IX. Dark Matter

Reading: Chapter 8

Circular velocity and mass

An object of mass m moving at speed v_c in a circular orbit of radius r around a central object of mass M has an acceleration $a = v_c^2/r$.

Using $F = ma = \frac{GMm}{r^2}$ implies

$$v_c = \sqrt{\frac{GM}{r}},$$

where the subscript c indicates circular velocity.

Rotation curve of the solar system

The earth, 1 AU from the sun, moves at 30 km s^{-1}. How fast does Jupiter, at 5 AU, move? $M = M_\odot$, so $v_J = v_E/\sqrt{5} = 30/\sqrt{5}$ km s^{-1}.

If we plot orbital speed against orbital radius, we get the *rotation curve* of the solar system:

$$v_c = 30 \left(\frac{1 \text{ AU}}{r}\right)^{1/2} \text{ km s}^{-1}.$$

Rotation curve of a spiral galaxy

The disks of spiral galaxies rotate. How do we know? Doppler shifts.

We can use Doppler shifts to measure the rotation speed at different distances from the center.

We can use the circular velocity formula to estimate the mass of the galaxy, but there is one complication: the mass of the galaxy is not all in a point at the middle.

If the galaxy is spherical, then the formula still holds, but M now represents the mass interior to the radius of the orbit.

$$v_c(r) = \sqrt{\frac{GM_{\text{int}}(r)}{r}}.$$

Real galaxies are not perfectly spherical, but the formula remains reasonably accurate, and it will be sufficient for our purposes.

Suppose we plot rotation speed against distance from the center in a spiral galaxy. What do we expect?

Near the center: Mass rises as we move out. r grows, but not as fast as the mass. $v_c(r)$ rises.

At large distances: The amount of light stays nearly constant. r grows. v_c should fall, roughly $v_c(r) \propto r^{-1/2}$, like in solar system.

What we actually observe for spiral galaxies is

Near the center: rotation curve rises, as expected.

At large distances: rotation curve becomes flat.

What does this imply? Mass is still growing, even after light dies out.

Much of the mass in the galaxy is not in stars.

This extra mass is in an extended *dark halo.*

It is made of dark matter, but we don't know what the dark matter is.

Four Questions about Dark Matter

Original attribution, Martin Schwarzschild, at Princeton meeting on Dark Matter in the Universe, 1985.

- Do we need it (to explain what we observe)?
- What is it?
- Where is it?
- How much is there?

Do we need it?

Empirical evidence for the existence of dark matter:

- Motions of galaxies in clusters. Galaxies move fast enough that clusters would fly apart if not held together by dark matter.

  Can estimate mass using

$$M_{\rm int}(R) \approx \frac{2\sigma_v^2 R}{G},$$

where σ_v is the spread of galaxy velocities. Like the formula based on rotation, but with $2\sigma_v^2$ in place of v_c^2. Accuracy of formula depends on density profile of cluster and distribution of galaxy orbits (radial vs. tangential vs. isotropic), but should be good at the 50% level in any case.

This evidence dates back to Fritz Zwicky's study of 20 galaxies in the Coma cluster in 1933, and it has been confirmed in many other clusters.

- Flat rotation curves of spiral galaxies. These show that the mass continues to rise after the light has faded out.
- Motions of stars in elliptical galaxies. (Similar idea, but harder to use because of difficulty in measuring to large distances, uncertain distribution of stellar orbits).
- Relative velocities in binary galaxy pairs, small groups.

- Strong gravitational lensing of background galaxies by foreground galaxies, probes inner region of foreground galaxies. Good method for elliptical galaxies, which don't have circular motions for rotation curve analysis.
- Weak gravitational lensing of background galaxies by foreground galaxies, can be used to map dark matter out to large distances.
- Hot gas in galaxy clusters, between the galaxies, at temperatures $T \sim 10^7 - 10^8$ K. Detected by X-ray satellites.

  Can estimate pressure of gas, mass required to hold it in cluster.

  Results similar to mass estimated from galaxy motions.

  Hot gas itself accounts for $\sim 25\%$ of mass in rich clusters (several times the mass of stars in cluster galaxies), but not 100%.
- Gravitational lensing by galaxy clusters.

Do we need it?

Answer is clearly "yes" IF General Relativity is correct.

Lots of evidence for GR on solar system scales.

Only evidence for GR on galaxy and cluster scales invokes DM, otherwise the predictions of GR fail.

Evidence for GR on cosmic scale: successes of the big bang theory, especially big bang nucleosynthesis (which relies on GR prediction of expansion rate in the hot early universe).

Seems unlikely that an alternative theory of gravity would retain these cosmological successes and solve the DM problem.

But until we detect DM directly, this remains a possibility, actively pursued by a few people.

Recent analyses of the "Bullet Cluster," formed from a recent merger of two clusters, shows that the mass producing lensing is not distributed like the dominant mass of baryons (traced by X-ray gas). This is a big challenge for modified gravity models, since the peaks of the (lensing inferred) mass distribution are not at the center of the cluster.

Where is it?

We know DM resides in

- Extended halos of galaxies.
- Common halos of groups and clusters.

However, we do not have a direct way of measuring DM in "cosmic voids" – large regions (> 10 Mpc across) devoid of bright galaxies.

These occupy a lot of volume, so they could potentially harbor a lot of DM even if the density within them is low.

How much is there?

Nearly equivalent to the question "What is $\Omega_{m,0}$?"

Why? Most mass is dark. If we know $\rho_{\rm DM}$, we can compare to ρ_c, measure $\Omega_{m,0}$.

