V. The Cosmic Microwave Background

My treatment of the CMB doesn't map neatly onto Huterer's book, but sections 13.1 and 13.2 cover much of the same ground in a useful way. Sections 9.1-9.3 of Ryden's book are also good. I am going to separate the discussion of CMB anisotropy from the discussion of the CMB as a homogeneous phenomenon.

Other readings are suggested below.

Evolution of blackbody radiation

Suppose that the universe at time t_1 is filled with blackbody radiation of temperature T_1, and that photons are not created or destroyed, just redshifted by the cosmic expansion.

A volume V_1 contains

$$dN_1 = V_1 \frac{8\pi\nu_1^2 d\nu_1}{c^3} \frac{1}{e^{h\nu_1/kT_1} - 1}$$

photons in the frequency range $\nu_1 \longrightarrow \nu_1 + d\nu_1$.

At time t_2 these photons occupy the frequency range

$$\nu_2 \longrightarrow \nu_2 + d\nu_2, \quad \nu_2 = r\nu_1, \quad d\nu_2 = r d\nu_1, \quad r \equiv a(t_1)/a(t_2).$$

[NB: wavelengths get bigger, frequencies get smaller.]

These photons are contained in a metric volume $V_2 = V_1/r^3$ (this won't actually contain the same photons, but we appeal to homogeneity to say that the ones that leave the volume are replaced by ones with an identical distribution of properties). Thus

$$\begin{aligned} dN_2 = dN_1 &= \frac{V_1}{r^3} \frac{8\pi r^3 \nu_1^2 d\nu_1}{c^3} \frac{1}{e^{hr\nu_1/krT_1} - 1} \\ &= V_2 \frac{8\pi\nu_2^2 d\nu_2}{c^3} \frac{1}{e^{h\nu_2/kT_2} - 1}, \end{aligned}$$

where $T_2 = rT_1$. Since this argument applies to any frequency interval, we see that the radiation spectrum at time t_2 is just that of a blackbody with temperature $T_2 = T_1 a(t_1)/a(t_2)$.

Since $T \propto 1/a$ and $\rho_r = a_r T^4$ (where a_r is the radiation energy constant), this is another way of showing that

$$\rho_r = \rho_{r,0}(a_0/a)^4.$$

However, the energy scaling does not require that the radiation have a blackbody spectrum.

Implications:

- If the universe was once opaque to radiation and in thermal equilibrium (likely if density was high in the past), it should still contain radiation with a blackbody spectrum, unless processes injected or removed photons or altered photon energies (differently from cosmic expansion redshift) since that time.

- If the radiation density is non-zero today, then at sufficiently early times, $(1 + z) = a_0/a(t) > \rho_{m,0}/\rho_{r,0}$, it was the dominant energy component.
- The early universe was hot!

Discovery of the CMB

Good recaps of the remarkable history of the discovery of the CMB appear in Peebles, *Principles of Physical Cosmology*, pp. 139-151, and in S. Weinberg, *The First Three Minutes*, chapter 1.

In brief, the idea of a hot early universe was motivated by the goal of explaining the origin of chemical elements. In the early universe, age and temperature are uniquely related by the Friedmann equation, and for a given density of matter, one can calculate what abundances come out. Requiring the right abundances determines the matter density at that time, and comparison to today's matter density yields the redshift, hence the present day temperature. However, the primordial nucleosynthesis problem has subtleties that were not appreciated in the early papers by Gamow and collaborators in the 1940s.

Although the rediscovery of this idea in the 1960s led a group at Princeton to search for the CMB, it was actually discovered "by accident" by Penzias and Wilson, as part of their characterization of a new microwave antenna at Bell Labs. "By accident" is in scare quotes, because it was only the high sensitivity of the detector that Penzias and Wilson built and the extreme care they took in their experimental procedures that allowed them to make the discovery.

Some of the key papers:

Gamow (1946, Phys Rev): Equilibrium abundances of nuclei don't match observations. Must have built up out of equilibrium. Plausible if formed in early universe because expansion rate would have been high. Assumes matter domination. Also assumes that *all* elements are produced in hot early universe.

