LECTURE 7 (1/22): PRINCIPLES OF STELLAR STRUCTURE
II. LEAKING PHOTONS AND STELLAR LUMINOSITIES

7.1 BASIC MODEL OF A STAR

At the most basic level, our theory of a star is that it is a ball of gas held together by gravity. We will see that this theory, suitably extended, can explain the observed properties of the sun and other stars.

In this picture, a star is entirely gaseous and has no solid surface. However, there is a radius at which the gas density drops rapidly to zero, and we can reasonably call this radius the "surface" of the star (think of the ocean).

Stars are nearly opaque. We only see light escaping from near the surface; we cannot see directly to the interior.

7.2 ENERGY IN A STAR

The center of a star is hot, so it contains a great deal of *heat energy.* For the star to be luminous, this energy must get out.

Most of this energy is held by the atoms, which are moving rapidly and bumping into each other. However, the atoms are trapped in place by other atoms above them, so this energy cannot move out to the star's surface.

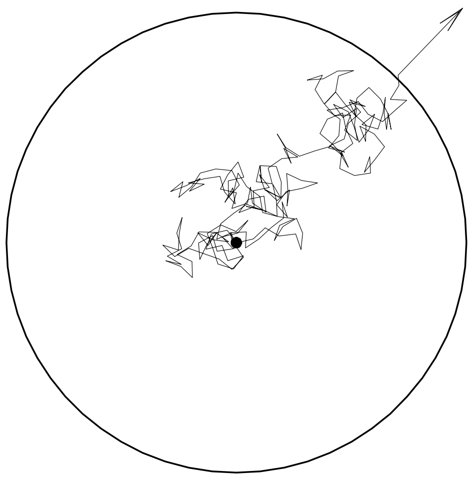
Some of the energy is held by the photons, which can carry energy to the surface.

The supply of photons in the center of the star is continually replenished.

7.3 RANDOM WALKING PHOTONS

At the speed of light, it would take 2.3 seconds for a photon to go straight from the center of the sun to its surface.

But a photon in the sun can go only a short distance (about 1 centimeter) before running into an atom. It then changes direction randomly.



A path with random changes of direction is called a random walk. If each step in a random walk is length l, then it takes, on average, $(d/l)^2$ steps to go a distance d from the starting point.

[A "direct walk" would take only d/l steps.]

On average, it takes a photon 30,000 years to random walk from the center of the sun to the surface of the sun.

A complication: When a photon changes direction, it can also divide. During the walk from the center of the sun, a single gamma-ray photon splits into a large number of visual photons.

Another complication: If random-walking becomes too slow, a star will transport energy by convection instead of random-walking photons. Convection is important in some parts of some stars (for example, the outer 30% of the sun).

7.4 LUMINOSITY OF A STAR

To calculate the luminosity of a star (theoretically), we divide the amount of energy stored in the photons by the average time required for a photon to escape.

The luminosity of a star is thus controlled by the central temperature and by the rate at which photons can "leak out" of the star.

7.5 THEORY OF STELLAR LUMINOSITY

Given the mass M and radius R of a star, one can estimate the luminosity with the following steps:

(1) Require that pressure balance gravity to get the central pressure.

(2) Use the equation for pressure of an ideal gas to get the central temperature.

[(1) and (2) together yield equation # 5, $T_c = GMm_a/kR_{\text{avg}}$.]

(3) From the central temperature, determine the energy stored in the photons.

(4) From the number of atoms and the volume of the star, find the average density of atoms. Then find the typical distance l that a photon travels before colliding with an atom and changing direction.

(5) From l, R, and the speed of light, find the time needed for photons to escape from the center of the star.

(6) Divide the energy of the photons by the time required for escape to get the luminosity.

We still have to learn what determines the radius of a star.

7.6 AGE OF THE SUN

If the sun produced its luminosity by chemical burning, it could last about 2000 years.

In fact, the center of the sun contains an enormous reservoir of heat energy because it is very hot.

This energy is transferred from the atoms to the photons, and the photons carry it off into space as they "leak out."

If we divide the heat energy of the sun's atoms by the observed luminosity of the sun (the rate at which this energy escapes), we find that the sun's energy supply would be exhausted in about 30 million years.

The sun can replace some of this energy by gravitational contraction (which increases the central pressure and therefore heats up the gas in the middle), but this process can only increase the lifetime by about a factor of two.

19th century physicists estimated the sun's age in this way; their estimate conflicted with geological evidence for an age of the earth exceeding 1 billion years.

Physicists' answer: geologists were wrong.
Actual answer: physicists were wrong.

7.7 SUMMARY

Basic model: a star is a ball of gas held together by gravity.

A star's heat energy is held partly by the atoms and partly by the photons.

The atoms cannot move far, so they cannot transport energy. Photons can transport energy, but rather slowly because they must random walk from the center to the surface.

An average photon takes 30,000 years to get from the center of the sun to the surface.

The luminosity of a star can be found by dividing the energy of the photons by the time required for them to "leak out" of the star.

Given the mass and radius of a star, we can compute the luminosity using the principles of hydrostatic equilibrium and "leaking photons."

With its stored heat energy, the sun could produce its luminosity for about 30 million years, but geological evidence indicates that the earth is much older (about 4.5 billion years). Our picture is still missing a key ingredient.