IX. Large Scale Structure at Low Redshift

This topic is covered in §§9.5-9.7 of Huterer.

Tracers of large scale structure

Galaxies – angular clustering, 3-d clustering in redshift surveys

Clusters of galaxies – detectable in hot gas as well as galaxies themselves

Redshift-space distortion (RSD) – anisotropy of galaxy clustering in redshift space gives a statistical measure of peculiar velocities

Direct peculiar velocity measurements – less influential, but still used

Lyman-alpha forest – $z = 2 - 5$

Weak gravitational lensing – directly responsive to mass distribution

Redshifted 21cm emission or absorption – not yet useful, but could become important

Two-point statistics

To measure the galaxy power spectrum, conceptually. Define the galaxy overdensity field

$$\delta_g(\mathbf{x}) = \frac{n_g(\mathbf{x}) - \bar{n}_g}{\bar{n}_g}$$

Fourier transform and compute $\langle \delta_k^2 \rangle$.

There are many details to doing this well; Feldman, Kaiser, & Peacock (1994) is a foundational reference.

In configuration space (as opposed to Fourier space), one can define the two-point correlation function (a.k.a. auto-correlation function)

$$\xi_{gg}(\mathbf{r}) = \langle \delta_g(\mathbf{x}) \delta_g(\mathbf{x} + \mathbf{r}) \rangle \tag{9.1}$$

Note that ξ is dimensionless.

If there were no RSD, we would expect ξ_{gg} to depend on $r = |\mathbf{r}|$ alone.

Because of RSD, it depends on r and $\mu = \cos \hat{\mathbf{z}} \cdot \hat{\mathbf{r}}$, the cosine of the angle between the galaxy separation vector and the line-of-sight.

Alternatively, one can compute and tabulate ξ as a function of transverse and line-of-sight separations.

If one has only angular positions, one can compute the analogous angular correlation function, usually written $w(\theta)$.

Although it looks different, it is equivalent to define ξ_{gg} in terms of the probability of finding a galaxy 2 in a volume element dV_2 separated by r_{12} from a randomly selected galaxy 1:

$$dP(2|1) = n_g[1 + \xi(r_{12})]dV_2 \tag{9.2}$$

If $\xi = 0$, so galaxies are unclustered, the probability is just $n_g dV_2$ as expected.

With this definition, one can see that a way to estimate the correlation function is through pair counting:

$$\xi(r) = \frac{\#\text{pairs observed}}{\#\text{pairs random}} - 1 \ .$$

Again, there is artistry in how to do this (see Huterer §9.5.7).

Famously, the observed correlation function for "typical" galaxies at low redshift is well approximated by a power-law

$$\xi(r) = (r/r_0)^{-1.8}$$

for $0.1h^{-1}\,\text{Mpc} < r < 10h^{-1}\,\text{Mpc}$.

The correlation length depends somewhat on the type of galaxies selected but is typically about $5h^{-1}\,\text{Mpc}$.

The correlation function is the Fourier transform of the power spectrum (Huterer §9.5.2).

Initial conditions: Isentropic vs. Isocurvature

Different early universe physics can produce qualitatively different kinds of primordial fluctuations.

If fluctuations are present equally in all species (e.g., radiation, dark matter, baryons, neutrinos) then there is no perturbation to the entropy.

These are usually referred to as adiabatic fluctuations, though isentropic is a more accurate term.

There are fluctuations in the energy density, so fluctuations in the spacetime curvature.

Alternatively, fluctuations in the energy density of one species can be compensated by fluctuations in the energy density of another species, e.g., matter vs. photons.

In this case, there is initially no perturbation to the energy density, hence no perturbation to the curvature.

Energy density fluctuations can develop on scales smaller than the horizon as the energy densities of different species evolves differently.

Figure 9.3 of Huterer illustrates these two forms of fluctuations.

Observations, especially CMB anisotropies, imply that primordial fluctuations are mostly or purely isentropic.

Quantum fluctuations during inflation are a natural mechanism for producing isentropic (a.k.a. adiabatic, a.k.a. curvature) fluctuations.

Matter-radiation equality: A characteristic scale

CMB observations imply $\Omega_m h^2 \approx 0.143$.

The combination of the observed CMB temperature for photons and standard neutrino physics implies $\Omega_r h^2 = 4.15 \times 10^{-5}$.

(I took these numbers from Huterer table 3.1.)

The matter and radiation densities were equal at

$$1 + z_{\rm eq} = \frac{\Omega_m}{\Omega_r} \approx 3400 \ . \tag{9.3}$$

At this redshift, the growth rate of fluctuations changed as the universe went from radiation-dominated to matter-dominated.

