Precision Cosmology With Large Scale Structure
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Lecture 2: Theoretical Approaches

Dark Matter Clustering

If collisionless dark matter were the only component of the universe, its evolution could be modeled using gravity alone.

Baryons also produce gravity, and their evolution is affected by hydrodynamics, star formation, feedback.

Thus, these processes also have some impact on dark matter clustering.

However, on scales larger than ~ 1 Mpc, it is usually a good approximation to follow gravitational dark matter clustering, "paint" the baryons on top.

Importance of baryonic effects on total mass distribution and dark matter clustering itself depends on scale and desired level of accuracy.

Linear perturbation theory

In Eulerian linear perturbation theory, density fluctuations just "grow in place" in proportion to the linear growth factor.

Equations (from Lecture 1)

$$\delta(\mathbf{x},t) \;\equiv\; \frac{\rho_m(\mathbf{x},t)-\bar{\rho}_m(t)}{\bar{\rho}_m(t)} \;=\; \delta(\mathbf{x},t_i)\times\frac{G(t)}{G(t_i)}$$

where

$$\ddot{G}_{\rm GR} + 2H(z)\dot{G}_{\rm GR} - \frac{3}{2}\Omega_m H_0^2 (1+z)^3 G_{\rm GR} = 0$$

and

$$f_{\rm GR}(z) \equiv \frac{d\ln G_{\rm GR}}{d\ln a} \approx [\Omega_m(z)]^\gamma$$

implying

$$\frac{G_{\rm GR}(z)}{G_{\rm GR}(z=0)} \approx \exp\left[-\int_0^z \frac{dz'}{1+z'}[\Omega_m(z')]^\gamma\right] \;.$$

One can also use linear perturbation theory to compute the displacements and peculiar velocities of particles given the linear density field.

$$\Delta\mathbf{x}(\mathbf{q},t) \;\equiv\; \mathbf{x}(\mathbf{q},t)-\mathbf{q} \;=\; \mathbf{g}_0(\mathbf{q})\frac{G(t)}{G(t_0)}\left(\frac{3}{2}\Omega_{m,0}H_0^2 a_0\right)^{-1}$$

$$\mathbf{v} \;=\; a(\dot{\Delta\mathbf{x}}) \;=\; \frac{\dot{G}}{G}(a\Delta\mathbf{x}) \;\approx\; [\Omega_m(z)]^\gamma H(a\Delta\mathbf{x})$$

$G(t)$ = growing mode growth rate of linear perturbations

$\mathbf{q}$ = comoving position in an unperturbed universe

$\mathbf{g}_0$ = gravitational acceleration at $t = t_0$

Instead of calculating the gravitational accelerations, it can be easier to just think in terms of the continuity equation, which in linear theory gives

$$\vec{\nabla} \cdot \Delta \mathbf{x}(\mathbf{q}, t) = -\delta(\mathbf{q}) \ .$$

Lagrangian linear perturbation theory, a.k.a. the Zeldovich approximation, is surprisingly accurate, provided the linear density field is smoothed over a scale comparable to the scale of non-linearity (where $\sigma \approx 1$).

In the Zeldovich approximation, particles follow straight paths in comoving coordinates, in the direction of their gravitational acceleration, and their peculiar velocities are proportional to their Lagrangian displacements.

Higher order perturbation theory

I will skip this topic because it will be treated more expertly and in considerable detail in the Zaldarriaga lectures.

Basically, one can expand the equations of motion and keep terms of higher than linear order in δ and $\mathbf{v}$.

Both Eulerian higher order perturbation theory and Lagrangian higher order perturbation theory can be useful for some purposes.

Higher order perturbation theory can be used to compute non-linear corrections to the power spectrum or redshift-space distortions.

For Gaussian initial conditions, N-point correlations or Nth reduced moments of the density field vanish at order $N-2$ perturbation theory.

