2. Newton's Laws of Motion

This material is not well covered in Thorne's book, but it is covered in just about any standard introductory astronomy textbook. For example, in the current A161/162 text (Astronomy Today, by Chaisson & McMillan, 7th edition), it is in §§2.5-2.8, especially 2.7 and 2.8.

Isaac Newton (1642-1727)

Brief biography:

- Premature baby, raised by grandmother.
- As an undergraduate at Cambridge, invented calculus!
- Became Cambridge professor at age of 26.
- Later jobs included Member of Parliament, Warden of the Mint.
- Lived at time of great political upheaval and scientific progress.
- The leader among many great contemporary scientists and mathematicians, in England and on the Continent.
- Embroiled in numerous disputes over priority of discoveries, most notably with Leibniz over invention of calculus.

Contributions and publications:

- Created mechanics (physics of motion) in its modern form.
- Discovered law of gravity and many of its applications.
- Major contributions to theory of light and optics.
- Invented reflecting telescope, still the preferred type today.
- Outstanding experimenter. His optics experiments essentially established modern practice of experimental physics.
- Two major publications:
- 1687: *Philosophiae Naturalis Principia Mathematica* (Mathematical Principles of Natural Philosophy), usually known just as *Principia*
- 1704: *Opticks*
- Other letters and shorter publications.
- Enthusiastic alchemist, numerous experiments and unpublished writings, most of them now considered completely bogus.
- Wrote enormous treatises on theology, the bible, early Christian history. Extreme Anti-Trinitarian. Some writings published after his death.

Newton's achievements built on a wealth of experimental and theoretical progress by others over the preceding two centuries, especially Copernicus, Galileo, and Kepler.
Theory of gravity and theory of motion (a.k.a. "Newton's laws") developed simultaneously, drawing on astronomical observations and terrestrial experiments.
For simplicity of explanation, we will treat motion first, then turn to gravity.

Velocity

Velocity is the rate of change of position.
For an object moving in a straight line at fixed speed: $v = d/t$
Equivalently, $d = vt$.
But velocity has a magnitude *and* a direction. In mathematical terms, represented by a vector (roughly speaking, an arrow).
The magnitude of the velocity (length of the vector) is called the speed.
Objects moving with the same speed in different directions have different velocities.

Acceleration

Acceleration is the rate of change of velocity.

It is also a vector, with a magnitude and a direction.

If you start from rest, the direction of the acceleration tells which direction you will go, and the magnitude tells how rapidly you will reach a certain speed.

For motion in a straight line, starting from rest, with constant acceleration for time t,

velocity at beginning $= 0$

velocity at end $= at$

average velocity $= \frac{1}{2}at$

distance traveled $=$ (avg. velocity) $\times t = \frac{1}{2}at \times t = \frac{1}{2}at^2$.

Units

In physics, we often quote distances in meters (m) and time in seconds (sec, or sometimes just s).

Since velocity = distance/time, the corresponding units of velocity are meters-per-second (m/sec).

Since acceleration = (change of velocity)/time, the corresponding units of acceleration are meters-per-second-per-second, usually written (m/sec^2).

For example, the acceleration of falling objects near the surface of the Earth is 9.8 m/sec^2.

If an object starts at rest and falls for one second, its velocity is $v = at = (9.8\,\mathrm{m/sec}^2) \times (1\mathrm{s}) = 9.8\,\mathrm{m/sec}$.

The distance it travels is $d = \frac{1}{2}at^2 = \frac{1}{2} \times (9.8\,\mathrm{m/sec}^2) \times (1\mathrm{s})^2 = 4.9\,\mathrm{m}$.

In astronomy, it is sometimes convenient to use other units for distances, velocities, and accelerations.

Newton's Laws of Motion

In Newton's *Principia*, he summarizes his theory of mechanics with three "laws":

First Law: A body remains at rest, or moves in a straight line at a constant speed, unless acted upon by a net outside force.

