Astronomy 142: Assignment 4

This assignment is due at the beginning of class on *Friday*, March 6. You may consult with others in the class when you are working on the homework, but (a) you should make a first attempt at everything on your own before talking to others, and (b) you must write up your eventual answers independently.

You are welcome to come to my office hours or David Nataf's office hours for advice (remember that you can make an appointment if need be). Office hours are listed on the syllabus; note in particular that I have office hours after class on Wednesday. Please be sure that your name is on your assignment, and please staple or paper clip all sheets together.

Assignments that are late will be marked down 10 points (out of 100) if they are in my mailbox by 5 pm Friday, or by 20 points if they are in my mailbox by 5 pm Monday. Assignments later than that will not be accepted unless you have made prior arrangements with me.

Part I: Short Questions

Questions 1-3 are worth 5 points; question 4 is worth 10 points.

1. Who invented the term "black hole" and when?

2. What was Karl Jansky's contribution to astronomy?

3. For this question, refer back to Homework 1, Part III, where you obtained the equation:

$$\frac{GM}{4\pi^2} = \frac{r^3}{P^2}. \tag{1}$$

You observe a visible star that is orbiting around an X-ray source, and by monitoring its Doppler shifts you conclude that the visible star is moving in a circular orbit of radius $r = 1$ AU and $P = 100$ days. What is the mass (in $M_\odot$) of the compact object that is producing the X-rays? Is it a neutron star or a black hole?

Hint: To make the calculation easy, scale it to the solar system (like you did for Jupiter). Remember that the earth's period is 365 days. Don't worry that this equation isn't perfectly accurate for a star around a black hole — it's close enough for our purposes.

4. Highly ionized iron atoms emit X-ray photons with a specific energy $E = 6.4$ keV. (Here "keV" stands for "kilo-electron-volt," but for our purposes you don't really need to know what that means.) When we observe black holes with X-ray telescopes, we detect these photons coming from iron atoms, but *some* are observed to have energies higher than 6.4 keV and *most* are observed to have energies lower than 6.4 keV.

(a) Why do *some* of the observed photons have energies above 6.4 keV?

(b) Why do *most* of the observed photons have energies below 6.4 keV?

Part II: The Luminosity of an Accretion Disk Around a Black Hole

Each part of the question is worth 5 points.

The *luminosity* of an object is the rate at which it radiates energy, and its units are energy per time. For example, the luminosity of a 100-watt light bulb is 100 joules/second, and the luminosity of the sun is 4×10^{26} joules/second.

One joule is equal to 1 kg-m^2/s^2, i.e., it is twice the energy of motion of a one kg mass moving at one m/s. (Twice because the energy of motion is $\frac{1}{2}mv^2$.)

(a) Over its lifetime $t_\odot = 10^{10}$ years, the sun will convert 10% of its mass from hydrogen to helium in the core. When a mass m of hydrogen is fused to helium, the energy released is $7 \times 10^{-3} mc^2$. Argue that the average luminosity of the sun (during the time that it is a main sequence star fusing hydrogen to helium) is

$$L_\odot = \frac{7 \times 10^{-4} M_\odot c^2}{t_\odot}. \tag{2}$$

(In other words, explain in a sentence or two why equation 2 is correct.)

(b) Using the above definition of the joule, calculate the value of $L_\odot$ from equation (2) in joules and check that it is close to the luminosity of the sun listed above. (Useful numbers: $M_\odot = 2 \times 10^{30}$ kg, $c = 3 \times 10^8$ m/s, 1 year $= 3 \times 10^7$ sec.)

(c) As stated in class, the luminosity of a non-spinning black hole that is accreting mass at a rate $\dot{M}$ through a thin disk is

$$L = \frac{1}{12}\dot{M}c^2. \tag{3}$$

Suppose that over the course of $t_X = 5 \times 10^6$ years, a black hole accretes a mass $m = 0.5 M_\odot$ from a binary companion that has become a red giant. Using equation (3) and your result from (a), show that the average luminosity of the accretion disk around the black hole is

$$L_{\rm disk} = 1.2 \times 10^5 L_\odot.$$

(d) Suppose that you *observe* an X-ray source with a total luminosity of $1.2 \times 10^5 L_\odot$, but you do not know initially whether it is powered by gas accreting onto a neutron star or onto a black hole.