For example, 3-point correlations, the bispectrum, and skewness are all zero in linear perturbation theory, but they are non-zero at second order, with $\langle \delta^3 \rangle \sim \langle \delta^2 \rangle^2$.

N-body simulations

Cosmological N-body simulations can be thought of as a Monte Carlo solution to the collisionless Boltzmann equation + Poisson equation in an expanding universe.

Initial conditions: create Gaussian random field with desired linear power spectrum. Perturb particles from a grid or "glass" using the Zeldovich approximation or 2nd-order Lagrangian perturbation theory.

In comoving coordinates and Newtonian approximation, equations of motion are:

$$\begin{aligned}
&\dot{\mathbf{x}} = \mathbf{u} \\
&\dot{\mathbf{u}} + 2\frac{\dot{a}}{a}\mathbf{u} = -\frac{\nabla\phi}{a} \\
&\nabla^2\phi = 4\pi G a^2 \overline{\rho}\delta(\mathbf{x}).
\end{aligned}$$

$a(t)$ is determined by integrating the Friedmann equation.

The Poisson equation can be expressed in terms of Fourier components:

$$\nabla^2 \int d^3k\, e^{2\pi i\mathbf{k}\cdot\mathbf{x}}\, \phi_{\mathbf{k}} = (4\pi G a^2 \overline{\rho}) \int d^3k\, e^{2\pi i\mathbf{k}\cdot\mathbf{x}}\, \delta_{\mathbf{k}}$$

$$\Longrightarrow \quad -4\pi^2 k^2 \int d^3k\, e^{2\pi i\mathbf{k}\cdot\mathbf{x}}\, \phi_{\mathbf{k}} = (4\pi G a^2 \overline{\rho}) \int d^3k\, e^{2\pi i\mathbf{k}\cdot\mathbf{x}}\, \delta_{\mathbf{k}}$$

$$\Longrightarrow \quad \phi_{\mathbf{k}} = -(4\pi G a^2 \overline{\rho}) \frac{\delta_{\mathbf{k}}}{4\pi^2 k^2}.$$

N-body codes compute $\phi(\mathbf{x})$ by direct summation over particles or particle groups, by FFT using the result above, or by a combination. Combinations of Tree+FFT algorithms can be made very efficient.

Interparticle forces are softened on small scales to suppress 2-body relaxation and allow larger timesteps.

Particle positions and velocities are integrated forward in time, typically with a leapfrog algorithm.

High resolution simulations, attempting to resolve the inner regions of halos, may use individual timesteps – shorter timesteps for particles in denser regions – to increase efficiency.

N-body simulations, some key results

For an initial power spectrum with a sharp small scale cutoff, first non-linear structures are sheets and filaments.

For a CDM-like power spectrum, or a power-law spectrum with $-1 \gtrsim n > -3$, halos form along similar filamentary structures, beads on a string.

Largest halos form at intersections of filaments, fed by filamentary flow.

Small scale structures form by non-linear collapse of larger scale perturbations. Information flows from large scales to small scales, but small scale details have little back reaction on large scales.

Halos are roughly in virial equilibrium inside r_{200}, radius within which mean interior overdensity is $\approx 200\bar{\rho}$, but best choice of halo "boundary" remains a matter of debate.

Halos form with NFW-like profiles (Navarro, Frenk, & White 1996, 1997), $\rho(r) = \rho_0/[(r/r_s)(1 + r/r_s)^2]$ bending from r^{-1} at small radii to r^{-3} at large radii. (Detailed inner form still somewhat debated, affected by baryons.)

Concentration r_{200}/r_s increases with decreasing halo mass.

Halo mass function has roughly Press-Schechter-ish form, a power law with an exponential-of-power-law cutoff at characteristic scale where $\sigma(M_*) = \delta_c$ and $\delta_c \approx 1.68$ is the linear density contrast at which a spherical perturbation would collapse.

More accurate analytic arguments and numerically calibrated forms for dn/dM are now available.

More massive halos are more strongly biased. Bias is roughly unity at $M \approx M_*$.