This (slow) transition imprints a characteristic scale on the power spectrum of fluctuations at comoving wavelength

$$\lambda_{\rm eq} \sim c t_{\rm eq}(1 + z_{\rm eq}) \sim \frac{c}{H(z_{\rm eq})}(1 + z_{\rm eq}) = \frac{c}{H(z_{\rm eq})} \frac{\Omega_m}{\Omega_r} \ .$$

We can evaluate this using the Friedmann equation

$$\frac{3H^2}{8\pi G} = \rho_m(z) + \rho_r(z) = \left[\Omega_m(1+z)^3 + \Omega_r(1+z)^4\right] \frac{3H_0^2}{8\pi G}$$

At $z_{\rm eq}$ the two terms in [...] are equal so we can write

$$H(z_{\rm eq}) = H_0 \left[2\Omega_m(\Omega_m/\Omega_r)^3\right]^{1/2} = H_0 \left[2\Omega_m^2/\Omega_r\right]^{1/2} (\Omega_m/\Omega_r) \ .$$

Putting these results together,

$$\lambda_{\rm eq} = c H_0^{-1} \left[2\Omega_m^2/\Omega_r\right]^{-1/2} \approx 95 h^{-1}\,{\rm Mpc} \ .$$

The Inflation + Cold Dark Matter Power Spectrum

The scale-invariant power spectrum

Scale-invariant fluctuations "enter the horizon" with a constant fluctuation amplitude δ_H.

This is approximately what inflation predicts, though the idea of scale-invariant fluctuations already appears in (separate) papers by Harrison, Peebles, and Zeldovich around 1970.

Consider fluctuations that enter the Hubble radius during the matter-dominated era, when $D_1(a) \propto a$.

Scale-invariance implies

$$4\pi k^3 P(k, t_c) \approx \delta_H^2 \approx \text{ const.}, \tag{9.4}$$

where t_c is the time when the comoving scale $\lambda \sim 1/k$ "crosses" (is equal to) the Hubble radius.

The condition for crossing the Hubble radius is:

$$a(t_c)\lambda \sim ct_c \implies a(t_c)/k \propto [a(t_c)]^{3/2} \implies [a(t_c)]^{1/2} \propto 1/k,$$

where we have used $a \propto t^{2/3}$ for a matter dominated universe.

Since $P(k,t) \propto D_1^2(t) \propto a^2$, we find

$$P(k,t) = P(k,t_c)\frac{a^2(t)}{a^2(t_c)} = \frac{\delta_H^2}{4\pi k^3}\frac{a^2(t)}{a^2(t_c)} \propto \frac{\delta^2 H}{k^3}\frac{a^2(t)}{1/k^4} \propto \delta_H^2 a^2 k.$$

Bottom line: scale-invariant fluctuations produce $P(k,t) \propto k$ on scales that reenter the Hubble radius during the matter dominated era.

The transfer function

But comoving scales smaller than $\lambda_{\rm eq}$ enter the Hubble radius when the universe is radiation dominated.

These fluctuations grow slower than $a(t)$ until the universe becomes matter dominated.

For very small scales, which reenter when the universe is strongly radiation dominated, we get essentially no growth after reentering the Hubble radius.

For wavelengths in this regime, at *fixed time*

$$4\pi k^3 P(k,t) \approx 4\pi k^3 P(k,t_c) \approx \text{const.} \implies P(k,t) \propto k^{-3}$$

(in practice, there is logarithmic growth, see Huterer eq. 9.39, hence an additional factor of $\log k$.)

The transition between scales that enter in the radiation dominated regime and scales that enter in the matter dominated regime is a slow one, so the change from $P(k) \propto k$ to $P(k) \propto k^{-3}$ is gradual.

The linear theory inflation+CDM power spectrum is written

$$P(k,t) \propto k^n T^2(k) D_1^2(t), \tag{9.5}$$

where $T(k)$ is the *transfer function.*

Here CDM stands for "cold dark matter," and it entered our calculation implicitly through the assumption that fluctuations that enter the horizon after matter domination can grow with $\delta(t) \propto a(t)$, unaffected by pressure.

Perfectly scale-invariant fluctuations correspond to $n = 1$, but typical inflationary models predict a small "tilt" with $0.9 < n < 1$, the exact value depending on the potential of the field that drives inflation.

$T(k)$ describes the growth of fluctuations after they re-enter the horizon. It depends on the energy density of components with different equations of state, ($\Omega_{\rm CDM}h^2$, $\Omega_b h^2$, $\Omega_\Lambda h^2$, $\Omega_r h^2$, $\Omega_\nu h^2$, ...), but it applies regardless of what spectrum of fluctuations comes out of inflation (or any other mechanism that produces primeval fluctuations).

$T(k)$ does depend on whether fluctuations at horizon crossing are adiabatic (matter and radiation fluctuate together) or isocurvature (opposite matter and radiation fluctuations), since this determines the interplay between gravity and pressure.

From our above analysis, we see that $T^2(k) = $const. at large scales and $T^2(k) \propto k^{-4}$ at small scales.

The transition where $P(k)$ turns over is about $0.02h\,{\rm Mpc}^{-1}$ (see Huterer Fig. 9.6), which roughly corresponds to our earlier calculation of $\lambda_{\rm eq} \approx 95h^{-1}\,{\rm Mpc}$.

