VII. The Early Universe

This section goes through some but not all of the topics covered in chapters 4-6 of Huterer, especially chapter 6. If you want to go deeper on these topics, Kolb & Turner's *The Early Universe* is the go-to reference; the material covered here is mostly in their section 3.5, for which 3.3 and 3.4 are useful background.

Interaction rates

A particle experiences interactions at a rate

$$\Gamma = n\sigma v$$

where σ is the cross-section for the interaction, n is the number density of other particles available for interaction, and v is the characteristic relative velocity.

The units of Γ are inverse time, e.g., s^{-1}.

If $\Gamma > H$, and thus $\Gamma^{-1} < H^{-1}$, reactions are "fast" compared to the expansion of the universe.

When reactions are fast in this sense, particle number densities and energy distributions can often reach equilibrium.

Particles and anti-particles in the early universe

If the rates of reactions that exchange energy between particles are fast compared to the expansion rate, the populations should relax to thermal equilibrium.

Consider a particle of mass m_K. If $kT > 2m_K$ and the rate of reactions that can create $K\overline{K}$ pairs from other particles (e.g., photons) are fast compared to the expansion rate, maximizing entropy $\Longrightarrow$an abundance of K and $\overline{K}$ particles roughly equal to the photon abundance (up to statistical weight factors).

If the temperature falls below $2m_K$ while the $K\overline{K}$ annihilation rate is high, the particles will annihilate and dump their energy into the background of still-coupled particles. This happens to electrons and positrons.

If the $K\overline{K}$ annihilation rate drops below the expansion rate while $kT > 2m_K$, the particle decouples and redshifts thereafter independent of other species. This happens to neutrinos, and it may happen to other weakly interacting particles. For these species, the abundance should be comparable to the abundance of CMB photons.

"Natural" units

For early universe problems, it is often convenient to adopt "high energy physics" (a.k.a. "natural") units in which $\hbar = c = k_B = 1$ ($k_B =$Boltzmann's constant) and the fundamental dimension is energy (see Appendix A of Huterer for discussion). A traditional and convenient unit of energy is 1 GeV$=10^9$ eV, and in the high energy system of units:

$$1\ \mathrm{GeV} \ = \ 1.16\times10^{13}\ \mathrm{K} \ = \ 1.78\times10^{-24}\ \mathrm{g} \ = \ (1.97\times10^{-14}\ \mathrm{cm})^{-1} \ = \ (6.58\times10^{-25}\ \mathrm{s})^{-1}.$$

Newton's gravitational constant enters into calculations via the Planck mass,

$$m_{Pl} \equiv (\hbar c/G)^{1/2} = G^{-1/2} = 2.18 \times 10^{-5} \text{ g} = 1.22 \times 10^{19} \text{ GeV}.$$

Number and energy densities

A particle species of rest mass m will be relativistic when $kT \gg mc^2$.

Huterer (sections 4.2.1 and 4.2.2) gives expressions for the number density and energy density of relativistic particles, which are slightly different for bosons (particles with integeer spin) and fermions (particles with half-integer spin) because of their different statistics.

Here I list the expressions for zero chemical potential μ, which is often but not always an adequate assumption.

For bosons:

$$n = \frac{\zeta(3)}{\pi^2} g \left(\frac{k_B T}{\hbar c}\right)^3 = 0.122 g T^3 \tag{7.1}$$

where $\zeta(3) \approx 1.202$, and

$$u = \rho c^2 = \frac{\pi^2}{30} \frac{k_B^4}{(\hbar c)^3} g T^4 = \frac{\pi^2}{30} g T^4, \tag{7.2}$$

where g is the number of statistical degrees of freedom. For photons there are two spin states, so $g = 2$, and the above formula is equivalent to the usual $u = a_B T^4$.

For fermions, the right-hand-sides of equations (7.1) and (7.2) are multiplied by factors of 3/4 and 7/8, respectively.

If the energy density of the universe is dominated by relativistic particles and T_γ is the temperature of the photons, the total energy density is

$$u = \frac{\pi^2}{30} g_* T_\gamma^4, \tag{7.3}$$

where

$$g_* \equiv \sum_{i=\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{i=\text{fermions}} g_i \left(\frac{T_i}{T}\right)^4, \tag{7.4}$$

the sums are over all species of particles relativistic at temperature T_i, and we have allowed for the possibility that each species i is characterized by a different temperature T_i.

(When in doubt, take T to be the temperature of the photons T_γ.)