At masses well below M_*, halo bias has significant dependence on formation time: oldest halos are more strongly clustered. Effect is weaker, and maybe opposite sign, above M_*.

The dependence of halo bias on formation time, or more generally on properties other than mass, is referred to as assembly bias.

Self-similar clustering, for power-law initial conditions and an $\Omega_m = 1$ universe, is a useful source of intuition and test of numerical accuracy. CDM on M_* scales corresponds roughly to $n \approx -1$ to -1.5.

Non-linear clustering is surprisingly insensitive to Ω_m, Λ, w, if one considers the same $P(k)$ and uses $G(t)$ as the time variable (see Z. Zheng et al. 2002, ApJ 575, 617).

These quantities primarily affect matter clustering through their influence on the $P(k)$ shape and amplitude, the history of $G(t)$, and the amplitude of peculiar velocities ($\propto [\Omega_m(z)]^\gamma$).

Halo model of dark matter clustering

Reading: A. Cooray & R. Sheth 2002, Phys Rep 372, 1

A useful conceptual and practical model of non-linear matter clustering is:

• all matter resides in halos, distributed according to mass function

• distribution of matter within halos is NFW-like, with numerically calibrated mass-concentration relation

• power spectrum of halos of a given mass is $b^2(M)$ times the linear matter power spectrum

Allows matter power spectrum to be calculated as sum of one-halo and two-halo terms.

Variations in the details, including linear vs. perturbative matter power spectrum, scale-dependent bias, treatment of halo exclusion.

Can be generalized to clustering of gas, given model for gas distribution within halos.

Closely related to halo occupation distribution (HOD) modeling discussed below, but I use the term "halo model" specifically to refer to an approximate method of computing non-linear matter clustering, and HOD to refer to a model for the relation between galaxies and mass.

A Sketch of Galaxy Formation Theory

Reading: S. Cole et al. 1994, MNRAS 271, 781; A. Benson 2010, Phys Rep 495, 33

Galaxies are made of stars, which formed from gas that dissipated its energy and condensed within dark matter potential wells.

For the most part, gas and dark matter collapse together and accrete onto halos together. Corrections due to pressure support, especially for halos with circular velocity below $\sim 30\,\mathrm{km\,s^{-1}}$.

Gas could be pressure supported in DM halo if it were at the virial temperature $kT_{\rm vir}/m_p \sim GM_h/r_{\rm vir} \sim v_c^2$.

Gas within the cooling radius, where $t_{\rm cool} \lesssim t_{\rm dyn}$, can radiate its energy and sink to the center of the halo.

Makes a disk, size determined by angular momentum conservation. $r_d \sim \lambda r_{vir}$ where $\lambda \approx j/(v_c r_{\rm vir})$.

N-body simulations show $\lambda \sim 0.03 - 0.07$ for dark matter, but the specific angular momentum of baryons that form the disk may be different.

In halos with $M \lesssim 10^{11.5} M_\odot$, hydrodynamic simulations and analytic arguments suggest that there is little or no hot gas halo, and most gas accretes cold, with $T \ll T_{\rm vir}$, often along filamentary streams.

More massive halos have pressure supported hot gas halos, which may be penetrated by cold streams.

Ellipticals could form from rapid, chaotic collapse or from dissipative mergers of disks.

Galaxy formation is extremely inefficient. Only 5-10% of baryons are in the stellar components of galaxies. Even maximally efficient halos ($M \sim 10^{12} M_\odot$) have only 20-30% of the halo baryons in stars.

Current lore:

• For galaxies with $M_h \lesssim 10^{12} M_\odot$, stellar feedback ejects gas from the disk, as much or more as forms into stars. Increasingly efficient outflow driving at lower halo mass.

• For more massive galaxies, which are typically no longer forming stars, stellar feedback is insufficient. AGN feedback from accreting black holes may be what prevents gas from cooling in massive halos.

This picture accounts for a lot, but the feedback physics is still only sketchily understood.

