5 Accretion Flows

Reading: Pringle 1981, Ann Rev Astron Astrophys 19, sections 1-4
Optional reading: Shu, chapter 7 ; Ryden chapters 8-10 (especially 9)
Introduction to advection-dominated accretion flows: Narayan et al., astro-ph/9803141

5.1 General Introduction

Accretion is important because
(a) it is a way for objects to grow
(b) it is a way for gravitational energy to be released

If accreting gas can dissipate energy so that it is not pressure supported, then it will settle into a disk, the minimum energy configuration for fixed angular momentum.

In order for gas to accrete, some of it must lose angular momentum and move inward. Since total angular momentum must be conserved, other gas must gain angular momentum and move outward.

Viscous dissipation, necessary to transport angular momentum, will also heat the gas. This heat can either be radiated away near where it is generated or "advected" inward with the accreting gas.

5.2 Equations for a thin accretion disk

The equations governing a thin accretion disk with surface density profile $\Sigma(R)$ can be obtained from the Navier-Stokes equations in cylindrical symmetry, integrating over the height of the disk. They are:

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{R}\frac{\partial}{\partial R}\left(R\Sigma V_R\right) = 0,$$

and

$$\frac{\partial}{\partial t}\left(\Sigma R^2 \Omega\right) + \frac{1}{R}\frac{\partial}{\partial R}\left(R \cdot \Sigma R^2 \Omega \cdot V_R\right) = \frac{1}{R}\frac{\partial}{\partial R}\left(R \cdot \Sigma R \cdot \nu R \frac{\partial \Omega}{\partial R}\right),$$

representing mass conservation and angular momentum conservation, respectively.

Here $V_R =$ radial velocity [cm s^{-1}]
$\Omega =$ angular velocity [rad s^{-1}] $\Longrightarrow$ $R\frac{\partial \Omega}{\partial R}$ =velocity shear
$\nu =$ kinematic viscosity [cm^2 s^{-1}].

The left hand side of the second equation is the convective derivative of the angular momentum per unit area. The product of ν and the velocity shear gives the torque of one annulus on the next, and it is the derivative of this torque that drives changes in angular momentum.

Recall that for "molecular viscosity" $\nu \sim v_T \lambda$, where v_T is the thermal velocity and λ is the mean free path.

For a point mass potential, $\Omega = \left(\frac{GM}{R^3}\right)^{1/2}$, these equations can be combined to yield

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left[R^{1/2}\frac{\partial}{\partial R}\left(\nu \Sigma R^{1/2}\right)\right],$$

a diffusion equation for the surface density.
A ring that is initially thin (in ΔR as well as Δz) spreads over time, with the inner part giving angular momentum to the outer part.

The timescale for this evolution is $t_{\rm acc} \sim R^2/\nu$, as one can roughly see from the diffusion equation by taking $\frac{\partial^2}{\partial R^2} \longrightarrow \frac{1}{R^2}$ and $t_{\rm acc}^{-1} = \frac{1}{\Sigma}\frac{\partial \Sigma}{\partial t}$.

For molecular viscosity, $\nu \sim v_T \lambda$, this timescale is usually orders of magnitude too long to account for the observed phenomena, leading to the "anomalous viscosity" problem: what physical mechanism provides the viscosity that drives the evolution of real astronomical accretion disks?

The vertical thickness H of the disk follows from hydrostatic equilibrium:

$$\frac{1}{\rho}\frac{\partial P}{\partial z} = \frac{\partial}{\partial z}\left[\frac{GM}{(R^2+z^2)^{1/2}}\right] \approx \frac{-GMz}{R^3} \qquad \text{for } z \ll R,$$

implying

$$-\frac{1}{\rho}\frac{a^2\rho}{H} \sim -\frac{GMH}{R^3} \quad \Longrightarrow \quad H \sim a\left(\frac{GM}{R^3}\right)^{-1/2} \sim a\Omega^{-1} \sim \frac{a}{V_\phi}R.$$

The disk is thin if and only if the sound speed is $\ll$ rotational speed, which is the condition for pressure gradients to be small compared to centrifugal forces.

5.3 Steady-state accretion

Important simplifications occur if we require a steady state, $\frac{\partial}{\partial t} = 0$.
Integrating the mass conservation equation yields

$$\dot{M} \equiv -2\pi R \Sigma V_R = \text{constant},$$

i.e., constant mass flow *at every radius*.
The angular momentum equation can be manipulated to yield, in the case of a Keplerian disk,

$$\nu\Sigma = \frac{\dot{M}}{3\pi}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right],$$

where R_* is the inner radius of the disk.
The rate of viscous dissipation per unit area is

$$D(R) = \nu\Sigma\left(R\frac{\partial \Omega}{\partial R}\right)^2 = \frac{3}{4\pi}\frac{GM}{R^3}\dot{M}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right].$$

(See Pringle or Ryden for details.)

