VIII. Inflation

Reading: Chapter 11.

The Horizon Problem

In a universe with $a(t) \propto (t/t_0)^\alpha$, where $0 < \alpha < 1$, a photon that travels in a straight line from $t' = 0$ to $t' = t$ covers a comoving distance

$$r = \int_0^t \frac{c\,dt}{(t'/t_0)^\alpha} = ct_0 \int_0^t \left(\frac{t}{t_0}\right)^{-\alpha} d\left(\frac{t}{t_0}\right) = \frac{ct_0}{1-\alpha}\left(\frac{t}{t_0}\right)^{1-\alpha}.$$

At any time t, there is a *horizon* (in more technical language, a "particle horizon") that is the maximum distance over which any two points could have had causal contact with each other, using signals that travel no faster than light.

The comoving size of the horizon grows with time.

When we observe two widely spaced patches on the CMB, we are observing two regions that were never in causal contact with each other.

More quantitatively, at $t = t_{\rm rec}$, the presently observable universe was divided into $\sim 30,000$ causally disconnected patches.

How do all of these patches know that they should be the same temperature to one part in 10^5?

The Flatness Problem

The curvature radius is at least $R_0 \sim cH_0^{-1}$, perhaps much larger.

The number of photons within the curvature radius is therefore at least

$$N_\gamma = \frac{4\pi}{3}\left(\frac{c}{H_0}\right)^3 n_\gamma \sim 10^{87},$$

where $n_\gamma = 413{\rm cm}^{-3}$.

The universe is thus *extremely* flat in the sense that the curvature radius contains a very large number of particles.

Huge dimensionless numbers like 10^{87} usually demand some kind of explanation.

Another face of the same puzzle appears if we consider the Friedmann equation

$$H^2 = \frac{8\pi G}{3c^2}\epsilon_{m,0}a^{-3} - \frac{kc^2}{R_0^2}a^{-2}.$$

If the "energy" term and "curvature" term on the right hand side are similar in magnitude today, then at very early times (BBN, say, with $a \sim 10^{-9}$) the energy term was enormously bigger. (In the radiation dominated regime, the energy term scales as a^{-4}, so the discrepancy is worse.)

One might generically expect the universe to come into being with comparable energy and curvature terms, but if so it would have either have collapsed very quickly or become "empty" very quickly, and it would not have reached its present, large size with a matter density that is comparable to the critical density.

Another face of the same puzzle appears if we consider the evolution of the density parameter $\Omega(t)$ in a decelerating universe.

Equivalently, in a decelerating universe $\Omega(t)$ always evolves away from one as t increases unless it is exactly one.

In order to have $\Omega \sim 1$ today, Ω must have been tuned extremely close to one at epochs in the early universe.

This form of the flatness problem is treated more fully in the textbook.

Inflation in a nutshell

In the standard big bang model, the flatness and large scale uniformity of the universe are just accepted as initial conditions, not explained.

Inflation is an extension of the big bang model that attempts to give a causal explanation for the origin of flatness and homogeneity, much as the big bang model itself gives a causal explanation for the primordial helium abundance.

However, while BBN relies on standard nuclear and particle physics, inflation must invoke physical processes beyond those in the standard particle physics model, so it is more speculative.

Historically, inflation was also motivated by "the monopole problem," as discussed in the textbook.

In the inflation scenario, the early universe went through an accelerating phase in which it was dominated by vacuum energy with an extremely high energy density.

During this phase, the universe expanded exponentially in time, with an e-folding timescale

$$t_{\rm exp} = \frac{1}{H} = \left(\frac{8\pi G\epsilon_{\rm vac}}{3c^2}\right)^{-1/2}.$$

In typical inflation models, $t_{\rm exp} \sim 10^{-32}$s. (A very rough number; this depends on the energy scale at which inflation occurred and on the details of how it happened.)

If the universe expanded by at least a factor of e^{60} during inflation, then the entire volume of the presently observable universe was within one causally connected patch *before* inflation started, so causal processes could have established the homogeneity of the universe.

This exponential phase would have grown the universe by a very large factor, making space very flat (the curvature radius very large).

During this phase, $\epsilon =$const., so the energy term in the Friedmann equation grows relative to the curvature term.

Eventually, inflation ended, and the enormous energy that had been stored in $\epsilon_{\rm vac}$ was converted to photons and other particles, producing the very large number of particles within the curvature radius.

