LECTURE 5 (1/15): THE HERTZSPRUNG-RUSSELL DIAGRAM

IMPORTANT EQUATIONS

3: THE LUMINOSITY OF A SPHERICAL BLACKBODY

In symbols:

$$L = (4\pi\sigma)R^2T^4.$$

L = luminosity of a spherical blackbody [ergs/second]
R = radius of the body [centimeters]
T = temperature of the body (degrees Kelvin)
σ = the Stefan-Boltzmann constant

In words: The luminosity of a spherical blackbody is proportional to the square of its radius and to the fourth power of its temperature. The constant of proportionality is $4\pi\sigma$.

Note: A star is not a perfect blackbody, but it is close. Astronomers define the *effective temperature* of a star, T_e, by the equation

$$L_* = (4\pi\sigma)R_*^2T_e^4.$$

Examples:

(1) Star A and star B have the same radius, but star B is twice as hot. How much more luminous is star B?

The luminosity is proportional to T^4, so star B is $2^4 = 16$ times more luminous.

More formally,

$$\frac{L_B}{L_A} = \frac{4\pi\sigma R_B^2T_B^4}{4\pi\sigma R_A^2T_A^4} = \frac{T_B^4}{T_A^4} = 16.$$

(2) Two stars have the same spectral type, and they have the same apparent brightness (flux). However, star A has a parallax of 1", and star B has a parallax of 0.1". How big is star B relative to star A?

We have to pull together several different equations here. The distance to a star is $d = 1/p$, where p is the parallax, so star B is 10 times more distant than star A. The apparent flux of a star is $f = L/(4\pi d^2)$, so if the two stars have the same apparent flux, star B must be 100 times more luminous. Since the two stars have the same spectral type, they are the same temperature. But $L \propto R^2T^4$, so if T is the same and star B is 100 times more luminous, it must be ten times bigger than star A.

5.1 THE HERTZSPRUNG-RUSSELL DIAGRAM

The mass-luminosity and mass-radius relations can tell us a lot about stars, but they can only be plotted for the 100 or so stars with accurately measured masses.

We can estimate the temperature of a star from its spectral type or from its color. We can determine the luminosity of a star if we know its apparent flux and its distance.

The blackbody formula $L = 4\pi\sigma R^2 T^4$ suggests that we plot luminosity against spectral type, color, or inferred temperature. This plot is called the *Hertzsprung-Russell diagram* or HR diagram. It is the most important single tool in understanding the population of stars.

5.2 POPULATIONS IN THE HR DIAGRAM

In the HR diagram, most stars lie along a *main sequence* running from cool, faint stars to hot, luminous stars. The location of a main sequence star in the HR diagram depends primarily on its mass. More massive stars are hotter (bluer) and more luminous.

Above the main sequence are *red giant* stars. These are usually cool (spectral class K or M), but they are bright because they are very big. They have narrow absorption lines.

A red giant may be brighter than a more massive star and may be cooler than a less massive star.

Below the main sequence are *white dwarf* stars. These can be quite hot, but they are faint because they are very small. They have broad absorption lines.

We will learn that stars on the main sequence are burning hydrogen into helium in their cores. Red giants and white dwarfs have radically different structure from main sequence stars.

5.3 THE HR DIAGRAM OF STAR CLUSTERS

Normally, a plot of apparent flux against color does not look like an HR diagram, since the apparent flux depends on the distance of a star as well as its luminosity.

However, when we look at a star cluster, we observe stars that are all at (approximately) the same distance from us. The apparent flux is therefore proportional to the luminosity, with an unknown constant $1/4\pi d^2$. Thus, a plot of apparent flux against color provides an HR diagram of the cluster.

The HR diagrams of typical open clusters (loose, 10's or 100's of stars) have blue stars on the main sequence and relatively few red giants.

The HR diagrams of globular clusters (compact, $100,000 - 1,000,000$ stars) do *not* have blue main sequence stars. They have prominent red giant branches, and a *horizontal branch* of bright, blue stars.

Some open clusters have HR diagrams that roughly resemble those of globular clusters.

One goal of the theory of stars is to explain these differences.

5.4 SUMMARY

The Hertzsprung-Russell (HR) diagram is a plot of stellar luminosity against an indicator of stellar surface temperature (color or spectral type). It is motivated by the blackbody luminosity formula $L = (4\pi\sigma)R^2T^4$.

From the HR diagram of nearby stars, we learn of the existence of a main sequence, red giants, and white dwarfs. The position of a main sequence star on the HR diagram depends mainly on its mass.

For a cluster of stars at a common distance, we can plot an HR diagram of *apparent* brightness against color or spectral type. The HR diagrams of open clusters and globular clusters look quite different from each other.

Mass-luminosity, mass-radius, and HR diagrams look "sensible," and the sun fits smoothly onto the relations derived for other stars. This boosts our confidence in the assumption used to infer the properties of stars from observations — that the fundamental physical laws that apply on earth also apply to astronomical objects.