Modeling Galaxy Clustering

Hydrodynamic simulations

Reading: D. Weinberg et al. 2004, ApJ 601, 1; D. Weinberg et al. 2008, ApJ 678, 6

In principle, the "right" way to model galaxy clustering is to carry out hydrodynamic simulations that incorporate all of the physics of galaxy formation within an evolving DM distribution.

Hydro simulation methods will be discussed in Borgani's lecture.

The biggest difficulty with this approach is that one cannot simulate very large volumes while maintaining the resolution needed to model the formation of galaxies.

In addition, this approach is subject to numerical systematics — i.e., to incorrectly computing the result of the assumed input physics.

Even if the computations are correct, there are significant uncertainties in the galaxy formation physics itself, especially with regard to stellar and AGN feedback and their interaction with accretion.

Despite the two latter problems, this is probably the most reliable way to compute the small scale clustering of stellar mass-thresholded samples of galaxies. For color or morphology selection, the uncertainties of galaxy formation physics become more important because of their impact on galaxy properties.

Populating DM halos via semi-analytic models

Reading: Kauffmann, Nusser, & Steinmetz 1997, MNRAS 286, 795; A. Benson et al. 2000, MNRAS 311, 793; V. Springel et al. 2005, Nature 435, 629 (Millenium Simulation)

An alternative to running hydro simulations is to run DM-only N-body simulations and populate the halos with galaxies using some recipe.

The most physical of these approaches uses semi-analytic calculations of the cooling of gas, star formation, feedback, etc., implementing the physical processes sketched above.

Early versions of this approach used Monte Carlo merger trees, generated with the extended Press-Schechter formalism, to assign a merger history to each dark matter subhalo in the final output of the simulation.

As simulations achieved higher resolution, these merger trees could be constructed directly from the N-body simulations themselves.

Strengths: Incorporates a complete, albeit approximate, model of galaxy formation physics, tuned to reproduce observed galaxy properties. Can impose any desired galaxy selection.

Limitations: Uncertainties in galaxy formation physics may affect predictions. Subhalos may be artificially disrupted in the inner regions of large halos, either because of numerical effects or because influence of condensed baryons is not included. Need to correct for this in some way.

Abundance matching and age matching

Reading: Conroy, Wechsler, Kravtsov 2006, ApJ 647, 201; V. Simha et al. 2012, MNRAS 423, 3458; Hearin & Watson 2013, MNRAS 435, 1313; A. Hearin et al. 2014, MNRAS 444, 729

To a decent approximation, hydro simulations and semi-analytic models predict that more massive galaxies live in more massive halos.

Assume this is so, and put galaxies in halos of N-body simulation with luminosity assigned to match the cumulative number density of observed galaxies above that luminosity: $n_h(> M) = n_g(> L)$.

Subhalos, hosting galaxies that are satellites of larger galaxies, are a bit tricky. Works best to assign galaxy luminosity based on the mass that the subhalo has *at the time it is accreted* onto the larger halo, since the mass decreases after that because of tidal stripping.

Get interestingly different answers using halo mass or maximum circular velocity to assign galaxy luminosities.

Can extend to predict galaxy color by monotonically mapping observed distribution of color (redder = older) at fixed luminosity onto distribution of halo formation times at fixed halo mass.

Easily extended to incorporate some scatter between halo properties and luminosity or color.

Observed luminosity function and color distribution are matched by construction, but clustering is predicted.

For a specified cosmological model, there are no free parameters (unless scatter is incorporated), though there are choices (e.g., mass or circular velocity, what measure of formation time).

Strengths: Simple, and straightforward to describe. Given few or no free parameters, describes observations astonishingly well.

Limitations: Makes strong assumptions about the effect of galaxy formation physics. Same subhalo resolution issues that affect semi-analytic population approach.

Both semi-analytic population and abundance matching are very good for illustrating what *could* happen, generating plausible galaxy distributions to test methods.