Gamow (1948, Phys Rev): Temperature when deuterium formed $\sim 10^9$ K. Universe then radiation dominated. Deuterium formed at t ~few minutes. Requiring that ~half particles go into deuterium $\Longrightarrow vtn\sigma \sim 1$, gives n when $T = 10^9$ K. Galaxy formation cannot start until matter domination, but early universe was *radiation dominated.*

Alpher, Bethe, Gamow (1948, Phys Rev): Results of calculations based on previous ideas.

Gamow (1948, Nature): Essentially a recap of previous 3.

Alpher & Herman (1948, Nature): Corrections to previous. State that present radiation temperature should be ~ 5 K. No mention that it might be observable. While their value is impressively close to the true value, I think this is largely a coincidence of cancelling errors; the assumption of neutron-dominated initial conditions that underlies Gamow's calculation is not accurate enough to give an accurate value for the temperature.

Russians (Zel'dovich, Doroshkevich, Novikov, early '60s): Estimate expected temperature from helium abundance. Realize Bell Labs telescope can constrain. Misinterpret Bell technical report as implying $T < 1$ K, which seems too low.

Hoyle & Tayler (1964, Nature): Helium abundance in sun and other places where it is measurable is $\sim$ 10% by number ($\sim$ 25% by mass). Can't be made in situ, or in same stars that make heavy elements. Essentially correct version of primordial helium calculation, incorporating key improvements that came along subsequent to Gamow papers of 1940s: weak interactions determine ratio of neutrons to protons, not a pure neutron initial condition. Idea is to make helium in early universe, not all elements. Their calculation slightly overpredicts solar helium abundance, so they also consider idea of making helium in radiation-pressure dominated, supermassive stars.

Dicke, Peebles, Roll, & Wilkinson (1965, ApJ): Realize oscillating or singular universe might have thermal background. Build detector to search. Then they hear about the discovery of . . .

Penzias & Wilson (1965, ApJ): Holmdel antenna has isotropic excess noise of 3.5 ± 1.0 K. Careful experiment. Explanation could be that of Dicke et al.

Roll & Wilkinson (1965, Phys Rev Letters): Detect background at 3.2 cm, with amplitude consistent with Penzias & Wilson for *blackbody* spectrum. Isotropic to 10%.

Woolf & Field (1966): Interpret previously unexplained excitation of interstellar CN, known since 1941, as caused by CMB.

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COBE (Cosmic Background Explorer, 1990): The cosmic microwave background has a blackbody spectrum of temperature $T = 2.728 \pm 0.004$ K (Fixsen et al. 1996, ApJ, 473, 576). No spectral distortions were detected by COBE (the Cosmic Background Explorer satellite). Departures from a perfect blackbody are no more than 5×10^{-5} of peak intensity (noise-weighted rms). 1992: Anisotropies $\sim 10^{-5}$, except for dipole reflex of our peculiar motion, $\sim 10^{-3}$.

At redshift z, the temperature of the photon background is

$$T = 2.73 \times (1+z) \text{ K}, \qquad kT = 2.39 \times 10^{-4} \times (1+z) \text{ eV}.$$

The baryon-to-photon ratio

The CMB temperature determines the number density of CMB photons, $n_\gamma = 413$ photons cm^{-3}.

The baryon-to-photon ratio is

$$n_B/n_\gamma = 2.68 \times 10^{-8}\Omega_B h^2 = 5.9 \times 10^{-10}\left(\frac{\Omega_B h^2}{0.022}\right),$$

where $\Omega_B = \rho_B/\rho_c$ and $h \equiv H_0/100$ km s^{-1} Mpc^{-1}.

In round numbers, there are a billion CMB photons for every baryon.

"Re"combination

When did CMB photons last interact with matter?

When $kT \gg 13.6$ eV, expect hydrogen to be ionized, by photons and by collisions.

Naively, expect atoms to form when $kT \sim 13.6$ eV.

Actually occurs when $kT \sim 0.3$ eV because photon-to-baryon ratio is very high $\Longrightarrow$high-energy tail of Planck distribution can ionize.

A thermal equilibrium calculation implies that the ionization fraction should fall to 0.5 at $T \sim 3700$K, $z \sim 1360$, but there are several complications:

> Recombinations to ground state produce ionizing photon $\Longrightarrow$net recombination must go through excited state.
>
> Other recombinations produce photons that put other hydrogen atoms in excited states, making them easy to ionize.
>
> Expansion rate is not negligible $\Longrightarrow$mildly out of equilibrium, and resonance photons produced in recombination redshift out of resonance later on.

