Astronomy 5682
Problem Set 1
Due Tuesday, January 22, in class

Normally homework assignments will be handed out Thursday and due the following Thursday. However, I will be out of town for most of the next week, so I have pushed the deadline for this assignment to the following Tuesday in case there are questions (you can also send them by email). I think this assignment will be straightforward.

The lecture on Tuesday, Jan 15 will be given by Barbara Ryden. There will be no office hours on Wednesday, Jan 16. I *will* be in for office hours and lecture on Thursday, Jan 17.

Question 1

(a) The typical luminosity of a bright galaxy is $L \sim 10^{11} L_\odot$. The average space density of such galaxies is $n \sim 0.001$ galaxies/Mpc^3. If the universe is 15 billion years old, what is the total flux $F_{\rm gal}$ received at earth from all of the external galaxies in the universe? (Ignore the contribution from stars in the Milky Way.)

Express your answer in $\mathrm{erg\,s^{-1}\,cm^{-2}}$, remembering that the luminosity of the sun is $L_\odot = 3.9 \times 10^{33}\,\mathrm{erg\,s^{-1}}$ and $1\,\mathrm{Mpc} = 3.1 \times 10^{24}\,\mathrm{cm} = 3.26$ million light years.

(b) How does $F_{\rm gal}$ compare to the flux $F_\odot$ received from the sun at earth (i.e., what is the ratio $F_\odot/F_{\rm gal}$)?

Ignore all of the obvious potential complications, such as redshifting of light and evolution of the galaxy population. Just do the "naive" calculation for a static, non-evolving universe. Don't worry much about factors of two; aim for a simple calculation with order-of-magnitude accuracy. If you need guidance, see §2.1.

Question 2

Hubble's law says that a galaxy at distance d is moving away from us at a speed $v = H_0 d$.

A modern estimate of the expansion rate is $H_0 = 70\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$.

(a) If you assume that galaxies have always been moving at a constant speed, what do you infer for the age of the universe? (Explain your answer, and give a value in Gyr, where 1 Gyr = 1 billion years. You can find relevant unit conversions in the back of the book if you need them.)

(b) Suppose that the expansion of the universe has been slowing down over time because of the decelerating effects of gravity. Is the implied age of the universe larger or smaller than the one you gave in part (a)? Explain.

(c) If you look at Hubble's velocity-distance plot (Fig. 2.4 of the book, the one I showed in class), you'll see that he has galaxies going $1000\,\mathrm{km\,s^{-1}}$ at a distance of 2 Mpc. We now understand that Hubble's distances were systematically off because he was confusing one kind of variable star with another. However, if you take his measurements at face value, what is the implied value of H_0, and what is the implied age of the universe? If you had evidence that the age of the solar system was 5 Gyr, why would Hubble's measurements pose a problem for a big bang model of the universe?

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Question 3

Assume that the light in the galaxies of Question 1 is produced entirely by stars like the sun. This is a poor assumption in detail, but it gives roughly the right answer for the calculation in this problem.

(a) What is the total mass M, in $M_\odot$ ($= 2.0 \times 10^{30}$ kg), contained in a sphere of radius 100 Mpc?

(b) If the Hubble constant is $H_0 = 70$ km s^{-1} Mpc^{-1}, what is the recession speed v of an object at a distance of 100 Mpc?

(c) For a test mass m at a distance $R = 100$ Mpc, what is the ratio α of the kinetic energy $mv^2/2$ to the gravitational potential energy GMm/R, according to the numbers you just derived?

(It is not immediately obvious that you can ignore the gravitational effects of all matter *outside* the 100 Mpc sphere, but it turns out that you can, for reasons we will discuss in class.)

(d) What is the value of α at $R = 200$ Mpc? (If you think carefully about this part, it will be easy.)

(e) Show that the general result is

$$\alpha = \frac{3H_0^2}{8\pi G\bar{\rho}} \ ,$$

where $\bar{\rho}$ is the average density of matter in the universe.

(f) Based on your results, do you expect the expansion of the universe to continue forever, or do you expect gravity to eventually halt and reverse this expansion? Explain your reasoning.

(Note: We have ignored two important components of the real universe, dark matter and dark energy, so the answer based on your calculation here will not necessarily be the same as the answer for the real universe.)