

Astronomy 682
Problem Set 1
Due Wednesday, April 4, in class

Question 1

The typical luminosity of a bright galaxy is $L \sim 10^{11} L_{\odot}$. The average space density of such galaxies is $n \sim 0.001$ galaxies/Mpc³. If the universe is 15 billion years old, what is the total flux F_{gal} received at earth from all of the external galaxies in the universe? (Ignore the contribution from stars in the Milky Way.)

Express your answer in $\text{erg s}^{-1} \text{cm}^{-2}$, remembering that the luminosity of the sun is $L_{\odot} = 3.9 \times 10^{33} \text{ erg s}^{-1}$ and $1 \text{ Mpc} = 3.1 \times 10^{24} \text{ cm} = 3.26$ million light years.

How does F_{gal} compare to the flux F_{\odot} received from the sun at earth (i.e., what is the ratio F_{\odot}/F_{gal})?

Ignore all of the obvious potential complications, such as redshifting of light and evolution of the galaxy population. Just do the “naive” calculation for a static, non-evolving universe. Don’t worry much about factors of two; aim for a simple calculation with order-of-magnitude accuracy.

Question 2

Assume that the light in these galaxies (the ones described in Question 1) is produced entirely by stars like the sun. This is a poor assumption in detail, but it gives roughly the right answer for the calculation in this problem.

What is the total mass M , in M_{\odot} ($= 2.0 \times 10^{33} \text{ g}$), contained in a sphere of radius 100 Mpc?

If the Hubble constant is $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, what is the recession speed v of an object at a distance of 100 Mpc?

For a test mass m at a distance $R = 100 \text{ Mpc}$, what is the ratio α of the kinetic energy $mv^2/2$ to the gravitational potential energy GMm/R , according to the numbers you just derived?

(It is not immediately obvious that you can ignore the gravitational effects of all matter *outside* the 100 Mpc sphere, but it turns out that you can, for reasons we will discuss in class.)

What is the value of α at $R = 200 \text{ Mpc}$? (If you think carefully about this part, it will be easy.)

Show that the general result is

$$\alpha = \frac{3H_0^2}{8\pi G\bar{\rho}},$$

where $\bar{\rho}$ is the average density of matter in the universe.

Based on your results, do you expect the expansion of the universe to continue forever, or do you expect gravity to eventually halt and reverse this expansion? Explain your reasoning.

(Final note: We have ignored two important components of the real universe, dark matter and dark energy, so the answer based on your calculation here will not necessarily be the same as the answer for the real universe.)