Semi-analytic population method can give a sense of how the cosmological results depend on uncertainties in the galaxy formation physics.

Halo occupation distribution (HOD) modeling

1. Basic philosophy

Reading: R. Scoccimarro et al. 2001, ApJ 546, 20; Berlind & Weinberg 2002, ApJ 575, 587; Zheng & Weinberg 2007, ApJ 659, 1

N-body simulations can predict the population of DM halos with little uncertainty due to baryonic physics.

Treat assignment of galaxies to halos as a statistical problem, solved by fitting observed galaxy space density and clustering.

For specified cosmology, derived HOD teaches us about galaxy formation physics.

Marginalizing over parameters describing the HOD is a way to remove sensitivity to galaxy formation physics in deriving cosmological parameters.

2. Parameterization

Reading: A. Kravtsov et al. 2004, ApJ 609, 35; Z. Zheng et al. 2005, ApJ 633, 791

Key element is $P(N|M)$, probability that a halo of virial mass M contains N galaxies of a specified class.

Also need to specify the spatial and velocity distribution of galaxies within halos. The first only matters for small scale clustering; the second only matters for redshift-space measurements.

Each class of galaxies (defined by, e.g., luminosity or stellar mass range, color, morphology) has its own HOD, which depends on redshift.

For a class of galaxies defined by a luminosity or stellar mass threshold, a useful parameterization that accurately describes theoretical predictions is:

• A softened step function for $\langle N_{\rm cen}(M)\rangle$ for central galaxies, corresponding to Gaussian scatter of $\ln M_*$ at fixed $\ln M_h$. Every halo contains zero or one central galaxies.

• A power-law $\langle N_{\rm sat}(M)\rangle = (M/M_1)^\alpha$ for satellite galaxies, cut off at minimum mass for central galaxies. Distribution of $P(N|N_{\rm sat})$ is Poisson.

• Central galaxies located at halo center-of-mass (or potential minimum). Satellite galaxies have NFW profile, perhaps with different concentration than dark matter.

• Central galaxy has center-of-mass velocity $b_{v,\rm cen}\sigma_{\rm DM}$ where $\sigma_{\rm DM}$ is the dark matter velocity dispersion. Natural limit is $b_{v,\rm cen} = 0$.

• Satellite galaxies have velocity dispersion $b_{v,\rm sat}\sigma_{\rm DM}$. Expect $b_{v,\rm sat} \approx 1$.

Parameters: $M_{\rm min}$ and $\sigma_{\ln M}$ for central galaxies, M_1, α, and $M_{\rm cut}$ for satellites, concentration of satellites in halo, two velocity bias parameters.

For color or morphology selected samples, parameter values may be different, and one can no longer assume that all sufficiently massive halos have a central galaxy.

For luminosity ranges instead of luminosity thresholds, $\langle N_{\rm cen}(M)\rangle$ may be a log-normal instead of a softened step function.

HOD can be generalized to the "conditional luminosity function" describing a continuous relation between galaxy properties and halo mass.

3. Observational results

Reading: I. Zehavi et al. 2005 (ApJ 630, 1), 2011 (ApJ 736, 59)

HOD models applied to ΛCDM cosmology are impressively successful at fitting observed luminosity and color dependence of clustering in the Sloan Digital Sky Survey (SDSS).

Inflection in 2-point correlation functions reflects transition from 1-halo regime (both galaxies in same halo) to 2-halo regime (galaxies in separate halos).

Luminosity dependence largely reflects increase of $M_{\rm min}$ for central galaxies and M_1 for satellites with increasing luminosity threshold.

Color dependence largely reflects greater satellite fraction of red galaxies (lower M_1, higher α).

The ratio $M_1/M_{\rm min}$ is typically $10-20$. While the halo mass of the Milky Way is $\sim 10^{12}M_\odot$, you need a halo mass of $\sim 1.5\times 10^{13}M_\odot$ before you are likely to get two galaxies more luminous than the Milky Way.

