

Astronomy 682
Problem Set 2
Due Wednesday, April 11, in class

Question 1: Gravitational Frequency Shift

The lower figure on the attached page shows you standing in your “laboratory,” a steel box of height l (I’d call it h , but we need that for something else later in the problem). The laboratory is sitting on the earth, so you and everything in the laboratory feel a downward gravitational acceleration of magnitude $g = 10 \text{ m/s}^2$ (don’t use the numerical value in this problem, just call it g).

(a) If you drop a ball from rest at the top of the box, it hits the bottom after a time t , where

$$\frac{1}{2}gt^2 = l.$$

What is its kinetic energy $E_{\text{bot}} = \frac{1}{2}mv^2$ at the bottom of the box?

(b) Your friend is in a similar laboratory in space (upper figure), far enough away from the earth that the earth’s gravity is negligible. She has attached a laser to the ceiling of her laboratory, pointing down towards a light detector on the floor. The laser emits photons (i.e., light) that have frequency ν_{top} . The detector at the bottom can measure the frequency (and hence the color) of photons that it receives. If the laboratory were floating freely in space, then the frequency of the photons received at the bottom would be $\nu_{\text{bot}} = \nu_{\text{top}}$.

How long does it take photons to go from the top of the box to the bottom? (Give an equation for the time t in terms of l and c .)

(c) In fact, your friend’s laboratory is not floating freely, it is being towed by a stiff cable behind a rocket ship, which is accelerating upwards (in the frame of the diagram) with acceleration $a = 10 \text{ m/s}^2$. By the time the light reaches the bottom, therefore, the box is moving upwards with a speed $v = at$.

What is the frequency ν_{bot} of photons measured by a detector bolted to the floor at the bottom of the box? (Give a formula expressing ν_{bot} in terms of ν_{top} , a , l , and c .)

Are the photons at the bottom bluer or redder than those emitted at the top?

(d) Remember that the energy of a photon with frequency ν is $E = h\nu$ (where h is Planck’s constant). Show that the photons received at the bottom of the box have greater energy than those emitted at the top of the box by an amount

$$\text{energy gained} = E_{\text{top}} \times \frac{al}{c^2}.$$

(e) You install an identical laser and light detector in your laboratory on the earth. The laser again emits photons with frequency ν_{top} . Apply the equivalence principle discussed in class to answer the following questions:

Are the photons received at the bottom bluer, redder, or the same color as those emitted at the top?

Do photons gain energy, lose energy, or keep the same energy as they fall through the box?

What is the amount of energy that a photon gains or loses as it falls through the box? (If the energy doesn’t change, answer zero.)

Explain (briefly) the reasoning behind your answers.

(f) Compare your answers to part (e) and part (a). What does this comparison suggest?

Question 2: Bending of Light

Light passing near a star (or planet, or other massive body) will be bent by the star's gravity. You can calculate the amount of bending approximately (within a factor of 4 or so) using the same "steel box" example as in Question 1. The top diagram on the attached page will be useful for this purpose. It shows a light ray passing at a distance r from a star of mass M . We can make the approximation that the light ray is not affected by star's gravity when it is much further than r (because gravity falls off as $1/r^2$), and that the light ray changes direction while it passes through an imaginary "box" of size r whose center is a distance r away from the star. The dashed line shows the path that the light ray would follow if it were not deflected by the star's gravity.

(a) What is the gravitational acceleration g in the box? (Note that the gravitational acceleration is different at different locations in the box. You should give a typical value, such as the one that applies at the center of the box.)

(b) How long does it take the light ray to cross the box?

(c) What is the distance x by which the light ray has "dropped" by the time it reaches the far side of the box? (Figure this out using the equivalence principle; this is the key step of the problem.)

(d) What is the angle θ by which the light is bent away from its original straight path (see the figure for an illustration)? Your answer should be a formula that gives the value of θ in terms of M , r , and Newton's constant G . The formula does not have to be exact, since we have already made some approximations in setting up the solution. Be sure to specify the units that you are using for θ .

(e) Using your result from (d), calculate the angle by which light rays from distant stars are bent if they pass close to the surface of the sun (i.e., set $M = M_{\odot} = 2 \times 10^{33} \text{g}$, $r = R_{\odot} = 7 \times 10^{10} \text{cm}$, and use $G = 6.7 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2}$). Express your answer in *arc-minutes*, by remembering that there are $60 \times 180/\pi = 3438$ arc-minutes in a radian.

(f) Suppose that we look at a distant star, and that there is an intervening star exactly along our line of sight to the background star, halfway in between. This situation is illustrated in the bottom diagram on the attached page. Light rays are shown emanating from the background star. Based on your result from (d), draw on this diagram the continuation of these light rays. You should make the (somewhat unrealistic) assumption that the intervening star is massive enough to produce bending that is detectable on the scale of this drawing.

If you could observe this star with a perfect telescope and no blurring of the image by the atmosphere, what would you see?