Detailed calculations show that the fraction of ionized hydrogen drops rapidly from 1 to $\sim 10^{-5}$ at $z \approx 1100$, $T \approx 3000$K.

A good discussion is in Peebles, pp. 165-175, roughly following the seminal paper of Peebles 1968 (ApJ, 153, 1). The state of the art calculation is Ali-Haimoud & Hirata (2011, Phys Rev D, 83, 04350) updated by Lee & Ali-Haimoud (2020, PRD, 102, 083517).

The last scattering surface

For density at $z = 1100$, photon mean free path to Thomson scattering is $\lambda_f \ll ct$ if the free electron fraction $x_e \approx 1$.

$\Longrightarrow$

Before recombination, universe is opaque.

At recombination, photon mean free path increases dramatically, universe becomes transparent. Photons travel freely, redshifting in cosmic expansion.

In "standard recombination" the probability of last scattering peaks at $z \approx 1100$, with width $\Delta z \approx 80$.

But the weakness of Lyman-alpha absorption in quasar spectra $\Longrightarrow$most hydrogen in the universe was reionized by $z = 5$.

If reionization occurred early, optical depth could be high enough that a large fraction of CMB photons rescattered of free electrons at lower redshift. Optical depth is $\tau = 1$ if universe is reionized at $z \approx 50$.

Constraints from the Planck experiment imply $\tau = 0.054 \pm 0.007$.

Hence, a map of the CMB is basically a map of the $z = 1100$ "last scattering surface," with only 5% of CMB photons rescattered.

Explaining the central value $\tau = 0.054$ requires reionization at $z \approx 8$ if it is an instantaneous transition.

Reionization is more likely an extended process, ending between $z = 5$ and 6.

Understanding the sources of ionizing photons and the topology of reionization (e.g., inside-out vs. outside-in) is a major topic of high-z galaxy and AGN research and a goal of future redshifted 21cm (radio HI) experiments.

Significance of thermal spectrum

Blackbody spectrum $\Longrightarrow$universe once in thermal equilibrium, as predicted by big bang theory.

Energy density of CMB is ~ 1 eV/cm^{-3}, comparable to that of starlight *in* the Milky Way. Much greater than energy density in intergalactic space.

Difficult but not inconceivable to create background of observed energy density from young stars, dust reprocessing.

But seems impossible to create perfect thermal spectrum in this way. Superposition of blackbodies is not a blackbody.

No viable explanation for the CMB other than the big bang (more specifically, a hot, dense early universe) has been proposed.

What redshifts does the CMB probe?

Generic term for $e^- + \gamma \longrightarrow e^- + \gamma$ is Compton scattering.

But energy exchange only occurs to $\mathcal{O}(v_e^2/c^2) \sim \mathcal{O}(kT_e/m_e c^2)$.

In limit $v_e^2/c^2 \ll 1$, process is usually called Thomson scattering. Changes photon direction, but not photon energy.

Since last scattering occurs at $z = 1100$ (for standard recombination), seems like CMB only tests big bang model back to this redshift.

But energy injected into background can only be thermalized if Compton scattering is able to redistribute energies.

For $\Omega_B h^2 = 0.01$, this is only possible at $z > z_c \sim 10^5$.

Energy injected after that time would distort the blackbody spectrum.

Compton scattering does not change photon number, so even at $z > z_c$ Compton scattering can only relax a distorted spectrum to a Bose-Einstein form (with non-zero chemical potential μ), *not* a Planck distribution.

In the cosmological context, the most effective number changing processes are:

Bremmstrahlung: $e^- + X \longrightarrow e^- + X + \gamma$ ($X =$ an ion)

Double Compton scattering: $e^- + \gamma \longrightarrow e^- + \gamma + \gamma$.

For $\Omega_B h^2 = 0.01$, these processes become efficient at

$$z > z_{\rm Br} \sim 10^7 \quad \text{and} \quad z > z_{\rm dc} \sim 3 \times 10^6,$$

respectively.

Bottom line: The absence of spectral distortions in the CMB strongly constrains processes affecting the radiation background back to $z \sim 10^7$!

For discussion, see Hu & Silk 1993 (Phys Rev D, 48, 485).