Accurate calculations of the transfer function can be made with public codes such as `CAMB` and `CLASS`.

Useful analytic approximations exist, of which the most famous are the ones of Bardeen, Bond, Kaiser, & Szalazy (1985), which does not include baryonic effects or massive neutrinos, and the more complex but more accurate formulation of Eisenstein & Hu (1999).

Primordial fluctuations could be much more complicated than this description, but in practice it seems to describe observations quite well.

Baryon acoustic oscillations (BAO): Another characteristic scale

Before recombination, baryons are tightly coupled to photons. The baryon-photon fluid has very high pressure (sound speed $\sim c/\sqrt{3}$).

One can think about this phenomenon in the Fourier domain or the real space domain. Here we'll describe the real space view.

An overdensity that starts as a δ-function at the origin launches a spherical pressure wave that propagates through the photon-baryon fluid at the speed of sound.

The comoving distance this pressure wave propagates by the epoch of recombination is called the acoustic scale,

$$r_s = \int_0^{t_{\rm rec}} \frac{c_s(t)}{a(t)} dt = \int_{z_{\rm rec}}^{\infty} \frac{c_s(z)}{H(z)} dz \ , \tag{9.6}$$

where the sound speed is

$$c_s(z) = \frac{c}{\sqrt{3}} \left[1 + \frac{3}{4}\frac{\rho_b(z)}{\rho_\gamma(z)}\right]^{-1/2} \quad .$$

When recombination occurs, the baryons decouple and the sound speed drops to effectively zero.

An overdensity at the origin (in dark matter and baryons) is accompanied by an overdensity in baryons at distance r_s.

Subsequently, this baryon overdensity attracts dark matter.

BAO imprint a bump in the linear theory matter correlation function at a scale of $r_s \approx 150$ Mpc. The exact value can be calculate if one knows the values of $\Omega_m h^2$ and $\Omega_b h^2$ (with a small dependence on $\Omega_\nu h^2$).

The BAO scale can be measured in the galaxy or Lyman-α forest correlation function and used as a standard ruler to infer $D_A(z)$ (from the transverse scale) or $c/H(z)$ (from the line-of-sight scale).

The Fourier transform of an off-center δ-function is a sine wave, so the bump in the correlation function corresponds to the damped oscillations in its Fourier transform, the power spectrum.

Physically, pressure causes a Fourier mode to oscillate as a sound wave instead of growing steadily, imprinting fluctuations on the transfer function $T(k)$ and thus on $P(k)$.

If there were no dark matter, these oscillations in $P(k)$ and the bump in $\xi(r)$ would have a large amplitude.

But because BAO only affect the baryons, the amplitude is reduced by a factor $\sim \Omega_b/\Omega_m$.

BAO were first detected in the clustering of galaxies in the Sloan Digital Sky Survey (SDSS) and 2-degree Field Redshift Survey (2dF), reported by Eisenstein, Zehavi, Hogg et al. (2005) and Cole, Percival, Peacock et al. (2005).

Cosmological measurements using BAO as a standard ruler were a primary goal of the Baryon Oscillation Spectroscopic Survey (BOSS) of SDSS-III and eBOSS of SDSS-IV, and now of the Dark Energy Spectroscopic Instrument (DESI).

For deeper introductions of BAO I recommend section IV of Weinberg et al. (2013) and all of Aubourg et al. (2015).

The cutoff scale: hot, warm, and cold dark matter

If the dark matter is mildly relativistic at the time that a scale reenters the Hubble radius, then it will stream preferentially out of overdense regions and into underdense regions, ironing out fluctuations on that scale.

If the dark matter consisted of neutrinos with rest mass $\sim 10 - 30$ eV, then they would be mildly relativistic at $t_{\rm eq}$ (essentially by definition, since their number densities are similar to those of photons), and fluctuations on scales smaller than $ct_{\rm eq}$ would be strongly suppressed, radically changing $T(k)$.

This gives a "hot dark matter" power spectrum. A pure hot dark matter model fails because the absence of small scale fluctuations means that small objects are unable to form at high redshift.

If the dark matter particle has a mass ~ 1 keV, then primordial fluctuations are suppressed on sub-galactic scales but not on galactic scales.

Galaxies can still form fairly early, but they are not preceded by generations of smaller collapsed structures.

This "warm" dark matter model offers some attractive features. But it's squeezed by constraints from the Lyman-alpha forest and dwarf galaxies.

However, the "cold" dark matter model, in which particles are massive enough that fluctuations are not erased on any cosmologically interesting scale, seems more natural, because the particle mass does not have to be fine-tuned and lies more in the range expected from particle physics models.

If the dark matter is a WIMP that decoupled in thermal equilibrium, then there is still a cutoff scale associated with the relic thermal velocities. For a WIMP mass of 100 GeV, the cutoff means that the first dark matter halos to collapse have roughly an earth mass.