Magically, the viscosity vanishes from this expression. This is an important advantage of steady-state accretion, since we don't know what the viscosity is. This result emerges because we have implicitly *assumed* that the viscosity will adjust itself to accommodate a constant mass accretion rate, and in this case many properties of the disk follow from the requirements of mass, angular momentum, and energy conservation alone.

The total disk luminosity is

$$L_{\rm disk} = \int_{R_*}^{\infty} D(R) 2\pi R dR = \frac{1}{2}\frac{GM\dot{M}}{R_*},$$

i.e., half the gravitational energy released in accreting the gas to radius R_*. The remaining gravitational energy goes into rotational energy, which may be either dissipated in a boundary layer or sucked into a black hole.

At large R, $D(R) \propto R^{-3}$, so if this energy is radiated away with emissivity $\sigma_{\rm SB}T^4$ then $T \propto R^{-3/4}$. Integrating the blackbody spectrum over radius gives the predicted spectrum of an optically thick, geometrically thin, steady-state accretion disk

$$S_\nu \propto \int_{R_{\rm in}}^{R_{\rm out}} B_\nu[T(R)] 2\pi R dR,$$

which rises steadily from a frequency $kT_{\rm out}/h$ to $kT_{\rm in}/h$, cutting off fairly sharply at higher frequency and gently at lower frequency.

5.4 Viscosity

Even in steady state, a complete solution of disk structure (e.g., surface density profile, scale-height profile, vertical structure) requires specification of the viscosity.

Shakura & Sunyaev (1973) introduced the "α-disk" parameterization

$$\nu = \alpha\, a\, H, \tag{39}$$

where a is the sound speed, H is the disk scale height (a function of radius), and α is a dimensionless constant.

This at least has the right units, and if one imagines that the viscosity arises from collisions of "clouds" in a turbulent medium moving in eddies of lengthscale $l \lesssim H$ at velocity $v \lesssim a$, then one expects $\nu \sim l \cdot v \sim \alpha a H$ with $\alpha \lesssim 1$.

Typical models of disks have $\alpha \sim 0.01 - 0.1$.

For specified α, one can completely solve for the structure of a (steady-state or time-dependent) accretion disk. However, the assumption that α is constant with radius, with time, or from one accretion disk to another is nothing more than an assumption.

While the notion of “turbulent viscosity” is intuitively appealing, detailed studies suggest that hydrodynamic mechanisms alone will not produce sustained turbulence in differentially rotating disks (see, e.g., Hawley, Balbus, & Winters, astro-ph/9811057).

In recent years, a clear “leading contender” has emerged as the viscosity mechanism in most astrophysical accretion disks: magneto-rotational instability. First proposed by Chandrasekhar, the case for this mechanism has been made mainly by Balbus & Hawley, with a combination of analytic and numerical work.

Rough idea: MHD instabilities in a differentially rotating, magnetized disk drive turbulence (tapping the rotational energy), which in turn produces viscosity.

Three-dimensional MHD simulations that follow magneto-rotational instability in realistic situations are just now becoming possible. They support the basic picture, but the details are not yet fully understood. Among other things, it is not yet clear how similar a disk with this viscosity mechanism is to a steady-state α-disk.

A different mechanism may be required in proto-stellar disks, which might have very small ionization fractions (and thus insufficient anchoring of magnetic fields to the disk).

5.5 Some open issues in accretion physics

For black hole accretion, the big questions are:

If gas falls onto a black hole of mass M at a rate $\dot{M}$,

What is the radiative efficiency $\epsilon \equiv L/\dot{M}c^2$, where L is the luminosity?
What is the spectral energy distribution of the emerging radiation?

Some of the relevant questions that must be addressed along the way are:

5.5.1 What is the inner boundary condition?

The energy radiated in reaching the innermost stable orbit around a Schwarzschild black hole corresponds to efficiency $\epsilon \sim 0.08$.
However, the gas at the innermost stable orbit still has a lot of rotational energy.
To calculate the overall radiative efficiency at the factor of two level, we need to know what happens to this energy.
The conventional assumption is that gas decouples after going inside the innermost stable orbit, so it takes its rotational energy into the black hole with it.
However, it is possible that gas inside $R_{\rm in}$ remains viscously or magnetically coupled to gas in the accretion disk, in which case the efficiency could be higher (since there is more gravitational energy to be extracted).

5.5.2 What generates the hard X-ray luminosity?

While a thin accretion disk can account for the bolometrically dominant portion of a typical quasar SED, the hard X-rays (and the radio emission, where present) must come from something else.

Typical models invoke a "corona" above the accretion disk that produces the hard X-rays.
However, models of the corona and the mechanism that produces it are only slightly above the cartoon level.

