Astronomy 682
Problem Set 5
Due Wednesday, May 9, in class

Question 1: A Lambda-Dominated Universe (40 points)

Consider a spatially flat ($k = 0$) universe in which the only energy component is a cosmological constant, with energy density $\epsilon_\Lambda = \epsilon_{c,0}$ that *does not change* as the universe expands.

(a) Show that the solution to the Friedmann equation in this case is

$$a(t) = a_0 e^{H_0(t-t_0)}$$

(b) Does this universe have a "big bang"?

(c) What is the age of the universe (in terms of H_0 and t_0) at redshift $z = 1$?

(d) What is the comoving distance to an object at $z = 1$?

(e) What is the physical separation of two objects separated by $d\Omega = 0.01$ radians at $z = 1$?

(f) Consider a photon emitted today (at t_0). What comoving distance r has it traveled to by time $t_f > t_0$?

(g) Suppose that this model is a good description of our universe (it can't be a perfect description, because it has no matter). If a supernova goes off in our galaxy today, will an observer in a galaxy that is presently 6000 Mpc away from us ever be able to see it?

Question 2

Consider a universe in which $a(t) = (t/t_0)^\alpha$, with $0 < \alpha < 1$.

(a) Consider a photon emitted at time $t = 0$. What comoving distance r has it traveled to by time t_0?

(b) Consider a photon emitted today (at time t_0). What comoving distance r has it traveled to by time $t_f > t_0$?

(c) What is a fundamental difference between your result above (2b) and your corresponding result for 1f?

Question 3

Carefully examine the diagrams on the following page, which plot the expansion factor $a(t)$ against time t. Which diagram corresponds to each of the cases below?

- Matter-dominated universe, $k = +1$
- Matter-dominated universe, $k = 0$
- Matter-dominated universe, $k = -1$
- Flat ($k = 0$) universe with equal energy contributions from matter and from a cosmological constant

(Here "matter-dominated" means that non-relativistic matter is the only form of energy in the universe.)

What is wrong with the two diagrams you didn't pick for any of the above?