10. Supermassive Black Holes
Reading: Thorne, Chapter 9

Luminosity of an Accreting Black Hole
The luminosity of an accreting black hole is proportional to the rate at which it is gaining mass.
A non-spinning black hole accreting gas at a rate $\dot{M}$ through a thin disk has a luminosity

$$L \approx \frac{1}{12} \dot{M} c^2.$$  

Note that $\dot{M}$ is a rate of change of mass, with units of, for example, $M_\odot \text{yr}^{-1}$ (or kg s$^{-1}$).
$Mc^2$ is energy, so $\dot{M}c^2$ is energy/time, the units of luminosity (or power).
The factor $1/12$ comes from calculating the amount of gravitational potential energy gained by
the gas as it moves from a large distance to the orbit at $3R_{\text{Sch}}$, after which it spirals in without
radiating more energy.
For example, a non-spinning black hole accreting at $\dot{M} = 10^{-7} M_\odot \text{yr}^{-1}$ through a thin disk has a
luminosity

$$L \approx \frac{1}{12} \dot{M} c^2 = 1.2 \times 10^5 L_\odot,$$

where $L_\odot$ is the luminosity of the sun.

For a spinning black hole, the gas can get closer before it spirals in, so it can gain more energy and
the factor in this equation would be bigger than $1/12$.
However, if there is too little “friction” in the flow of accreting gas, then it will just heat up instead
of radiating the energy that it gains, and it may eventually carry that heat energy with it into the
black hole.
In this case we would get a “puffy” disk with lower luminosity.

The Eddington Luminosity Limit
The mass of the black hole itself doesn’t enter our luminosity formula, just the growth rate.
But photons radiating from the inner accretion zone carry momentum and exert pressure. The
pressure pushes back on the accreting gas.
There is a maximum luminosity above which outward pressure exceeds the inward pull of gravity,
in which case the black hole cannot accrete.
This is called the Eddington luminosity.
A black hole that is twice as massive exerts twice as much gravitational force, so its luminosity
limit is twice as high:

$$L_{\text{Edd}} = 3 \times 10^4 L_\odot \left( \frac{M}{M_\odot} \right),$$

where $M$ is the black hole mass.

Basic Cosmology
In 1921, Edwin Hubble demonstrated that the “spiral nebulae” seen in telescopes are in fact distant
galaxies, like the Milky Way.
Galaxies range widely in size, but a “typical” galaxy is roughly 100,000 light years across and
contains 10-100 billion stars.
Over the next decade, Hubble’s colleague Milton Humason showed that the spectral lines of most
galaxies are shifted to the red, which suggests that they are moving away.
Hubble (1929): A galaxy’s redshift is proportional to its distance (twice as far away $\implies$ receding twice as fast). The universe is expanding.

This discovery, combined with theoretical calculations based on General Relativity, led to

The Big Bang Theory: The universe has expanded from a very hot, very dense state, which existed at some finite time in the past (about 14 billion years ago).

Key points for our purposes:

- Stars are point sources of light, and their spectra show only small redshifts or blueshifts (up to a few hundred km s$^{-1}$).
- Galaxies are extended sources of light, and their spectra are (nearly) always redshifted, often by thousands or tens of thousands of km s$^{-1}$.
- The higher the redshift the more distant the galaxy.

The Discovery of Quasars

Early radio telescopes detected many sources over the sky.

Initially hard to identify with optical sources because of poor resolution of radio telescopes.

Resolution gradually improved by using linked networks of radio telescopes.

Many sources appeared to coincide with galaxies.

Better resolution showed emission often coming from “lobes” on either side of galaxy, hundreds of thousands of light years from the stars.

A few sources appeared to coincide with stars.

Weird, “radio stars” not previously known.

Astronomers obtained visible light spectra for some of these sources.

Spectra showed lines at completely unknown wavelengths.

Maarten Schmidt (1962): Lines in the source 3C 273 are from hydrogen, but redshifted by 48,000 km s$^{-1}$, 16% of the speed of light!