Second Law: Force on a body causes it to accelerate in the direction of the force. The acceleration is proportional to the strength of the force and inversely proportional to the mass of the body.

Third Law: Whenever one body exerts a force on a second body, the second body exerts an equal and opposite force on the first body.

The first law is the principle of inertia, a (much) more clearly stated version of the discovery of Galileo and his predecessors.

The third law is sometimes phrased: "For every action, there is an equal and opposite reaction."

Note that Newton's "laws" really constitute a *theory* of motion and forces.

This theory is highly successful in explaining a wide range of phenomena.

F=ma

Newton's second law is best summarized by a fundamental equation:

$$F = ma$$

F = force
m = mass
a = acceleration
Note that this is equivalent to $a = F/m$, which is more obviously a translation of the words to an equation.
F and **a** are really vectors, with a magnitude and a direction.
m is just a number, with no direction.

Mass, Weight, and Force

The *mass* of a body is, effectively, its "amount of stuff." It can be measured, from Newton's second law, by measuring the body's resistance to acceleration. The standard, metric system unit of mass is the kilogram (kg).
The *weight* of a body is the force that gravity exerts on it. For objects on the Earth, the weight is proportional to the mass. The same object on, say, the Moon, would have the same mass, but it would weigh less (1/6 as much).

The standard, metric system unit of force is the newton:

1 newton = 1 kg-m/s^2 = the force required to accelerate 1 kg at 1 m/s^2

On earth, a 1 kg mass *weighs* 9.8 newtons.
In English units, the pound is actually a unit of force (weight) rather than mass. The English unit of mass is the slug.

Newton's Laws in Action

First law, inertia:
A bowling ball is rolling towards me. Why do I have to push to stop it?
A tricky question: Why does the ball eventually stop even if I don't push it?

Second law, $F = ma$.
Which is harder to get moving, a billiard ball or a bowling ball?
Do I push harder to get something moving slowly, or to get it moving fast?

Third law, action and reaction:
I step off of a rowboat onto the shore. What happens to the boat?
Standing on a skateboard, I fire a bullet from a high powered rifle. What happens to me?

Second and third law combined:
A 500-pound man and I collide on our skateboards. Who bounces back further?
Back to the gun: Why don't I fly backwards as fast as the bullet flies forward?

Circular Motion

Going in a circle at constant speed requires acceleration.
Q: Why?
A: Because the *direction* of the velocity is changing, even though the magnitude (speed) is not.

Consider an object moving at speed v in a circle of radius r.
Q: As it goes halfway around the circle, how much does its velocity change?
A: From $+v$ (forward) to $-v$ (backward): change is $2v$.
Q: How long does it take to go halfway around?
A: distance = $\pi \times r$. Therefore, time = distance/velocity = $\pi r/v$.
The *average* acceleration over this time is
(change in velocity) / time = $\frac{2v}{\pi r/v} = \frac{2}{\pi} \times \frac{v^2}{r}$.

The *instantaneous* acceleration in circular motion is $a = v^2/r$.
Q: What is the direction of this acceleration?
A: Toward the center of the circle.

Circular Orbits

Q: In what direction do you need to apply force to keep an object moving in a circle at constant speed?
A: Toward the center of the circle.
Analogy: whirling an object on a string.
Q: What happens if there is no force?
A: The object continues on in a straight line.
Analogy: let go of the string!

Conclusion: Planets can orbit in circles around the Sun if the Sun exerts a force that pulls them toward it.
Planets are always falling toward the Sun, but because they are moving, they "miss" and continue in a circle.
Explaining the motions of planets requires a force that attracts them to the Sun.
Similarly, the Moon is always falling towards the Earth.

More precisely, a planet moves in an ellipse, and its speed increases as it gets closer to the Sun.
The principles are the same as for circles, but with an ellipse, the attractive force can change the speed of the planet along its path as well as changing the direction of the path.
The Moon also moves in an ellipse around the Earth.