Fluctuations about $\langle N\rangle$ are significantly sub-Poisson for $\langle N\rangle < 2$, which greatly reduces the number of small scale pairs compared to Poisson $P(N|\langle N\rangle)$. This is a crucial ingredient in explaining observed galaxy correlation functions.

4. Assembly bias

Reading: Gao, Springel, & White 2005, MNRAS 363, L66; A. Zentner et al. 2014, MNRAS 443, 3044;

In its standard form, HOD modeling assumes that the galaxy content of a halo of fixed mass is independent of the halo's large scale environment.

To the extent that this assumption holds, the HOD allows a complete description of the relation between galaxies and mass, from large scales into the fully non-linear regime.

(Caveat, a given parameterization of the HOD may or may not be sufficiently flexible to describe this relation.)

But the clustering of halos of fixed mass depends on halo formation history – e.g., formation time – at least in some regimes.

If galaxy properties are correlated with halo formation history – e.g., redder galaxies in halos that form earlier – then galaxies can inherit "assembly bias" from their host halos.

In this case, $P(N|M)$ varies with large scale environment, and one must include this variation to correctly predict galaxy clustering.

One can include environmental parameters in the HOD, e.g. through a dependence of $M_{\rm min}$ and M_1 on the overdensity in the halo environment.

Hydrodynamic simulations predict little galaxy assembly bias (i.e., they predict an environment-independent HOD) because of stochastic relation betwen galaxy properties and halo formation history.

However, abundance matching models tied to halo circular velocity, and age matching models for galaxy color, do predict galaxy assembly bias.

Understanding how important galaxy assembly bias is, and how to incorporate it into modeling of galaxy clustering, is a frontier of the field.

Galaxy-galaxy lensing and cluster-galaxy lensing

Reading: R. Mandelbaum et al. 2006 (MNRAS 368, 715), 2013 (MNRAS 432, 1544); J. Yoo et al. 2006 (ApJ 652, 26); Zu & Mandelbaum 2015, arXiv:1505.02781

When we think of weak lensing, we typically think of cosmic shear, the correlated ellipticities of galaxies as a function of separation.

Cosmic shear directly probes dark matter clustering; in particular, the amplitude of the cosmic shear power spectrum is proportional to the amplitude of the total matter power spectrum, with a redshift weighting that depends on the source redshift distribution.

One can also measure the mean tangential shear of background galaxies around foreground galaxies or clusters (i.e., the stretch of the background galaxies perpendicular to the line of sight of the foreground galaxies).

This is called galaxy-galaxy lensing (GGL, or cluster-galaxy lensing for clusters).

The GGL signal is proportional to the product of $\Omega_m \xi_{gm}$, where ξ_{gm} is the galaxy-matter cross-correlation function.

This method can be used to probe the dark matter properties of galaxies of different stellar mass, luminosity, or color.

By combining GGL with measured galaxy clustering, one can constrain the amplitude of matter clustering.

In linear theory, we expect

$$\xi_{gg} = b_g^2 \xi_{mm}, \qquad \Omega_m \xi_{gm} = \Omega_m b_g \xi_{mm}.$$

Dividing GGL measurement by $\sqrt{\xi_{gg}}$ cancels the galaxy bias yielding $\Omega_m \xi_{mm}^{1/2} \propto \sigma_8 \Omega_m$.

In practice one does a joint fit to the observables, but the above relation captures the concept.

In the non-linear regime, one can use HOD modeling to describe both ξ_{gg} and ξ_{mm}, marginalizing over HOD parameters to constrain Ω_m and ξ_{mm}.

Galaxy assembly bias seems to pose a risk to this program, as the HOD description may be insufficient to capture all the relevant effects of non-linear galaxy bias.

However, our preliminary experiments on abundance matching catalogs (J. McEwen & D. Weinberg, in prep.) suggest that even in the presence of assembly bias, the value of

$$r_{gm} \equiv \frac{\xi_{gm}}{\sqrt{\xi_{gg} \xi_{mm}}}$$

is accurately predicted (at the 2% level) by an HOD model with parameters fit to the observed ξ_{gg}.

