9. Measuring Cosmological Parameters via Expansion

Reading: Chapter 7

Cepheids and Type Ia supernovae as standard candles

If you observe the apparent flux f of an object of known luminosity L, you can infer its distance from the relation

$$f = \frac{L}{4\pi d^2}.$$

Cepheid variable stars exhibit a tight correlation between period and luminosity. Measure P, infer L.

Type Ia supernovae (supernovae without hydrogen absorption lines, produced by thermonuclear explosions of white dwarfs) rise and fall over the course of about a month.

Their peak luminosities (luminosity at maximum) are similar, with a scatter of about 40% from one supernova to another.

In the early 1990s, it was recognized that supernovae that rise and fall slower are more luminous, and vice versa.

If you measure the shape of the light curve (the time required to rise and fall), you can "standardize" the Type Ia "candle," leaving residual scatter of 10-15% in peak luminosity (0.1-0.15 magnitudes).

This makes Type Ia supernovae powerful tools for cosmology.

Measuring H_0

At first glance, measuring H_0 looks easy: find an object of known distance d, measure its redshift, and infer $H_0 = v/d$.

The first step is the hard one. You have to *calibrate* your standard candles.

In rough outline, the best direct route to H_0 is currently:

• Measure the distance to the LMC, using stars that can be matched to stars in the Milky Way.

• With the known distance to the LMC, calibrate the period-luminosity relation for Cepheids.

• Find Cepheids in somewhat more distant galaxies (few Mpc) that have also hosted Type Ia supernovae. Use the periods and fluxes of the Cepheids to infer the distance to the galaxies, and thereby calibrate the luminosity of Type Ia supernovae.

• Find Type Ia supernovae in galaxies that are far enough away (50-200 Mpc) that one can ignore their peculiar velocities. Use the peak fluxes of the supernovae to get the distances to the galaxies, and measure the galaxy redshifts.

• Infer $H_0 = d/v$.

The LMC step can be circumvented by measuring Cepheid distances via parallax, but there are only a few Cepheids close enough to have measurable parallaxes.

The expansion history and cosmological parameters

What about measuring distances beyond the regime of Hubble's law?

The Friedmann equation is

$$H^2(t) = \frac{8\pi G}{3}\frac{\epsilon(t)}{c^2} - \frac{kc^2}{R_0^2 a^2(t)}.$$

We earlier showed (in our discussion of critical density, see also equation 4.31 of the textbook) that

$$\frac{kc^2}{R_0^2} = H_0^2(\Omega_0 - 1),$$

where $\Omega_0 = \epsilon_0/\epsilon_{c,0}$.
We can therefore recast the Friedmann equation in the form

$$\frac{H^2(t)}{H_0^2} = \frac{\epsilon(t)}{\epsilon_{c,0}} + \frac{1-\Omega_0}{a^2(t)}.$$

The value of $\epsilon(t)$ can be expressed in terms of the present day values of the matter and radiation densities and the value of the cosmological constant energy.

$$\epsilon(t) = \epsilon_{r,0}a^{-4} + \epsilon_{m,0}a^{-3} + \epsilon_\Lambda.$$

With the definition $\epsilon_{x,0} = \Omega_{x,0}\epsilon_{c,0}$, we have

$$\frac{H^2(t)}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{(1-\Omega_0)}{a^2}.$$

The $\Omega_{\Lambda,0}$ term could be adjusted to allow for dark energy that varies in time.

This version of the Friedmann equation is one of the most useful for practical calculations.
For forms of dark energy that are not a cosmological constant, one needs to make the substitution

$$\Omega_{\Lambda,0} \longrightarrow \Omega_{\mathrm{DE},0}\frac{\rho_{\mathrm{DE}}(z)}{\rho_{\mathrm{DE},0}} = \Omega_{\mathrm{DE},0}(1+z)^{3(1+w)},$$

where the last equality holds for constant w.
Note that throughout this discussion I have adopted our standard convention that the expansion parameter at the present day is $a_0 = 1$, and therefore $a \equiv (1+z)^{-1}$.

Comoving distance

Following a line of reasoning we have used before:

$$\frac{1}{a}\frac{da}{dt} = H \quad \Rightarrow \quad dt = \frac{1}{H}\frac{da}{a}.$$

For propagating photons

$$dr = \frac{c\,dt}{a(t)} = \frac{c}{H}\frac{da}{a^2} = \frac{c}{H_0}\frac{H_0}{H}\frac{da}{a^2}.$$

With our most recent version of the Friedmann equation, we can write the comoving distance as

$$r = \frac{c}{H_0}\int_{a_e}^{1}\frac{da}{a^2\left[\Omega_{r,0}/a^4 + \Omega_{m,0}/a^3 + \Omega_{\Lambda,0} + (1-\Omega_0)/a^2\right]^{1/2}},$$

with $\Omega_0 = \Omega_{r,0} + \Omega_{m,0} + \Omega_{\Lambda,0}$.