5.5.3 What happens when the accretion rate is far below Eddington?

The Eddington accretion rate is the accretion rate for which the black hole radiates at the Eddington luminosity, $\dot{M}_{\rm Edd} = L_{\rm Edd}/\epsilon c^2$.
It is generally thought that when the accretion rate is $\sim 0.01 - 1\dot{M}_{\rm Edd}$, thin disk accretion is a reasonable approximation.
With a high accretion rate, the gas density is high, so the gas is able to radiate efficiently and stay geometrically thin.

However, if the gas density is low, the gas may be unable to radiate energy at a rate that balances viscous heating.
In this case, the heat generated by viscosity will be "advected" inwards with the flow instead of being radiated. The disk becomes hot, hence geometrically thick (though perhaps optically thin), hence low density, and radiatively inefficient.
Such "Advection Dominated Accretion Flows" (ADAFs) were studied by Lightman & Eardley, Rees, and others in the 1970s. They were revived in the 1990s by the work of Narayan & Yi and others.

Relative to thin accretion disks, advection dominated flows have different structure (quasi-spherical, though angular momentum remains important), lower bolometric efficiency, and very different spectral energy distributions, with emission over a wider range of wavelengths.
ADAFs are also expected to have a 2-temperature structure, with hot ions holding most of the energy and transferring it inefficiently to the electrons that produce most of the radiation.

Narayan and collaborators have argued, based on a combination of theoretical models and observational studies of stellar mass black holes, that the inner regions of accretion disks are replaced by ADAFs when the accretion rate falls below $\sim 0.01\dot{M}_{\rm Edd}$.
In stellar mass black holes (X-ray binaries), which sometimes cycle between different states, the "high soft state" (high luminosity, soft spectrum) may correspond to thin disk accretion and the "low hard state" to something like an ADAF. AGN might go through similar kinds of cycles on a longer timescale.

5.5.4 What is the physics of radiatively inefficient flows?

A follow-on from the above.

In a true ADAF, most of the energy gained by the gas ultimately disappears down the black hole, being advected all the way into the event horizon.

Blandford & Begelman have argued that most of the gas may be driven out after accreting to small radius. They call this solution ADIOS (Advection Dominated Inflow-Outflow Solution). A small fraction of the gas actually accretes onto the black hole, providing the energy to drive the outflow.

Quataert & Gruzinov have argued instead that advection dominated flows are convectively unstable. They propose CDAFs (Convection Dominated Accretion Flows), in which a small fraction of the gas accretes onto the black hole, providing energy to drive convection that returns most of the gas to large radius (but not to infinity).
A CDAF cannot be indefinitely steady-state, since gas will build up at large radius.

Note that an ADAF is radiatively inefficient because most of the energy goes "down the hole," while ADIOS and CDAF flows are inefficient because the rate at which the black hole actually gains mass is much smaller than the "large scale $\dot{M}$."

Which, if any, of these solutions is the most physically realistic is a hotly debated theoretical and observational question.
On the theoretical side, one can ask what actually happens if one allows gas to accrete at a low rate onto a black hole (but a complete solution is tough).
On the observational side, one can ask which of the models makes predictions for SEDs and outflow rates that are in good agreement with observations.

5.5.5 What happens when the accretion rate is super-Eddington?

Presumably, a galaxy doesn't "know" that it shouldn't dump gas onto its central black hole above the Eddington rate. In understanding black hole growth and the quasar luminosity function, therefore, it is important to know what happens when the accretion rate is super-Eddington.

Note that the Eddington accretion rate is usually (though not always) defined for a radiative efficiency $\epsilon \sim 0.1$.

One possibility is that the gas accretes and radiates with high efficiency, and that the black hole therefore shines at a super-Eddington luminosity. This would be impossible in spherical symmetry and steady state, but Begelman has recently argued that luminosities $L \sim 10-100 L_{\rm Edd}$ are possible, with radiation escaping in "photon bubbles" that percolate out through the infalling medium.

A second possibility is that the gas accretes at a super-Eddington rate but radiates with low efficiency, so that the luminosity is $\leq L_{\rm Edd}$. This would be an advection dominated accretion flow, and there is indeed a class of such high accretion rate ADAF models. These are quite different from the low accretion rate ADAFs mentioned earlier – in these ADAFs, radiation pressure is dominant, and the radiation produced is trapped by the optically thick flow and pulled into the black hole with the gas. What is not clear is whether these high accretion rate ADAF solutions are stable.

A third possibility is that the black hole refuses to eat: if gas is dumped on at a super-Eddington rate, the resulting flow may drive most of the gas out, so that the actual accretion rate is sub-Eddington.

At present, any of these seems like a live possibility.

5.5.6 Can the accretion tap the spin energy of the black hole?

A spinning black hole has a lot of rotational energy. It may be possible for accreting gas to extract some of this energy, in addition to its own gravitational energy.
The leading model for such extraction is called the Blandford-Znajek mechanism, a messy (or, if it's to your taste, elegant) piece of relativistic MHD.
It is generally thought that this mechanism may be important in the production of relativistic jets, and that these represent the transformation of black hole spin energy into kinetic energy of the outflowing gas.