One can therefore infer ξ_{mm} from observations of ξ_{gg} and ξ_{gm}.

These kinds of investigations are an area of active research, with great promise in the era of the Dark Energy Survey, LSST, Euclid, and WFIRST.

One can do analogous things for cluster-galaxy lensing, where the theoretical complications are probably smaller because of the expected correlation between cluster properties and halo mass.

Theory of redshift-space distortions (RSD)

I am mostly skipping this topic for lack of time.

However, I'll write down the linear theory formula (Kaiser 1987). the galaxy power spectrum in redshift space depends on $\mu = \cos\theta$ where θ is the angle between the wavevector and the line of sight:

$$P_g(k,\mu) = b_g^2 (1 + \beta\mu^2)^2 P_m(k) = \left[b_g + \mu^2 f(z)\right]^2 P_m(k)$$

where $f(z) \approx [\Omega_m(z)]^\gamma$ is the fluctuation growth rate and $\beta = f(z)/b_g$.

With measurements of $P_g(k,\mu)$, or its Fourier transform $\xi(s,\mu)$, one can constrain the parameter combination $\sigma_8 f(z)$ even without knowing b_g.

Unfortunately, linear theory for RSD is not a good approximation on any scale where RSD is precisely measured. Small scale dispersions in non-linear structures are a particular nuisance.

A variety of approaches are being tried to get the most out of RSD measurements down to at least mildly non-linear scales.

The smaller scale one can get to, the higher the S/N of the measurements, but the harder the predictions are and the more accurate they have to be.

Numerical predictions using HODs are one potential way forward.

The Lyα forest

Basic phenomenology and physics

Reading: D. Weinberg et al. 1999, astro-ph/9810142; M. Peeples et al. 2010, MNRAS 404, 1281

The Lyα forest is a pattern of fluctuating absorption observed in the spectra of high redshift quasars, caused by absorption by intervening concentrations of neutral hydrogen.

The phenomenon was recognized observationally in the 1970s, but the underlying physics was not well understood until 3-d cosmological hydro simulations in the mid-1990s.

While it took sophisticated simulations to recognize it, the basic physics can be described in approximate terms rather simply.

The IGM is highly photoionized, so the neutral hydrogen density is

$$n_{\rm HI} \propto \frac{n_H^2 \alpha(T)}{\Gamma_{\rm HI}}$$

where n_H is the total hydrogen density, the recombination coefficient is $\alpha(T) \propto T^{-0.7}$ in the relevant temperature range ($T \approx 10^4$ K), and $\Gamma_{\rm HI}$ is the photoionization rate due to the background of UV photons produced by quasars and star-forming galaxies.

For densities near the mean density at $z = 2.5$, $n_{\rm HI}/n_H \approx 10^{-5}$, and $1/\Gamma_{\rm HI} \sim 3 \times 10^4$ years.

The interplay between photoionization heating and adiabatic cooling (because of the expansion of the universe) produces a tight relation between gas temperature and density through most of the volume, $T \propto \rho^\alpha$ with $\alpha = 0 - 0.6$.

Pressure is low, so the gas mostly traces dark matter, though pressure support becomes significant on sub-Mpc scales.

The optical depth for Lyα absorption is proportional to n_{HI}.

One can therefore view the Lyα forest as a non-linear map of the matter density along the line of sight to the quasar, with the continuum-normalized flux

$$\frac{F}{F_c} = e^{-\tau_{\rm HI}} \approx \exp\left[-(1+\delta_{\rm DM})^{2-0.7\alpha}\right].$$

In more detail, one needs to worry about peculiar velocities and smoothing by the thermal velocity of the atoms.

This *Fluctuating Gunn-Peterson Approximation* is a useful intuitive guide, and it can be used as a practical tool to paint the Lyα forest onto dark matter simulations, though one must be cautious of its approximations.