Huterer Fig. 4.1 shows the value of g_* as a function of t and T (beware a sign-flip typo on the t axis).

For example, at $T \approx 1 - 10$MeV, the boson species is photons with $g = 2$ and the fermion species are electrons and positrons each with $g = 2$ and three species of neutrinos and three species of anti-neutrinos, all with $g = 1$ because in the standard model relativistic neutrinos are only created as left-handed.

This gives

$$g_*(T \approx 1 - 10\text{MeV}) = 2 + \frac{7}{8}(2 \times 2 + 6) = 10.75 \ ,$$

just before electron-positron annihilation.

Combining equation (7.3) with the Friedmann equation gives

$$H = 1.66 g_*^{1/2} \frac{{T_\gamma}^2}{m_{\text{Pl}}} \tag{7.5}$$

during the radiation-dominated era.

A particle species will be non-relativistic when $kT \ll mc^2$.

The expressions for a non-relativistic particle species are (Huterer 4.23, 4.24, but I am ignoring chemical potential μ)

$$n = g \left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} e^{-mc^2/kT} = g \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T} \tag{7.6}$$

and

$$u = \rho c^2 = (mn)c^2 \ . \tag{7.7}$$

Temperature evolution

Huterer section 3.4 uses entropy conservation to show that the quantity

$$g_{*S}(T) T^3 a^3 = \text{const.} \tag{7.8}$$

throughout the expansion history of the universe.

Here

$$g_{*S} \equiv \sum_{i=\text{bosons}} g_i \left(\frac{T_i}{T_\gamma}\right)^3 + \frac{7}{8} \sum_{i=\text{fermions}} g_i \left(\frac{T_i}{T_\gamma}\right)^3 , \tag{7.9}$$

is almost the same quantity we defined in equation (7.4) but has $(T_i/T)^3$ instead of $(T_i/T)^4$ because it tracks number density instead of energy density. In standard cosmology, g_* and g_{*S} only differ (slightly, 3.94 vs. 3.38) after neutrinos decouple at $t \approx 1$s.

If the number of relativistic particle species is not changing, then $T \propto a^{-1}$.

However, when particle annihilation changes the number of species in equilibrium, the temperature evolution briefly slows down because of the change in g_{*S}.

This effect causes the photon temperature to rise above the neutrino temperature when electrons and positrons annihilate, because the neutrinos are already decoupled and follow $T_\nu \propto a^{-1}$ while the photon temperature drops more slowly.

After electron-positron annihilation, $T_\nu = (4/11)^{1/3} T_\gamma$.

WIMP dark matter

In one of the leading scenarios for dark matter, WIMPS (Weakly Interacting Massive Particles) are first suppressed in number as the temperature falls below their rest mass, then decouple once the typical annihilation time (determined by the weak interaction rate) exceeds the age of the universe.

The dark matter would then consist of equal numbers of WIMPS and anti-WIMPS.

For reasonable assumptions about the interaction cross-section and its dependence on mass, the predicted density of dark matter is about right (at the order of magnitude level), making this an attractive model.

In the densest regions of the universe today (at the centers of galaxies), WIMPS and anti-WIMPS might annihilate and produce gamma rays (and other particles) at a detectable rate.

Searching for this signature was one of the key motivations for the Fermi satellite.

Although there are gamma rays from the center of the Milky Way, it is hard to decide whether these are entirely explained by astrophysical sources (e.g., pulsars) or include a contribution from WIMP-annihilation.

The centers of nearby dwarf galaxies are a “cleaner” place to look, but the predicted signal is weaker.

There is not yet a convincing detection of WIMP-annihilation, though there are plenty of claims.

Baryon asymmetry

The local universe has baryons but almost no anti-baryons.

This asymmetry is small in the sense that the baryon-to-photon ratio is $n_B/n_\gamma = 2.68 \times 10^{-8}\Omega_B h^2$, where $\Omega_B = \rho_B/\rho_c$ and $h \equiv H_0/100$ km s^{-1} Mpc^{-1}. Since the particle density in the relativistic era was $\sim n_\gamma$, *almost* all relativistic baryons were annihilated by anti-baryons.

The baryon asymmetry (and corresponding e^+/e^- asymmetry) may be an initial condition of the universe.

Alternatively, the baryon asymmetry may have arisen after the Big Bang. Three conditions are necessary for this to be possible (Sakharov 1968):

(1) Processes exist that do not conserve baryon number (otherwise every B would be produced with an accompanying $\overline{B}$.

