LECTURE 22: GENERAL RELATIVITY (GR)

22.1 EINSTEIN VS. NEWTON

Newtonian gravity:
Matter tells gravity how to exert force.
Force tells matter how to accelerate.

For a body of mass m accelerated by a body of mass M:

$$\begin{aligned} \text{force} &= GMm/R^2 && (1) \\ \text{acceleration} &= \text{force}/m && (2) \\ \Longrightarrow \text{acceleration} &= GM/R^2. && (3) \end{aligned}$$

Thus, the acceleration does not depend on m.

Why was Einstein dissatisfied with this description?

- Equation (1) implies instantaneous knowledge of R, which can only exist if information propagates faster than light.
- The appearance of the *same* mass m in equations (1) and (2), and hence the disappearance of m from the final acceleration, seems like a coincidence.
- He knew that generalizing special relativity to freely falling observers must lead him to a new theory of gravity.

Einstein invented a new theory of gravity for theoretical reasons, not because of known empirical failures of the existing theory. Scientific progress rarely occurs this way.

22.2 CURVATURE AND SHORTEST PATHS

On a flat surface, the shortest path between two points is a straight line. Parallel lines do not converge or diverge.

On a curved surface, there are no "straight" lines, but there are still shortest paths. Shortest paths that start parallel can converge or diverge further along.

On a sphere, the shortest path between two points is a *great circle*, a circle whose center is the center of the sphere. For example, circles of constant longitude are great circles.

Two lines of longitude that start at the north pole first diverge, then become parallel at the equator, then converge again at the south pole.

One can also have curved 3-dimensional spaces, or even a curved 4-dimensional spacetime, in which shortest paths converge or diverge.

22.3 EINSTEIN'S THEORY OF GRAVITY (1915)

Mass curves spacetime (like the curvature of a sphere or a saddle).

Freely falling objects follow shortest paths in curved spacetime.

In flat spacetime (no gravity, no curvature), free objects move in straight lines.

Summary:
Matter tells spacetime how to curve.
Curved spacetime tells matter how to move.

The earth orbits the sun because it is moving "straight ahead" in a spacetime curved by the sun's gravity.

22.4 TESTING GR: 1. THE ORBIT OF MERCURY

Planets in the solar system move in ellipses around the sun, as predicted by Newtonian gravity.

When speeds are much smaller than the speed of light, the predictions of GR are nearly identical to those of Newtonian gravity $\Longrightarrow$ GR also predicts (nearly) elliptical orbits.

How could Einstein, in 1915, know that he was right?

The orbit of Mercury is not a perfect ellipse. The direction of the long axis shifts very slowly, at 570 arc-seconds per century.

Most of this shift is caused by the gravitational pull of other planets (e.g. Venus), but 43 arc-seconds/century remained unexplained as of 1915.

Einstein used GR to calculate Mercury's orbit and found that it exactly predicts the missing 43 arc-seconds/century.

Einstein's comment: "For a few days I was beside myself with joyous excitement."

22.5 TESTING GR: 2. BENDING OF LIGHT

Light also travels on shortest paths in curved spacetime $\Longrightarrow$ GR predicts that gravity bends light.

The predicted bending of the light of a background star by the sun's gravity was first confirmed during the 1919 solar eclipse.

Today we also see *gravitational lenses* — double (or triple, quadruple, etc.) images of quasars produced by light bending.

Gravitational lensing can also distort distant galaxies into rings and arcs.

22.6 TESTING GR: 3. THE BINARY PULSAR

Electromagnetism predicts: accelerating charges give off electromagnetic radiation (radio waves, light, X-rays)

GR predicts: accelerating masses give off gravitational radiation. This is a fundamentally non-Newtonian effect, but it is very weak because gravity is a very weak force.

1975: Hulse & Taylor discover a binary pulsar, a pair of orbiting neutron stars.

Precise pulse timing allows accurate measurement of the binary orbit, which has a period of 8 hours.

The long axis of the ellipse shifts like that of Mercury's orbit, but much faster — 4 degrees per year.

GR predicts that orbiting neutron stars give off gravitational radiation, so stars must move together to conserve energy.

Observations agree precisely! Spectacular confirmation of GR.

1994: Hulse & Taylor win Nobel prize for physics.

22.7 THE STATUS OF NEWTONIAN GRAVITY

Newton's equations can be derived as approximations to GR, which are accurate when velocities are much smaller than the speed of light.

In this regime (earth, solar system, stars, galaxies), physicists usually *think* in Newtonian terms.

GR is quantitatively different and it predicts qualitatively new phenomena. Theoretical cosmology relies heavily on GR.

GR also has a quite different conceptual basis from Newtonian gravity — curved spacetime instead of forces and accelerations.

Is Newtonian gravity "right" or "wrong"? The answer depends on your notion of how science advances and what a scientific theory is supposed to do.

22.8 SUMMARY

GR: generalizes special relativity to freely falling observers.

Freely falling objects follow shortest paths in curved spacetime.

Matter tells spacetime how to curve.
Spacetime tells matter how to move.

Empirical confirmations of GR:
  orbit of Mercury
  bending of light
  binary pulsar ($\longrightarrow$ gravitational radiation)