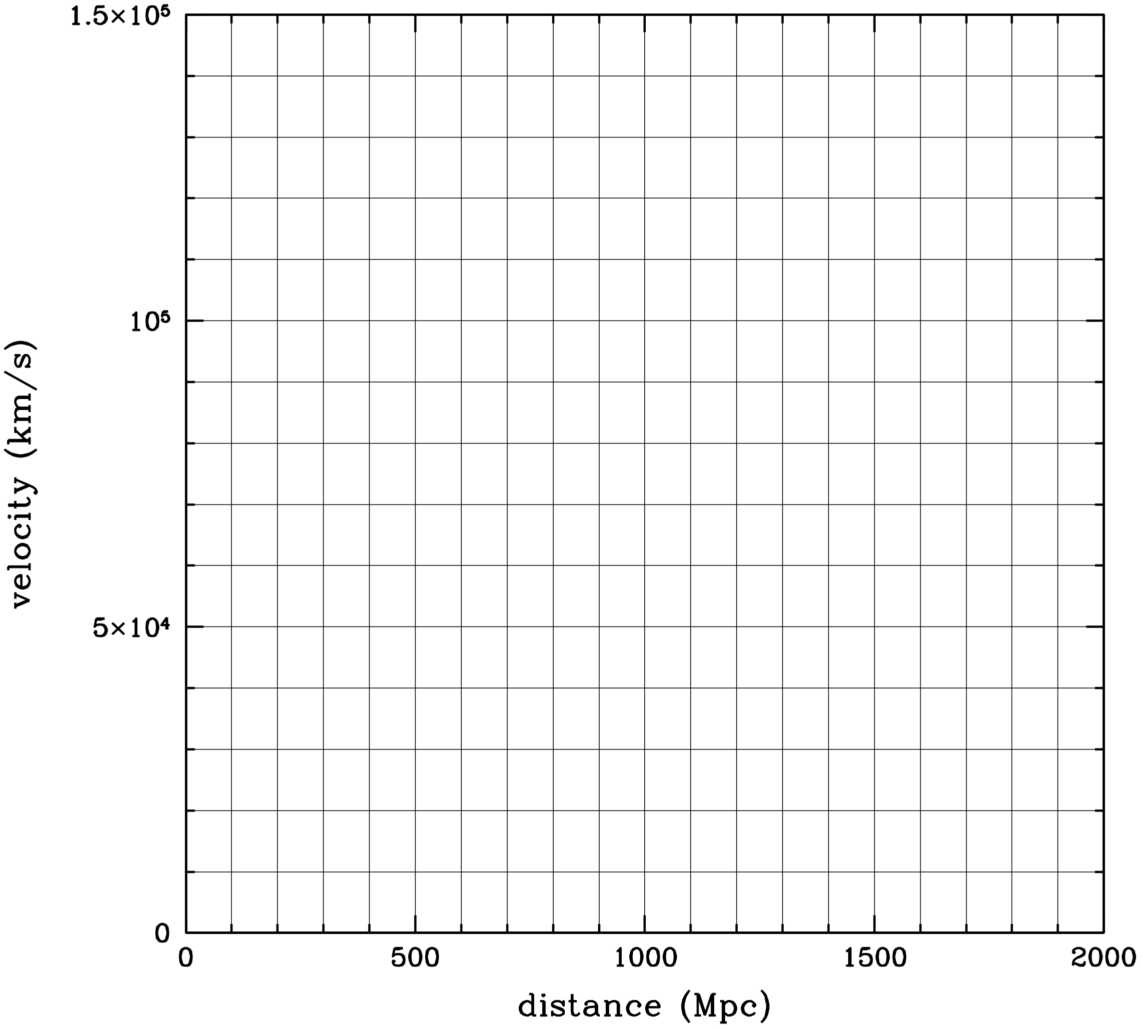
(d) Remember that 1 Mpc is 3.26 million light years. For a supernova that is 1000 Mpc away, how long has it taken its light to get here? More generally, light from the distant supernovae has taken longer to get here than light from nearby supernovae. Based on your plot, was the universe expanding *slower* in the past than today, or was it expanding *faster* in the past? Explain your answer.

(e) How could the existence of exotic energy, as discussed in class, help to explain this observation?

(f) Suppose that some of the light from the distant supernovae is being absorbed along its way to us by intergalactic dust. Would the true distances to these supernovae be larger or smaller than the estimates from the standard candle method, which were used for this plot? How might this affect your conclusion about exotic energy?

Hint: Think about what gets affected by absorption in $d = \sqrt{L/4\pi f}$.

(g) Why does the combination of this result with your result from Part II provide stronger evidence for exotic energy than the result from the supernovae alone?



Graph paper for Part III.