Tightly linked to the question "Where is it?"

Why? If all DM is in galaxy and cluster halos, we have measured it and can add it up. This gives $\Omega_{m,0} \approx 0.2 - 0.4 \approx 50\Omega_*$, where Ω_* represents mass in stars.

But if there is lots of DM in voids, then $\Omega_{m,0}$ is higher.

Could be the case if galaxies don't form where large scale matter density is low.

This idea is known as "biased galaxy formation."

As evidence built in the 1980s and 1990s for $\Omega_{m,0} \approx 0.3$, this appeared to pose a challenge to inflation, which naturally predicts $\Omega_0 = 1$.

But the discovery of dark energy, with $\Omega_{\Lambda,0} = 1 - \Omega_{m,0}$, appears to resolve this challenge.

The "measure it and add it up" approach to estimating $\Omega_{m,0}$ is plagued by the possibility of biased galaxy formation — even if we think galaxies *approximately* trace dark matter, the errors of that approximation cause inaccuracy of the $\Omega_{m,0}$ estimate.

The most precise estimates of $\Omega_{m,0}$ today come from combining different cosmological measurements that depend on $\Omega_{m,0}$ in different ways.

The combination of supernova and CMB data that you used in Problem Set 7 is one example, and one of the most powerful combinations.

Other methods involve the detailed structure of the CMB power spectrum, the masses of galaxy clusters, or weak lensing by large scale structure.

While these methods are indirect and rely on theoretical assumptions, there are enough approaches to allow cross-checks of these assumptions.

A current best guess would be $\Omega_{m,0} = 0.25 \pm 0.05$.

What is it?

Baryonic dark matter

There are various ways to "package" baryons in non-luminous (or at least low luminosity) forms. These include

- "Jupiters" or sub-stellar objects, gravitationally bound
- Frozen hydrogen snowballs
- Gas clouds
- Stellar remnants: white dwarfs, neutron stars, black holes

Each of these has difficulties – e.g., how do they form in an amount much greater than the amount of stars, and how do they keep invisible?

There are three generic arguments against baryonic dark matter of any form:

1. With the measured anisotropy of the CMB ($\Delta T/T \sim 10^{-5}$), structure cannot grow to the level that we see today without non-baryonic dark matter.
2. Big bang nucleosynthesis and the CMB imply $\Omega_{b,0} \approx 0.02h^{-2} \approx 0.04$. But dynamical evidence implies $\Omega_{m,0} \approx 0.25$. There aren't enough baryons to be the dark matter.
3. Searches for gravitational microlensing of (several million) stars in the LMC rule out a Milky Way halo made of compact objects in the range $10^{-7}M_\odot - 10M_\odot$.

Massive neutrinos ("hot dark matter")

The first suggested candidate for non-baryonic dark matter (around 1980) was neutrinos, which we know exist and interact only via gravity and the weak interaction.

If the neutrino mass were 10 eV, neutrinos would be abundant enough to be the dark matter.

But they would move fast in the early universe because of their low mass, erasing structure on small scales.

In this "hot dark matter" scenario, the first structures to form would be large scale, flattened "pancakes," which would then fragment into galaxies.

However, this "top down" picture does not seem to be how structure in the universe really formed. We see quasars and galaxies (small scale objects) at high redshifts, but giant superclusters appear to be dynamically young.

The best guess based on present evidence is that neutrinos do have mass but that $\Omega_\nu \sim 10^{-3} - 10^{-2}$.

Cold dark matter

People soon suggested that the dark matter might be some other weakly interacting massive particle (WIMP), perhaps associated with supersymmetry, an extension of the standard model of particle physics.

A sufficiently massive WIMP ($mc^2 \gtrsim 5$ keV; best guess based on particle physics is that mc^2 is more like 10 GeV) would be "cold dark matter," moving too slowly in the early universe to erase small scale structure.

In this "bottom up" scenario, sub-galactic objects collapse first, then merge into bigger systems.

With weak interaction cross sections, these WIMPS could fairly naturally "freeze out" at the right abundance in the early universe, when the density dropped too low for them to annihilate with each other.

The elegant model with inflation, cold dark matter, and $\Omega_{m,0} = \Omega_0 = 1$ was popular in the 1980s and had significant successes, but by the mid 1990s it was running into many problems in matching the observations.

The addition of a cosmological constant gives a model that explains a remarkably wide range of data on the expansion of the universe and the history of cosmic structure.

Outlook

However, we still don't know what dark matter or dark energy is, or why they exist in amounts similar to each other and to the amount of baryons.

For dark matter, there are plenty of reasonable ideas.

There are reasonable prospects for discovering the nature of dark matter in the next 5-20 years, by one or more means:

- Particles with the right properties to be dark matter could be discovered directly in accelerator experiments, such as the LHC.
- "Direct detection" experiments in deep underground laboratories try to detect the rare interactions between passing dark matter particles and normal atoms, via ionization trails or vibrations. These are analogous to neutrino detection experiments, but targeted at the mass range of dark matter.
- "Indirect detection" experiments look for gamma-rays or other particles produced by WIMP-antiWIMP annihilation, in regions where the dark matter density is high (the Galactic center or the centers of nearby dwarf galaxies). This is one of the major objectives of the recently launched *Fermi* satellite.

For dark energy, there are plenty of *unreasonable* ideas, but none that are compelling.

Precise measurements of the cosmic expansion may tell us more about the properties of dark energy. However, because dark energy does not cluster with normal matter, it has a smaller range of observable effects, and it is harder to pin down its properties through experiment alone.

True understanding will probably require a theoretical breakthrough.