Lines in the source 3C 48 are from other kinds of atoms, redshifted by 37% of the speed of light.

Implication: These “points of light” are enormously distant and therefore enormously luminous, 100-1000 times more luminous than an entire galaxy of stars.

Named “quasars” for quasi-stellar radio sources, though we now also know many similar objects with little or no radio emission.

What Powers Quasars?

Luminosities up to $5 \times 10^{13}L_\odot$ (or even more). (Total luminosity of all stars in Milky Way is $\sim 3 \times 10^{10}L_\odot$; brightest quasars are more than 1000 times more luminous.)

Vary substantially in brightness on timescales of days $\implies$ emission region must be at most a few light days across.

Some quasars associated with radio lobes and radio jets.

Total energy in lobes of $10^{54}$ joules, about $10^7M_\odot c^2$.

Argument by Donald Lynden-Bell (1969):

Suppose the energy in radio lobes came from nuclear fusion, which has a maximum efficiency of 0.01$m c^2$.

Implies total mass exceeding $10^9M_\odot$ in region $\sim 1$ light day $\sim 3 \times 10^{10}$ km across.

This is only $\sim 10$ times $R_{Sch}$ for $10^9M_\odot$. 

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Gravitational energy is \( E \approx \frac{GM^2}{R} \approx 0.05Mc^2 \), greater than the nuclear energy by a factor of five (or probably more, since stars do not fuse 100% of their hydrogen into heavy elements).

Implications:

- Energy must have come from gravity, not from fusion.
- Likely energy source is an accreting supermassive black hole of mass \( 10^8 - 10^9 M_\odot \).

**Quasars**

Now over 100,000 quasars known.

Brightest have luminosities equal to Eddington limit for \( 10^9 M_\odot \), or even a little higher.

Bright in X-rays, UV light, visible light, infrared light; some bright in radio waves and \( \gamma \)-rays.

Properties are what we expect for accretion disks around supermassive black holes.

For example (and especially significant): X-ray iron lines, which arise in hot gas fairly close to the event horizon, show large Doppler shifts and gravitational redshifts.

Some (maybe most) supermassive BHs appear to be rapidly spinning.

Jets can stay straight for millions of years because of the “gyroscope” provided by the spinning BH.

Finding quasars:

- With visible light images alone, look for “stars” (points of light) with very weird colors.
- Or look for visible-light “stars” that have counterparts in X-rays or radio.
- To confirm that an object is a quasar, and measure its distance (from redshift), have to measure its spectrum.

Most distant known quasars have light redshifted by a factor of 7 \( \Rightarrow \) light emitted when universe was 1 billion years old (compared to 14 billion years today).

Bright quasars were most common 10 billion years ago, much less common today.

The black holes themselves don’t go away, so it must be that they are “fed” less often at later times.

**Host Galaxies and Active Galactic Nuclei**

Sharp imaging from *Hubble Space Telescope*, some ground-based telescopes \( \Rightarrow \) quasars live at the centers of the galaxies.

Sharp images needed because quasar can be much brighter than host galaxy itself.

Hosts often appear messed up, suggesting mergers of galaxies may trigger galactic scale gas flows that feed quasars.

Brighter quasars generally reside in bigger host galaxies.

About 1-2% of nearby galaxies have bright central nuclei with spectra that look similar to quasars.

These “active galactic nuclei” (AGN) appear to be scaled down versions of quasars, less luminous because black hole is less massive or because a massive black hole is being fed slowly.

Higher percentage of galaxies have faint AGN.

**Masses of Active Black Holes from Reverberation Mapping**

Reverberation mapping (Homework 4, Part III) allows mass measurement for active black holes.

The accretion disk near the black hole varies in brightness because of gas flow variations.

The surrounding gas responds after some delay because of the light travel time.

Use time delay to get distance, from \( r = ct \).