One can also use $a \equiv (1+z)^{-1} \Rightarrow da = -dz(1+z)^{-2} = -a^2 dz$ to write this formula as

$$r = \frac{c}{H_0}\int_0^z \frac{dz'}{\left[\Omega_{r,0}(1+z')^4 + \Omega_{m,0}(1+z')^3 + \Omega_{\Lambda,0} + (1-\Omega_0)(1+z')^2\right]^{1/2}},$$

Note that if $z \ll z_{\rm eq} \sim 3600$, the $\Omega_{r,0}(1+z')^4$ term is negligible compared to the $\Omega_{m,0}(1+z')^3$ term.

Luminosity distance

Suppose we observe a standard candle of known luminosity L and redshift z.

From the metric

$$ds^2 = -c^2dt^2 + a^2(t)\left[dr^2 + S_k^2(r)d\Omega^2\right]$$

we can see that photons emitted isotropically by a source at comoving distance r are today (at t_0, with $a_0 = 1$) spread over a sphere of proper surface area

$$A_p(t_0) = 4\pi S_k^2(r),$$

since the solid angle is $\int d\Omega^2 = 4\pi$ steradians.
Recall that the comoving distance is defined so that it is equal to the proper distance at $t = t_0$, so for a flat universe with $S_k(r) = r$ this is just the usual Euclidean result.
For a positively curved space, the photons are spread over a smaller surface area than they would be in a flat universe, and for a negatively curved space they are spread over a larger surface area.

You might expect that the relation between flux, luminosity, and distance in an FRW universe would be

$$f = \frac{L}{4\pi S_k^2(r)} \qquad \text{(Warning: Incorrect Equation)}$$

However, there are two additional effects.
First, the energy of photons drops by a factor of $(1+z)$, and since f is an *energy* flux not a photon number flux, this reduces f by a factor of $(1+z)$.
Second, there is time dilation between the emitted frame and observed frame, so that photons emitted in a time interval Δt are received in a time interval $\Delta t(1+z)$. This reduces the flux by an additional factor of $(1+z)$.

Bottom line: The relation between flux, luminosity, and distance is

$$f = \frac{L}{4\pi S_k^2(r)(1+z)^2}.$$

The quantity

$$d_L = S_k(r)(1+z)$$

is often called the *luminosity distance*, because

$$f = \frac{L}{4\pi d_L^2}.$$

Cosmologists also often refer to the angular diameter distance d_A, for which the angular size of an object of physical length l is $\theta = l/d_A$.

If you find yourself in need of formulas for cosmological distance measures, a good general reference is Hogg (1999, arXiv:astro-ph/9905116).

Supernova cosmology

If we measure the redshift of such an object, we can calculate its luminosity distance *if* we specify the values of $\Omega_{m,0}$, $\Omega_{r,0}$, and $\Omega_{\Lambda,0}$.

Using the (approximately) known luminosity of a Type Ia supernova, we can *measure* the luminosity distance if we measure the apparent flux.

By comparing expected and measured values of the luminosity distance at a variety of redshifts, we can pin down the values of $\Omega_{m,0}$, and $\Omega_{\Lambda,0}$.

Improvements in digital detector technology in the 1990s made it feasible to start searching large areas of sky for high-redshift supernovae.

Two groups set out to do this, with the goal of measuring the deceleration of the universe and determining. $\Omega_{m,0}$.

Instead they found an accelerating universe and showed that $\Omega_{\Lambda,0} > 0$.

Today, we are trying to do this at higher precision to test whether "dark energy" really is a cosmological constant, with $w = -1$, or a new kind of field with a different value of w.

Baryon acoustic oscillations (BAO), discussed earlier in the course when I talked about the Sloan survey, offer a different way of doing the same kinds of investigations, using the angular diameter of a "standard ruler" instead of the apparent flux of a "standard candle."

Final Remark

It has been common practice for decades to summarize the impact of cosmological parameters in the quantity

$$q_0 \equiv -\left(\frac{\ddot{a}a}{\dot{a}^2}\right)_{t_0},$$

which is a dimensionless measure of the current deceleration rate of the universe.

Standard formulas for the luminosity or angular size distance are expressed in terms of q_0.

In my view, this practice is no longer useful, because there is no unique relation between the single parameter q_0 and the two parameters $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$, which are the quantities of physical interest.