It is still not entirely clear why the universe would have entered this accelerating, exponentially expanding phase and why it would have stopped accelerating once it started, but if it did, then we can understand how the universe became so homogeneous and so flat.

A natural prediction of inflation is that Ω_0 should be extremely close to 1.0.

A measurable departure from flat space would be a serious challenge to the inflation scenario; it could be accommodated, but only by fine-tuning the model to an uncomfortable degree.

A slowly rolling scalar field

In most implementations of inflation, the accelerated expansion is driven by a *scalar field* ϕ that fills space.

This field is assumed to have a potential energy $V(\phi)$ (analogous to the potential energy $B^2/8\pi$ of a magnetic field with magnitude B).

For a scalar field, the total energy density and pressure are

$$\begin{aligned}\epsilon_\phi &= \frac{1}{2}\frac{1}{\hbar c^3}\dot{\phi}^2 + V(\phi)\\ p_\phi &= \frac{1}{2}\frac{1}{\hbar c^3}\dot{\phi}^2 - V(\phi).\end{aligned}$$

ϕ has units of energy (e.g., GeV), while $V(\phi)$ has units of energy per unit volume.

If the field is changing slowly, so that the $\dot{\phi}^2$ terms are much smaller than $V(\phi)$, then we have $p_\phi \approx -\epsilon_\phi$ and thus a component that can produce exponential expansion if it dominates the total energy density.

Successful inflation thus requires a phase in which $V(\phi)$ dominates the energy and pressure budget for a sufficiently long time.

Smallness of the $\dot{\phi}^2$ terms will arise for suitable forms of the potential $V(\phi)$ and starting values ϕ_i of the field.

The requirement of slow evolution of ϕ is known as the *slow roll* condition.

If we go back (way back, §4 of the notes) to the fluid equation in an expanding universe,

$$\dot{\epsilon} + 3H(t)(\epsilon + P) = 0,$$

and substitute the above equations we get

$$\ddot{\phi} + 3H(t)\dot{\phi} = -\hbar c^3 \frac{dV}{d\phi}$$

as the "equation of motion" for the field ϕ.

This resembles the equation for a ball rolling down a potential hill $V(\phi)$ with a friction term $3H\dot{\phi}$ caused by the expansion of the universe.

The field reaches a "terminal velocity" for which $\ddot{\phi} = 0$ and

$$3H\dot{\phi} = -\hbar c^3 \frac{dV}{d\phi}.$$

This is known as the *slow roll* equation.

Inflation driven by a slowly rolling scalar field

If $\dot{\phi}$ is small and $V(\phi)$ is sufficiently flat, then the universe goes through a phase of nearly exponential expansion with

$$a(t) \propto e^{Ht}$$

and Hubble parameter

$$H = \left(\frac{8\pi G\, V(\phi)}{3c^2}\right)^{1/2}.$$

The expansion is "nearly" exponential because ϕ, and hence $V(\phi)$, are changing slowly as the expansion goes on.

Depending on the form of $V(\phi)$ and the value of $\phi(t_i)$ at the start of inflation, the exponential expansion may last long enough (at least 60 e-folds) to solve the horizon problem and the flatness problem.

Models of inflation differ in the assumed physical significance of the field ϕ, the energy scale and form of the potential $V(\phi)$, and the assumed starting value $\phi(t_i)$.

Many models associate inflation with the epoch at which grand unification breaks, expected to be at an energy scale $kT_{\rm GUT} \approx 10^{15}\,$GeV, or with the Planck epoch when quantum gravity effects become important, at an energy scale $kT_{\rm Planck} \approx 10^{19}\,$GeV.

Figure 11.3 of the book illustrates one commonly assumed form of the potential, which leads to inflation if ϕ starts near zero and eventually rolls to a minimum at some non-zero ϕ_0.

Another broad category of models, usually referred to as "chaotic inflation," has a potential something like $V(\phi) \propto \phi^4$, in which case the field must start at a *large* value, and inflation occurs while it rolls towards zero.

In either scenario, inflation ends when ϕ begins to oscillation about the minimum of $V(\phi)$, so that $\dot{\phi}$ terms dominate the energy density and pressure.

If there is some coupling between the field ϕ and other fields and particles (such as photons), then these oscillations will be damped and the energy will be dumped into these other fields and particles.

