LECTURE 4 (1/13): BINARY STARS –
STELLAR MASSES AND RADII

4.1 DETECTING BINARIES

Pairs of stars that are physically distant from each other can appear close on the sky because of projection. However, it's estimated that between 20% and 80% of stars are in *true binaries*, i.e. they are bound to each other by their gravitational attraction.

In *visual binaries* we see (with a telescope) both stars separately, and their paths in the sky show that they are orbiting around each other.

In *astrometric binaries* we see one star's position wobble periodically in response to the gravity of its unseen companion.

In *spectroscopic binaries* we detect the motion of one or both stars from Doppler shifts of their spectral lines.

In *eclipsing binaries* we see the total light from the pair of stars change periodically as one star eclipses the other.

4.2 VISUAL BINARIES

Two gravitationally bound stars will follow elliptical orbits around their center of mass; the center of mass may have a proper motion as well, but this can be removed.

By comparing the displacements of the two stars, we can learn the ratio of their masses, $M_1/M_2 = a_2/a_1$.

Newton's law of gravity ($F = GM_1M_2/R^2$) and Newton's second law of motion ($F = MA$) can be combined to obtain a generalized form of Kepler's third law (Seeds, p. 87, p. 189):

$$P^2 = \frac{4\pi^2}{G} \times \frac{a^3}{(M_1 + M_2)}.$$

P = period
G = Newton's gravitational constant
a = radius (semi-major axis) of orbit
M_1, M_2 = masses of stars

The period P can be determined from the orbit. *If* we know the distance to the binary, we can measure a and apply this equation to determine the sum of the masses $M_1 + M_2$. Since we also know M_1/M_2, we can determine the masses individually.

Bottom line: With a visual binary, we can determine the masses if (and only if) we can measure the orbits accurately and we know the distance.

4.3 SPECTROSCOPIC BINARIES

In most binaries, the two stars are too close together for us to see them separately. If we can measure Doppler shifts in the spectrum, we can still measure orbital motions of the stars.

A binary that is detected by Doppler shifts is called a *spectroscopic binary.*

We would like to apply the law of gravity and law of motion to determine the masses of the stars.
Problem: Doppler shifts only tell us the motion relative to the earth, and we don't know the *inclination* of the orbit, i.e. if it is close to edge-on or close to face-on.

4.4 ECLIPSING BINARIES

In an *eclipsing binary*, the brightness declines periodically as one star passes in front of the other. (See figure 10-17).

For an eclipse to occur, the orbit of the binary must be nearly edge-on. This solves the "problem" mentioned before. With good observations, we can determine the masses of both stars in an eclipsing binary, *without* knowing the distance.

From the shape and duration of the eclipses, we can find the radii of the stars.

4.5 THE MASS-LUMINOSITY RELATION AND THE MASS-RADIUS RELATION

After observing many visual binaries and eclipsing binaries, we can determine masses and radii for a large sample of stars (roughly 100). Then we can plot:

luminosity vs. mass – the mass-luminosity relation

radius vs. mass – the mass-radius relation

For *main sequence* stars, the luminosity and the radius both correlate well with the mass.

The luminosity is very sensitive to the mass, roughly $L \propto M^{3.5}$.

The radius is less sensitive to the mass. Roughly $R \propto M$ for low-mass ($M < 2M_\odot$) stars and $R \propto \sqrt{M}$ for high-mass ($M > 2M_\odot$) stars.

(Note: $M_\odot$ means the mass of the sun.)

Sensible results for the mass-luminosity and mass-radius relation for binary stars support our underlying assumption: that the laws of gravity, atomic physics, and light travel that apply in the solar system also apply to other stars.

The sun fits naturally onto the $M - L$ and $M - R$ relation derived for other stars. This continuity verifies the assumption that the sun is a "typical" star.

4.6 SUMMARY

Between 20% and 80% of stars are in true, gravitationally bound binaries. Binaries can be detected by positional measurements or by spectroscopic observations of Doppler shifts. Binaries can have many wonderful and weird properties, most of which we will ignore.

Binaries play an essential role in the study of stars because, in special cases, we can apply Newton's laws of gravity and motion to learn the masses of the stars.

We can determine the masses in a visual binary if we know its distance.

In an eclipsing, spectroscopic binary, we can determine the stellar masses and stellar radii without knowing the distance.

The existence of a mass-luminosity relation and a mass-radius relation show that the luminosity and radius of a main sequence star depend primarily on its mass. Approximately:
$L \propto M^{3.5}$
$R \propto M$ (low-mass stars)
$R \propto \sqrt{M}$ (high-mass stars)