Based on the Eddington luminosity limit discussed in class, what is the smallest possible mass (in $M_\odot$) of the compact object powering the X-ray source? From this result, would you say that the object is a neutron star or a black hole? Explain your answers.

Part III: Measuring the Mass of an Active Black Hole

Each part of the question is worth 5 points.

The two figures on the last page are from a paper by Kelly Denney (who gave the guest lecture on black hole mass measurements) and her collaborators (including several other OSU students and faculty). In the upper panel of the top plot, the solid curve shows a portion of the spectrum (intensity vs. wavelength) of visible light from an accreting black hole at the center of the galaxy NGC 4593. The broad peak centered at about 4900Å shows emission from hydrogen atoms. (1Å is 10^{-10} m, so 4900Å is 0.49 microns). It is broad because the atoms are moving with a range of velocities and thus have a range of Doppler shifts.

(a) To determine a characteristic velocity of the hydrogen gas surrounding this black hole, measure the width of the peak halfway down from its top. (Draw a line segment across the width of the peak with a pencil, halfway down from the top, and connect the ends of the line segment to the x-axis to measure the width.) Call this width $\Delta\lambda$, since it is a spread in wavelengths. What is the value of $\Delta\lambda$ in Å?

Convert $\Delta\lambda$ to a characteristic velocity v of the hydrogen gas with the Doppler formula

$$v = \frac{\Delta\lambda}{\lambda} \times c,$$

where $\lambda = 4900$Å is the central wavelength of the peak and $c = 3 \times 10^5\,\mathrm{km\,s^{-1}}$ is the speed of light. What is v in $\mathrm{km\,s^{-1}}$?

(b) In the lower plot, the top panel shows the observed variation of the light coming from near the black hole over a period of 40 days (the x-axis is marked in days). There is a sharp decline in brightness beginning just before day 3440 and ending about day 3450. The lower panel shows the observed variation of light emitted by the surrounding hydrogen gas. The pattern is similar, but, as Kelly explained in her lecture, it is delayed in time because the hydrogen gas is at some distance from the black hole, and it takes time for light coming from the black hole to reach the gas.

By carefully comparing the sharp decline in the two panels, measure the time delay Δt between the variation of the light from the black hole and the response of the hydrogen gas (i.e., the time shift between the two plots, which you should be able to measure to the nearest half-day). What is Δt in days?

What is the approximate distance between the black hole and the hydrogen gas *in light-days*?

(In detail, the answer to the second question depends on the geometry of the gas distribution around the black hole, but the simplest answer should be accurate to within a factor of two.)

(c) Recall from our discussion of circular orbits that

$$a = \frac{v^2}{r} = \frac{GM}{r^2},$$

where a is the acceleration of an object moving in a circle of radius r at speed v, with the acceleration produced by the gravity of a central object of mass M. From these equations, show that if you know the orbital speed v and the distance r, you can estimate the mass of the central object as

$$M = \frac{v^2 r}{G}.$$

(d) The distance from the earth to the sun, r_E, is 8 light-minutes, and the speed of the earth in its orbit is $v_E = 30\,\mathrm{km\,s^{-1}}$. If you plug these values into the above equation,