(2) There is an asymmetry between reactions involving particles and reactions involving the corresponding anti-particles (technical jargon: CP violation). Such asymmetry is observed

experimentally in weak interactions (K_0 mesons), and it is believed to occur in strong interactions.

This makes it possible to statistically favor particle production over antiparticle production.

(3) The universe must be out of thermal equilibrium, otherwise maximizing entropy would lead to equal numbers of particles and anti-particles. Departures from equilibrium can happen because the universe is expanding.

A number of specific models of "baryogenesis" have been proposed, none entirely compelling. Some place baryogenesis at the strong-electroweak ("GUT") symmetry breaking, others at electroweak symmetry breaking. Since the level of CP violation observed in the weak interaction is $\sim 10^{-9}$, it is not unreasonable to imagine creating a baryon asymmetry of the observed order.

Inflation (short version)

In the standard big bang model, the flatness and large scale uniformity of the universe are just accepted as initial conditions, not explained.

Inflation is an extension of the big bang model that attempts to give a causal explanation for the origin of flatness and homogeneity, much as the big bang model itself gives a causal explanation for the primordial helium abundance.

The basic idea and first concrete model of inflation was proposed in 1980, by Alan Guth, and quickly followed up by many others.

In the inflation scenario, the early universe went through an accelerating phase in which it was dominated by vacuum energy with an extremely high energy density.

During this phase, the universe expanded exponentially in time, with an e-folding timescale

$$t_{\rm exp} = t_H = \frac{1}{H} = \left(\frac{8\pi G\epsilon_{\rm vac}}{3c^2}\right)^{-1/2}.$$

If the universe expanded by at least a factor of e^{60} during inflation, then the entire volume of the presently observable universe was within one causally connected patch *before* inflation started, so causal processes could have established the homogeneity of the universe.

During inflation, $a(t)$ grew by a very large factor at constant ϵ, making the curvature radius very large and growing the energy term relative to the curvature term. After inflation, the universe is extremely flat.

Eventually, inflation ended, and the enormous energy that had been stored in $\epsilon_{\rm vac}$ was converted to photons and other particles, producing the very large number of particles within the curvature radius.

A natural though not completely unavoidable prediction of inflation is that Ω_0 should be extremely close to 1.0.

The great success of inflation came from the realization that quantum fluctuations of the energy field driving inflation would leave behind real fluctuations in energy density after inflation ended.

The predicted statistical properties of these fluctuations agree well with observations of large scale structure and CMB fluctuations, though the observed magnitude of $\sim 10^{-5}$ is not naturally explained.

The Kolb & Turner History of the Universe

Quantum gravity era ends — $t \sim 10^{-43}$ s $\sim (G\hbar/c^5)^{1/2}$, the "Planck time." Inflation?

Grand unification breaks? — $t \sim 10^{-34}$ s, $kT \sim M_{X,Y}$. Baryogenesis? Inflation?

Electroweak unification breaks — $t \sim 10^{-11}$ s, $kT \sim M_{W,Z}$. Baryogenesis?

Quarks combine into hadrons — $t \sim 10^{-6}$ s, $kT \sim$nucleon binding energy

Neutrinos decouple — $t \sim 1$ s $\sim (\sigma_\nu nc)^{-1}$

Electrons and positrons annihilate — $t \sim$ few s, $kT \sim 1$ MeV $\sim 2m_e$. Adds heat (entropy) to radiation background. Residual e^- keep universe opaque.

Light nuclei form — $t \sim 1$ minute, $kT \sim 0.1$ MeV $\sim B_{\rm deuterium}/20$.

Matter domination begins — $t \sim 10^3(\Omega_{m,0}h^2)^{-2}$ years, $(1+z) = \rho_{m,0}/\rho_{r,0} = 2.39 \times 10^4(\Omega_{m,0}h^2)$. Expansion changes from $a \propto t^{1/2}$ to $a \propto t^{2/3}$. Growth of instabilities possible.

Atoms form, photons decouple — $t \sim 10^5$ years, $(1+z) \sim 1100$, $kT \sim 0.3$ eV ~ 13.6 eV/45.

Stars, galaxies form — Later.

Λ-domination begins (?) — $(1+z) \sim (\Omega_{\Lambda,0}/\Omega_{m,0})^{-1/3}$. Expansion law changes again, towards $a \propto e^{Ht}$. Growth of instabilities slows down. We are in the middle of this transition now.

If dark energy is not a true cosmological constant, the transition may be slower, and the end state may be power-law rather than exponential expansion.