Hydro simulations of the Lyα forest are relatively straightforward (much more so than for galaxy formation), but it is hard to achieve the necessary ~ 100 kpc-scale resolution over large volumes.

1-d and 3-d measures of Lyα forest structure

Reading: R. Croft et al. 2002, ApJ 581, 20; N. Palanque-Delabrouille et al. 2015, JCAP 02, 045; T. Delubac et al. 2015, A&A 574, 59

The Lyα forest is a powerful probe of structure at $z = 2-4$, where Lyα is accessible to ground-based observations and forest absorption is not saturated.

Each well observed quasar probes dozens-to-hundreds of density fluctuations along its line of sight, depending on spectral resolution of observations.

On scales larger than ~ 1 Mpc, the power spectrum of the Lyα forest should be a biased version of the matter power spectrum.

The bias factor is < 1 because the flux is constrained to $0 < F/F_c < 1$, and the power spectrum shape is closer to the linear than the non-linear matter power spectrum because high density regions are not heavily weighted.

For galaxies, the RSD parameter $\beta = f(z)/b_g$, but for the Lyα forest it is not just $\beta_F = f(z)/b_F$ because saturation means that flux doesn't "add" in the way that galaxy or matter density does.

Nonetheless redshift-space distortions of Lyα forest clustering are very strong because the bias factor is small.

The 1-d power spectrum, computed for each line-of-sight individually and averaged over many lines of sight, is an integral over the 3-d power spectrum, $P_{1d}(k) = \int_k^\infty P(y)dy/2\pi$.

With large surveys like SDSS and BOSS, the 1-d power spectrum has been measured with exquisite precision.

Modeling at this level of precision requires hydro simulations, but the simulation volumes don't need to be enormous.

This is our best probe of the linear matter power spectrum at scales of 1 to a few Mpc.

It provides an important constraint on warm dark matter, which cuts off the Lyα $P(k)$ if the mass is less than about 2 keV (for a standard thermal relic).

It provides leverage on n_s and running of the spectral index.

It provides what is probably the tightest current constraint on neutrino mass, $\sum m_\nu < 0.15$ eV at 95% confidence, because neutrinos reduce small scale power relative to large scale power.

With 160,000 quasars over 10,000 deg^2, BOSS has a high enough density of sightlines to measure the Lyα forest in 3-d, with correlations across sightlines.

Think of it as a very oddly sampled non-linear map of the intergalactic hydrogen distribution, which traces the matter distribution.

The primary goal of the BOSS quasar survey was to measure BAO in the Lyα forest, which it has succeeded in doing, with precision of about 2%.

Structure can be measured at higher precision on scales of 10-80 Mpc. Potential applications: matter power spectrum shape and amplitude, Alcock-Paczynski test.

There is an observational challenge of removing subtle distortions imprinted by the data analysis method.

The theoretical challenge is to achieve sufficiently accurate modeling over enormous volumes. Hydro simulations are infeasible, and the Fluctuating Gunn-Peterson Approximation is probably not accurate enough.

Peirani et al. 2014 (ApJ 784, 11) present one possible route forward, based on applying conditional distributions derived from hydro simulations to large volume N-body simulations.

How accurate does theory need to be?

From the point of view of precision cosmology, the goal of modeling is to compute observables as a function of cosmological parameters accurately enough that any bias in the predictions is small compared to the statistical uncertainties in the measurements.

Theoretical uncertainties can sometimes be encoded by nuisance parameters and marginalized over in cosmological parameter fits. This typically increases the statistical error, but that is better than biasing the answer.

Ideally one would like to get theoretical priors on any such nuisance parameters tight enough that they don't degrade the cosmological parameter errors.

The requirements get tighter as surveys get bigger and measurement errors get smaller.

The level of the challenge is very different from problem to problem — e.g., BAO looks to be much easier than RSD or the Lyα forest power spectrum.