$$M = \frac{v_E^2 r_E}{G},$$

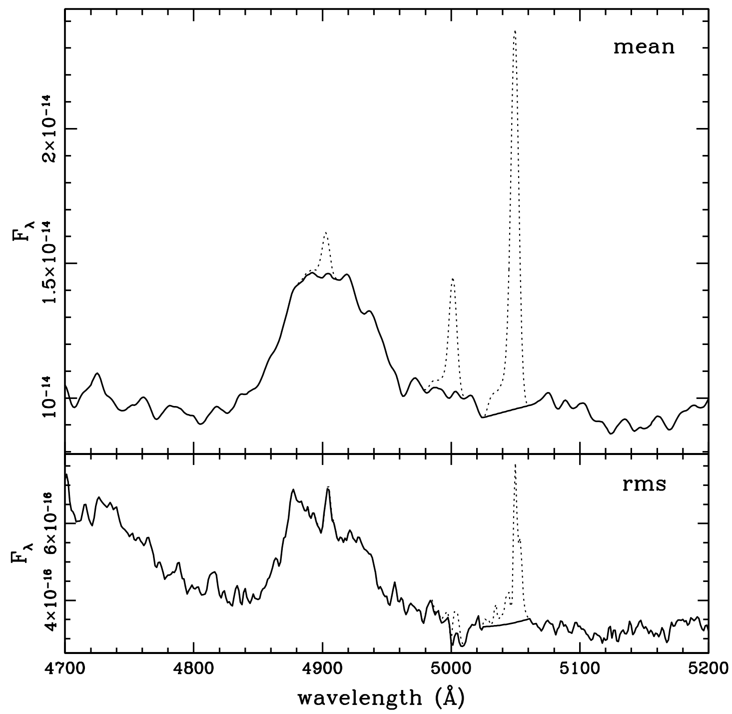
what is the value of M in units of $M_\odot$?

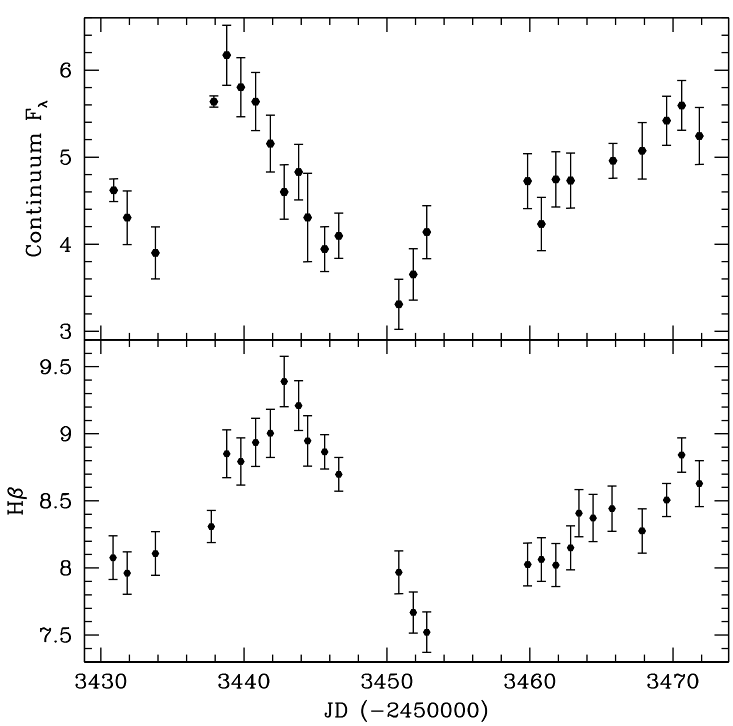
(Hint: You do not need a calculator for this part, only thought.)

(e) Using your velocity v in $\mathrm{km\,s^{-1}}$ from part (a) and your distance r in light-days from part (b), and using your result from (d) to scale your answer, what is the mass of the black hole in NGC 4593, in units of $M_\odot$?

(Hints: If you scale to your result from (d), you do not need to know the value of G. There are $24 \times 60 = 1440$ minutes in a day.)

Note: There are obviously several approximations here, in the way you estimate the distance and velocity and in the assumption of circular orbits, but they should be accurate to within a factor of two or three.


mean
rms
F_λ
2×10^{-14}
1.5×10^{-14}
10^{-14}
6×10^{-16}
4×10^{-16}
4700
4800
4900
5000
5100
5200
wavelength (Å)


Continuum F_λ
Hβ
JD (−2450000)
3430
3440
3450
3460
3470