7. GR and Black Holes

Reading: Thorne, Chapter 3; also Chapter 7 pages 290-294

The Schwarzschild Solution

Newton's equations, $F_g = GMm/r^2$ and $a = F_g/m$, are easy to write down.

There is an exact solution for two bodies orbiting around each other (e.g., a planet orbiting the sun, or two stars in orbit).

But even for three bodies, there is no exact solution that one can write down with equations. One must either use approximations or solve the equations numerically, e.g., on a computer.

The corresponding Einstein equations are substantially more complicated.

Einstein derived key results (bending of light, gravitational redshift, correspondence to Newtonian gravity, orbit of Mercury) using approximations that are very accurate for weak curvature.

He thought that no one would find exact solutions to the equations because they are so complicated.

But within three months of Einstein's publishing his curvature equation, Karl Schwarzschild discovered an exact solution describing the curvature of spacetime outside a spherical, non-rotating, star.

He also discovered the solution describing the curvature of spacetime inside the star.

Properties of the Schwarzschild spacetime

The properties of the Schwarzschild spacetime can be derived mathematically by looking carefully at his solution to Einstein's equations.

Light traveling on geodesic paths is bent as it passes by the star.

Light going outwards from radius r experiences gravitational redshift.

There is a "critical" radius $R_{\mathrm{Sch}} = \frac{2GM}{c^2}$, where M is the mass of the star. As the distance r approaches R_{Sch}:

- The redshift becomes infinite. Light cannot escape from inside R_{Sch}.
- The light-bending angle becomes large. Light can travel in a circle at $r = 1.5R_{\mathrm{Sch}}$.
- Clocks at r get infinitely out of sync with clocks at large distances.

R_{Sch} is the radius of the "event horizon" – events inside are forever shielded from view.

What the Schwarzschild Solution Describes

At distances much larger than R_{Sch}, the geodesic paths in Schwarzschild spacetime for objects moving with $v \ll c$ are just like the usual orbits in Newtonian gravity.

The Schwarzschild solution is an accurate description of spacetime curvature in our solar system, to the extent we can ignore the extra curvature caused by the planets themselves.

We don't see the unusual features of Schwarzschild spacetime in our solar system because the sun is much larger than $R_{\mathrm{Sch}} = \frac{2GM}{c^2} = 3\,\mathrm{km}$; once we are outside the sun, we are already in the regime of "weak gravity."

This is also true throughout the sun, since as we go to smaller radius we also have less interior mass.

The Schwarzschild solution also describes the spacetime around non-spinning black holes.

Reaction to the Schwarzschild Solution

Because the behavior near $R_{\rm Sch}$ is so strange, many great physicists (Einstein, Eddington) concluded that objects smaller than $R_{\rm Sch}$ could not exist, or could never form.

For example, Einstein showed that a star smaller than $R_{\rm Sch}$ would have to have its atoms move faster than light to support itself against gravity.

He concluded: nothing more compact than $R_{\rm Sch}$ can exist.

The correct conclusion: nothing more compact than $R_{\rm Sch}$ can support itself against gravity, so any star that becomes smaller than $R_{\rm Sch}$ must collapse completely.

Spinning black holes

As we will learn later, the collapse of a star that has exhausted its nuclear fuel can lead to the formation of a black hole.

If the star was not rotating, then the black hole is described by the Schwarzschild solution

The collapse of a spinning star produces a different spacetime than collapse of a non-spinning star. The spin "drags" nearby space into circulation around the black hole, and drags objects in that space along with it.

A ball dropped in towards a spinning black hole moving opposite to the direction of spin will reverse direction before it enters the event horizon, so that it is moving in the same direction as the black hole spin (Thorne Fig. 7.8).

The solution to Einstein's equations that describes a spinning black hole was discovered in 1963 by Roy Kerr.

Evidence for existence of black holes, 1920

By 1920, we have the following "theoretical" arguments for the existence of black holes:

Correctness of GR is supported by the orbit of mercury and the bending of light.

Schwarzschild solution shows that "black holes" (objects with event horizons; the term black hole hasn't been coined yet) could exist in GR, if they could form.

The interpretation of Schwarzschild's solution near or inside the event horizon is controversial, and many physicists believe such objects are impossible.

No one has proposed a way for black holes to form.

There is no direct *empirical* evidence for black holes.