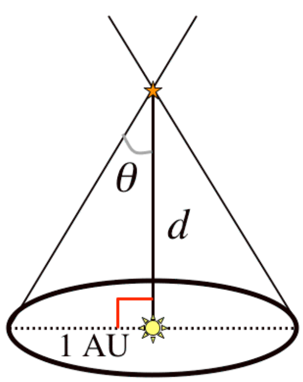
Astronomy 141 – Life in the Universe
Autumn Quarter 2008 – Prof. Gaudi

Homework #1 Solutions

Question 1 (40 points total)

Part 1 (20 points). One reason that the ancient Greeks didn't believe the heliocentric model is that it predicted that nearby stars should move relative to more distant stars due to the changing perspective arising from the motion of the Earth around the Sun. In other words, stars should exhibit annual *parallax*. Since they didn't observe these parallax motions, they reasoned the Earth must be stationary. Given that we now know that the Earth does indeed orbit the Sun, how far away must the nearest star be in order that the ancient Greek scholars failed to detect its parallax? *(Remember that the ancient Greeks did not have telescopes, so all of their observations were done with the naked eye.)*

First we need to figure out how the magnitude of the annular parallax of a star depends on its distance. This is easily derived from simple geometry:



$$\tan\theta = \frac{1\mathrm{AU}}{d}$$

We're dealing with very small angles, so

$$\tan\theta \approx \sin\theta \approx \theta$$

And so,

$$\theta = \frac{1\mathrm{AU}}{d}$$

Since the Earth moves 2 AU over the course of its orbit, the total annual parallax will be twice this angle. The second thing we need to know is how well the human eye can resolve angles. By poking around on the internet, you can find that the human eye can resolve angles of about 1 arcminute. According to the formula above, the distance at which a star's annual parallax is less than one arcminute is d=2 × 1 AU/1 arcminute, or $d = 2 \times 1.496 \times 10^{11}\text{m} \times 60 \times 180 / \pi = 1.029 \times 10^{15}$ m. Remember you have to convert from arcminutes to radians, where 1 arcminute / 180 / 60 × π = 1 radian. You can also have Google do this for you by typing "2*AU/arcminute=". One parsec is 3.086×10^{16} m, so the distance to the star is about 0.033 pc, or 1/30 of a parsec. That this value is 1/30 is not a coincidence: the definition of a parsec is the distance at which a star's parallax as it moves 1 AU would be one arcsecond. Since there are 60 arcseconds in an arcminute, the distance at which a star's parallax would be one arcminute as it moves 2 AU is just 2 divided by 60.

Answer: b) 1/30 parsec

Part 2 (10 points). How far from the Sun is the nearest star?

I said this in class, and it can be easily looked up on the net. The nearest star to the Sun is Proxima Centauri. Its distance can also be found on the net, for example go to Google and type "distance to Proxima Centauri". It will return 4.22 ly or 1.295 pc. You can convert this to AU using Google as well, type "1.295 pc/AU=", and it will return 267112.924. Converting to scientific notation, this gives the final answer of about 2.7×10^5 AU.

Answer: e) about 2.7×10^5 AU

Part 3 (10 points). How large is its annual parallax?

The distance to a star in parsecs is just one over its parallax in arcseconds, so in this case about 0.77 arcseconds.

Answer: c) about 0.77 arcseconds

We conclude from this that even the nearest star is so far away that its parallax is hopelessly undetectable with the human eye. In fact, the first stellar parallax wasn't measured until 1838!

Question 2 (20 points)

How do you express the numbers 535.25 and 0.000663 in scientific notation?

To convert to scientific notation, we want the number to be in the form of X.XXX $\times 10^{-X}$. In the first case, the significant digits in the number are "53525", so the first part is 5.3525. The number of decimal places we need gives the power of ten: this is 2, and the number is greater than one, so it must be positive. Therefore, we have $535.25 = 5.3525 \times 10^{2}$. Similarly, $0.000663 = 6.63 \times 10^{-4}$. So,

Answer: a) 5.3525×10^{2}; 6.63×10^{-4}

Question 3 (20 points)

A new planet is found in the outer solar system that has a distance from the Sun of 100 AU. Astronomers decide to call this planet Ernie. If you visited Ernie, what would be the apparent brightness of the Sun relative to its apparent brightness as seen from the Earth?

As one goes further away from the Sun, its luminosity is spread out over a larger and larger surface area. We can picture an imaginary sphere of radius d centered on the Sun, upon which the Sun's light is incident. The surface area of this sphere is $4\pi d^2$. The brightness B is the amount of light from the Sun that hits this sphere in a given unit area,

$$B = \frac{L}{4\pi d^2}$$

so the ratio of the brightness at the Earth to that at the distance of Fred is just the inverse square of the distances:

$$\frac{B_{Ernie}}{B_{Earth}} = \left(\frac{d_{Earth}}{d_{Ernie}}\right)^2 .$$

Here I've used a ratio in order to cancel out the Luminosity of the Sun ("L") and the "4π", both of which will be the same for Ernie and the Earth. So the ratio of brightness is $(100)^{-2}=1/(10{,}000)$.

Answer: b) Ten thousand times fainter.

In other words, Ernie (or any planet that far away from the Sun) would be a pretty dark and cold place!

Question 4 (20 points)

During one of your archaeological expeditions in Egypt, you find what appears to be a well-preserved mummy, who you decide to name Bert. By analyzing Bert's wraps, you discover that only 1/4 of the amount of original Carbon-14 in the cloth remains. How old is Bert? *(Assume the half-life of carbon-14 is 5,700 years.)*

The half-life is defined at the amount of time during which half of the original isotopic decays. Therefore, after four half lives, a fraction

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

remains. Thus Bert is 2 half lives old, or $2 \times 5700 = 11{,}400$ years old.

Answer: c) 11,400 years.