**Astronomy 142: Assignment 2**

This assignment is due at the beginning of class on *Friday*, Feb 3. You may consult with others in the class when you are working on the homework, but (a) you should make a first attempt at everything on your own before talking to others, and (b) you must write up your eventual answers independently.

Your answers should explain your reasoning. If you use an equation to get your answer, make sure you include that equation in presenting your solution. Be sure to give units for any answer that requires them.

You are welcome to come to my office hours or Ying Zu's office hours for advice. Office hours are listed on the syllabus, and so are our office locations. If you are unable to make any of the scheduled office hours, you can make an appointment.

*Please write your answers on separate sheets, not on the assignment sheets, where they are hard to read.* Please be sure that your name is on your assignment, and please staple or paper clip all sheets together.

Assignments that are turned in after class but before 5 pm on Friday (to my mailbox in McPherson 4055) will be marked down 10 points (out of 100) for lateness. Assignments turned in by classtime on Monday will be marked down 20 points. I will not accept assignments after class on Monday unless you contacted me *before* class on Friday to get permission to do so.

## Part I: Short Questions

Each question is worth 5 points. The last two questions rely on Chapter 2 and/or next week's lectures.

1. How did the discovery of the planet Neptune provide evidence for Newton's theory of universal gravity?

2. You have a box with mirrored interior walls, filled with hot atoms and light. For 30 minutes, you shine a laser into the box through a tiny shutter; then you close the shutter so that the box is perfectly lightproof once again. After you have shined the laser in, is the mass of the box higher than before, lower than before, or the same as before? (Assume you can measure the mass with arbitrary accuracy.)

3. True or False: In the 1880s, Michelson and Morley showed that the earth moves relative to the absolute rest frame defined by the "aether" through which electromagnetic waves propagate.

4. Who said that the concepts of space and time were "doomed to fade away into mere shadows," to be replaced by the concept of four-dimensional spacetime?

5. Since the moon orbits around the earth, it is clear that the moon is substantially affected by the earth's gravity. The moon is 250,000 miles from earth. The space shuttle orbits about 300 miles above the earth's surface, where earth's gravity is much stronger. However, astronauts in the space shuttle are "weightless," and loose objects in the space shuttle float rather than fall to the floor. Why?

**Part II: Energy and the Sun**

Each part of the question is worth 3 points, except part (f) which is worth 5 points. You may want to refer to the "Transitions" section of the notes (§4), especially the discussion of energy.

The metric unit of energy is the joule. The joule can be expressed in terms of more elementary units:

1 joule $= 1$ kg m$^2$ sec$^{-2}$

Power is a measure of the *rate* at which energy is produced (or consumed). The metric unit of power is the watt, which is equal to one joule/sec (i.e., producing or consume one joule per second).

(a) What is the kinetic energy of a 1 kg body moving at 1 m/sec?

(b) How many joules does a 100-watt light bulb use in ten seconds?

Sugar has an energy content of 4000 calories per kg (as you can see yourself by looking at nutritional information on a sugar packet). One calorie is the energy required to raise the temperature of one kg of water by one degree centigrade. One calorie is about 4000 joules, so the energy content of sugar is $16 \times 10^6$ joules/kg.

(c) Suppose that you "burn" one kg of sugar and use the energy to power a 100-watt light bulb. How long can you keep the bulb lit? (Give your answer in seconds, and convert to hours remembering that there are $60 \times 60 = 3600$ seconds per hour.)

(d) The luminosity of the Sun (the rate at which it radiates energy into space, mostly in the form of visible light) is $4 \times 10^{26}$ watts. The *mass* of the Sun is $2 \times 10^{30}$ kg. Suppose that the Sun were a burning ball of sugar. How long could it last at this luminosity? (Convert your answer from seconds to years recalling that there are $3 \times 10^7$ seconds in a year. In addition to your final number, clearly write down the numbers that you are multiplying and dividing in order to get it.)

The answer for other chemical fuels, such as gasoline or coal, would be roughly the same.

(e) The age of the oldest rocks on the Earth, estimated from radioactive dating, is 4 billion years. Why is this a problem for the "burning ball of sugar" theory of the Sun?

(f) When 1 kg of hydrogen undergoes nuclear fusion, it produces 0.99 kg of helium. (It's 0.993 kg if we're being exact, but we'll approximate this as 0.99 kg to make the math slightly easier.) If the Sun starts out as pure hydrogen and converts entirely into helium via nuclear fusion, how long can it last? (Remember that $c = 3 \times 10^5$ km/sec $= 3 \times 10^8$ m/sec. Express your answer in years. Write down the numbers you are multiplying and dividing as well as your final answer.)

In practice, only the central 10% of the Sun gets hot enough for fusion, the Sun is initially 75% hydrogen instead of 100%, which (together with the difference between 0.99 and 0.993) makes the lifetime of the Sun about 15 times lower than this.

**Part III: Relativistic Twins**

Each part of the question is worth 5 points. Parts (b) and (d) rely on next week's lectures.

Two twins are in separate rocket ships, floating in space. Twin A stays put. Twin B fires her rockets and accelerates to 99% of the speed of light, and travels a distance of one light year (as seen by Twin A). She then turns around, flies back, fires her engines in reverse to bring her rocket to rest, and rejoins Twin A.

(a) When they rejoin, who is older: Twin A or Twin B (or are they the same age)?

(b) Suppose the rockets were windowless and allowed no view of the outside world, and that the engines were soundless and their operations pre-programmed such that neither twin knew in advance what her spaceship was going to do. After Twin B returns but before she meets up with Twin A to compare ages, how can she know that she was the one who "went away and came back" instead of staying "at rest"? In other words, what about Twin B's experience of the trip distinguishes it from Twin A's experience of the same time interval?

(c) In the course of exploring the universe in a spaceship, you and a friend (who happens to have the same birthday), come across a black hole. You stay in the spaceship and orbit the black hole in a circular orbit at a large distance (several AU), where the orbital velocity is only 10 km/s. Your friend leaves in an auxiliary craft, flies down close to the black hole, orbits it a few million times at a speed of $0.99c$, then flies back to join you in the spaceship. When he gets back, which of you is older and why?

(d) Suppose that instead of orbiting the black hole, your friend flew close to the black hole but fired his rockets at just the level needed to hover above the black hole, spending the same amount of time as he would have spent for the few million orbits. He then returns to the spaceship. When he gets back, is there an age difference between you? If so, why?

If you google "twin paradox," you will find lots of material on parts (a) and (b) of this problem, which may or may not make it easier for you to answer.