We can get the typical gas velocity from the width of the spectral lines, caused by Doppler shifts of the moving atoms.
From the Doppler formula, we can relate the spread of a spectral line in wavelength $\Delta \lambda$ to the spread of velocities $\Delta v$ of the emitting gas: $\Delta \lambda / \lambda = \Delta v / c$.
Then we can apply the formula we found back in §3,

$$M \approx \frac{v^2 r}{G},$$

to infer the mass of the black hole. (The equation is exact for circular orbits.)

**Dormant Black Holes**

Stars and gas in galaxy move under influence of gravity from stars, gas, and extended halo of dark matter.
Not much mass in middle $\implies$ stars should move slower.
But at very center, observations often show stars and gas moving faster. Implies central dark mass, probably a black hole.
Roughly speaking, measure velocities, distance, apply

$$M \approx \frac{v^2 r}{G}.$$

The equation is approximate because stars are not actually on circular orbits.
Case became much more convincing with HST because sharp imaging made it possible to measure much closer to galaxy centers.
Can sometimes detect and measure rotating gas disks, for which the mass measurements are easier to interpret because we know how the gas is orbiting.

Current evidence suggests that all galaxies with central bulges of stars also have central supermassive black holes.
The BH mass and bulge mass appear to be tightly correlated, i.e., galaxies with more massive bulges have more massive central BHs.
Specifically, the mass of the black hole is typically about 1/1000 of the mass of stars in the bulge.
Reverberation mapping shows a similar trend with bulge mass for active black holes.

**The Milky Way’s Black Hole**

There is a peculiar radio source at the exact center of the Milky Way galaxy.
It also emits X-rays, well measured by Chandra satellite.
High-resolution measurements of motions of stars near Galactic Center, made in infrared light using clever techniques on large ground-based telescopes, allow mass measurement.
Mass is $4 \times 10^6 M_\odot$.
Total luminosity is not much larger than $L_\odot \implies$ very far below Eddington limit.
Must be getting very thin dribble of gas (but not zero).

**Black Holes and Galaxy Formation**

Quasars live in galaxies, appear to be associated with “big growth events” in galaxy life.
Omnipresent central black holes in local galaxies $\implies$ most galaxies had a “quasar phase.”
Quasars are much rarer than galaxies $\implies$ quasar phase only lasts a small fraction of galaxy’s life.
Close correlation between black hole mass and bulge stellar mass $\implies$ one affects the other, but we don’t know which way.
Option 1: Gravity provided by stars, or mass lost from stars, determines how big the BH can grow.
Option 2: When BH gets big enough, it drives gas out of galaxy, truncating further star formation. Influence of black holes on galaxies and *vice versa* is a big open issue in the theory of galaxy formation and quasar evolution.

**Summary of Empirical Evidence for Supermassive BHs**
- High luminosity, rapid variability of quasars $\implies$ enormous energy produced in a small volume. Accretion onto BH is only known mechanism to achieve this.
- Spectrum of radiation has properties expected for BH accretion.
- X-ray iron lines show large Doppler shifts and gravitational redshifts.
- Jets powering radio galaxies could come from BHs. No other ideas how to create such energetic jets or keep them so straight.
- Motions of stars at Galactic Center show large dark mass in small volume.
- Motions of stars and gas near centers of other galaxies show large dark mass in central regions.

**Imaging Black Holes?**
The angular diameter of the event horizon of a BH at distance $d$ is $2R_{Sch}/d$ (in radians).
The maximum angular resolution that can be achieved by a telescope of diameter $D$ observing radiation at wavelength $\lambda$ is $\lambda/D$ radians.
If radiation from multiple telescopes is combined precisely, $D$ can be the separation of the telescopes rather than individual diameters. (Hard with visible light, easier with radio.)
Nearly all BHs have an angular size far too small to be resolved by any existing or future telescope, BUT
The Galactic Center BH is marginally resolvable with mm-wave radio telescopes spread across the globe ($\lambda = 1$ mm, $D = 2R_{earth} = 12,000$ km).
The BH at the center of M87 is 500 times more massive and 1000 times further away, so also marginally resolvable.
Over the next decade, mm-wave observations might produce the first images of BH “silhouettes.”