During inflation, the temperature of, e.g., the radiation background was driven way down by the exponential expansion, but now it rises again to the temperature at the start of inflation.

This is known as the *reheating* epoch.

In effect, gravitational potential energy associated with repulsive gravity has been used to make the universe much larger, but with the same energy density as before inflation. Sometimes described as "the ultimate free lunch."

After this epoch, we return to a normal, radiation-dominated, hot big bang model, but with a universe that is much larger and flatter and causally connected over much larger scales.

Density fluctuations from inflation

Inflation "irons out" pre-existing inhomogeneities by stretching them out to enormous scales.

Soon after inflation was proposed, people realized that the scalar field driving inflation would experience quantum fluctuations in accordance with the Heisenberg uncertainty principle.

In an exponentially expanding universe, these fluctuations are stretched from microscopic scales to macroscopic scales, as different regions of the universe grow by slightly different factors and end up at slightly different densities.

Roughly speaking, at any time during inflation there are quantum fluctuations in the value of ϕ on scale cH^{-1} of magnitude $\delta\phi \sim H$.

These fluctuations cause inflation to end at slightly different times in different locations, with $\delta t \sim \delta\phi/\dot{\phi}$. This in turn leaves the universe with energy density fluctuations on this scale $\delta\epsilon/\epsilon \sim H\delta t$.

According to inflation, these quantum fluctuations from the very early universe are the source of density variations that produce anisotropy in the CMB and seed the gravitational growth of structure in the universe.

It turns out to be somewhat difficult to get the CMB fluctuations as low as 10^{-5}; the inflation potential has to be "fine-tuned" to do this. [Specifically, it has to be extremely flat, e.g., $\lambda \approx 10^{-15}$ for $V(\phi) = (\hbar c)^{-3}\lambda\phi^4$.]

However, once this fine-tuning is done, the statistical properties of the fluctuations predicted by inflation are in extremely good agreement with the observed properties of CMB anisotropies and large scale structure in the universe.

Specifically, in agreement with observations, the predicted fluctuations are

- Gaussian (bell curve distribution of $\delta\epsilon$)
- Nearly scale invariant (the amplitude of fluctuations on scale ct at time t, $\delta_H(t) \sim 10^{-5}$, is nearly independent of t)

- Present equally in all forms of energy – specifically, the cold dark matter, baryons, and photons all fluctuate together. This is crucial to getting the observed pattern of peaks and troughs in the CMB angular power spectrum, and to getting the observed polarization.

Assessment

Inflation is the best theory we have for explaining the flatness of the universe, the large scale uniformity of the universe, and the origin of inhomogeneity in the universe.

Inflation does not really change the "standard big bang" model, but it gives an explanation for things that in the standard model are just accepted as initial conditions.

As evidence for $\Omega_m \approx 0.2 - 0.3$ accumulated in the late 1980s and early 1990s, it looked like a serious challenge to inflation. Inflation models with an open universe are possible, but they require uncomfortable fine-tuning (exactly the right number of e-folds) and they don't really explain homogeneity any more.

The discovery of dark energy in the amount required to make the universe flat removes this objection (though it leaves us with the problem of explaining dark energy).

The measurements of fluctuations have given simple versions of inflation many chances to fail, but they haven't.

Inflation's successes are impressive, but it relies on physics that we do not fully understand, and the evidence for inflation is not nearly as strong as the evidence for the hot early universe implied by BBN and the CMB.

Departures from perfect scale-invariance (i.e., δ_H weakly dependent on scale) can give some insight into $V(\phi)$.

Departures from perfectly Gaussian fluctuations would be even more informative, but they are expected to be unobservably small in the simplest models.

In the long term, detection of a contribution of gravity waves to CMB anisotropy could provide stronger evidence for inflation. Alternatively, a specific model of inflation that is rooted in particle physics discoveries *and* naturally predicts that the level of fluctuations in the CMB should be $\sim 10^{-5}$ would be very convincing.

Because inflation requires an accelerating universe, it is tempting to connect it to today's cosmic acceleration. However, the energy and timescales are vastly different ($\sim 10^{-32}$ s vs. $10^1$0 yrs), and no convincing connection has been made.

The "larger scale setting" for inflation — i.e., what preceded it and how the universe entered an inflationary state in the first place — is far from clear.