4. Transition: Conservation Laws, Electromagnetic Radiation, and Reference Frames

Momentum and momentum conservation

Something with a lot of momentum is hard to stop.

In physics, the momentum of an object of mass m moving at a speed $v \ll c$ is $p = mv$.

The direction of the momentum (which is a vector) is the same as the direction of the velocity.

Since acceleration is the rate of change of velocity, $F = ma$ implies that the force gives the rate of change of momentum.

Newton's 3rd law, equal and opposite forces, therefore implies that the *total* momentum of a system doesn't change if there are no *external* forces. Momentum is conserved.

E.g., collision of skateboarders.

Energy and energy conservation

Roughly speaking, energy is ability to "do something."

Physics allows precise definition.

Energy comes in different forms, e.g., kinetic (energy of motion), thermal (energy of hot stuff), electrical.

Also *potential* energy, available to be tapped, e.g., gravitational or chemical.

The kinetic energy of an object of mass m moving at velocity v is

$$\mathrm{KE} = \frac{1}{2}mv^2.$$

Energy can be transformed, but total energy is always conserved.

Examples:

- Electric heater converts electrical energy to thermal energy
- Hydroelectric dam converts gravitational potential energy to electrical energy.
- Photosynthesizing plant converts energy of light from Sun to chemical potential energy.
- Coal burning plant converts chemical potential energy to electrical energy.

Electromagnetic Radiation

In the 1860s, James Maxwell postulated the existence of electromagnetic (EM) radiation: waves of electric and magnetic fields that propagate through space.

Existence first clearly demonstrated in the 1880s by Heinrich Hertz, who in effect built the first radio.

In empty space, all EM radiation propagates at the speed of light, $c = 300,000$ km/s.

Different forms of EM radiation are distinguished by their wavelength, or by their frequency.

Visible light is a form of electromagnetic radiation with wavelength of about 500 nano-meters (nm, 1 nm = 10^{-9} meters).

The wavelength of light determines its color.

From long wavelength to short wavelength:

- Radio and microwave (greater than 10^6 nm = 1 mm)
- Infrared ($\sim 10^6$ nm $- 10^3$ nm)
- Visible: R O Y G B I V ($\sim 700\,\mathrm{nm} - 400\,\mathrm{nm}$)
- Ultraviolet ($\sim 100\,\mathrm{nm} - 10\,\mathrm{nm}$)
- X-rays ($\sim 10\,\mathrm{nm} - 0.01\,\mathrm{nm}$)
- Gamma rays (less than $\sim 0.01\,\mathrm{nm}$)

EM radiation carries energy, e.g., sunlight heats the Earth, microwaves can cook food.

EM radiation also carries momentum.

E.g., if light bounces off a mirror, the mirror recoils.

With Maxwell's equations (1850s), which describe electromagnetism and electromagnetic radiation, can show that an amount of EM radiation with energy E has momentum $p = E/c$.

Photons

EM radiation has many wavelike properties — refraction, interference.

In early 20th century, Planck and Einstein showed that it travels in discrete packets of energy, now known as *photons*.

Photons can be thought of as "particles of light."

It's counter-intuitive for something to be both particle and wave, but that's the way nature is.

The energy of a photon is

$$E = \frac{hc}{\lambda}$$

E = energy of photon

λ = wavelength of photon

c = speed of light

h = Planck's constant (a universal number)

An individual short wavelength photon has *more energy* than an individual long wavelength photon.

Radio photon: not much energy.

X-ray or Gamma-ray photon: lots of energy.

Reference Frames

A *reference frame* is a system of coordinates and clocks (usually a "conceptual" system rather than a physical construction) that can be used to locate events and measure the positions and motions of objects.

For example, in a train station, the reference frame of the "fixed" platform and a "moving" train are different.

The rotating reference frame of riders on a merry-go-round is different from the reference frame of their parents standing on the ground watching them.

An *inertial frame* is a frame of reference in which the law of inertia applies. (No change in velocity unless acted on by a net external force.)

A train (or any other reference frame) moving at constant velocity is an inertial frame.

An accelerating train is not an inertial frame.

A rotating merry-go-round is not an inertial frame.

Momentum and energy conservation hold in any inertial frame.

A Thought Experiment

Case 1:

A box with mirrored interior walls contains a mix of hot atoms and EM radiation. Through a briefly opened shutter, the box emits an amount of EM radiation with energy E, upwards.

If the box is initially at rest, what is the velocity of the box afterwards?

Case 2:

Like before, but this time the box emits energy $E/2$ upward and an equal amount of energy $E/2$ downward.

Now what happens?

Case 3:

Like 2, but the radiation is emitted at a forward angle, so that after traveling a distance r out of the box it has moved forward a horizontal distance x.

What happens? Vertical momentum cancels, but horizontal momentum adds to $\frac{E}{c} \times \frac{x}{r}$.

Case 4:

Go back to case 2. However, observe it in a reference frame that is moving backward at speed v.

In this frame, after time t the light has gone a distance $r = ct$ and a horizontal distance $x = vt$. Thus $x/r = v/c$.

But from Case 2 I know that the box does not recoil, so its velocity after emitting the radiation is still v, momentum mv.

Looks like total momentum changed by Ev/c^2.

What could have gone wrong?

Momentum conservation fails in moving frame?

This would imply a preferred rest frame of the universe, the one in which momentum conservation holds. It would be different from Newtonian mechanics, where all frames in constant relative motion are equivalent.

What do I know about m? It's really only defined by Newton's laws.

I can restore momentum conservation if the mass of the box changes from m to $m' = m - E/c^2$, so that $m'v = mv - Ev/c^2$.

This would also be happening in Case 2, but there is no contradiction there because of symmetry.

Einstein goes through this reasoning in 1905.

(The specific form of the argument presented here is due to my colleague Andrew Gould; it is similar to Einstein's version but more transparent.)

His conclusion: Energy carries inertial mass $m = E/c^2$.

A body that emits radiation (e.g., a lump of radium undergoing radioactive decay) must decrease in inertial mass.

Couldn't be tested directly at the time, but turns out to be true.

The combination of the principle of relativity (frames in constant relative motion obey equivalent physical laws) and the behavior of EM radiation will lead to startling conclusions.

In the interest of drama, I have started out of order, beginning with the conclusion that Einstein reached last.

Now we will rewind to the beginning.