Astronomy 141 – Life in the Universe
Winter Quarter 2008 – Prof. Gaudi
Homework #3 Solutions

These questions concern the concept of the *habitable zone*, including the habitable zone for our Sun today, how the habitable zone of the Sun changes as the Sun's luminosity changes, and the concept of a *continuously habitable zone*. The extra credit questions deal with habitable zones around other stars.

For these questions, we will adopt the *conservative* definition of the habitable zone, which for the Sun with its current luminosity has an inner edge of 0.95AU and an outer edge of 1.4 AU.

Question 1 (10 points)

What is current width of the Sun's habitable zone?

a) 1.40 AU
b) 0.95 AU
c) 0.86 AU
d) 0.45 AU
e) 1 AU

This is an easy one: the current width of the habitable zone is just the outer edge minus the inner edge, i.e. 1.4 AU – 0.95 AU = 0.45 AU

Answer d) 0.45 AU

Question 2 (20 points)

When the solar system formed, the Sun's luminosity was only 70% of its current luminosity. Assuming that the inner and outer edges of the habitable zone are dictated only by the equilibrium temperatures at those distances, what was the width of the habitable zone when the solar system was formed? (*Hint: Assume that the equilibrium temperature at the inner edge of the habitable zone is constant. Then, you can write a scaling relation between the distance of the current inner edge of the habitable zone to the inner edge of the habitable zone when the solar system formed in terms of only the ratio of the*

luminosity of the Sun then versus the luminosity of the Sun today to some power. You can write a similar relation for the outer edge of the habitable zone.)

a) 0.45AU
b) 0.72 AU
c) 0.38 AU
d) 1.2 AU
e) 0.86 AU

The equation for the equilibrium temperature is,

$$T = \left(\frac{L}{16\pi\sigma}\right)^{1/4} d^{-1/2}.$$

We can solve for distance,

$$d = \left(\frac{L}{16\pi\sigma}\right)^{1/2} T^{-2}$$

So at fixed temperature, the distance scales with luminosity as, $d \propto L^{1/2}$. Therefore we can write the scaling relation for the distance d of a fixed equilibrium temperature when the luminosity is a given value L, relative to the distance d_{now} and luminosity L_{now} now as,

$$\frac{d}{d_{Now}} = \left(\frac{L}{L_{Now}}\right)^{1/2}.$$

In the case of the outer edge, we have d_{now}=1.4 AU, and the Sun's past luminosity is L=0.7 L_{now}. Thus,

$$d = 1.4\,\text{AU}\left(\frac{0.7L_{Now}}{L_{Now}}\right)^{1/2} = 1.4\,\text{AU}\left(0.7\right)^{1/2} = 1.17\,\text{AU}$$

Similarly, for the inner edge we can find,

$$d = 0.95\,\text{AU}\left(\frac{0.7L_{Now}}{L_{Now}}\right)^{1/2} = 0.95\,\text{AU}\left(0.7\right)^{1/2} = 0.79\,\text{AU},$$

The width is then 1.17 AU – 0.79 AU = 0.38 AU.

Answer c) 0.38 AU

Question 3 (20 points)

What is the width of the continuously habitable zone, defined as the range of distances that have been in the habitable zone during the entire evolution of the solar system, from formation until the present? (*Hint: here it may help to draw a diagram or refer to the diagram in the book*)

a) 0.22 AU
b) 0.45 AU
c) 0.1 AU
d) 0.01 AU
e) 1 AU

The range of distances where a planet will always be in the habitable zone throughout the Sun's evolution until today is just given by the *minimum* value that the outer edge of the habitable zone took anytime between the formation of the solar system and today, minus the *maximum* value that the inner edge of habitable zone took anytime between formation and today. This is then: minimum(1.4 AU, 1.17 AU) – maximum (0.95 AU, 0.79 AU) = 1.17AU – 0.95 AU = 0.22 AU.

There is a hidden assumption here that the evolution of the Sun's luminosity was *monotonic*, in other words that it has always been steadily increasing. This is, in fact, true.

Answer a) 0.22 AU

Question 4 (20 points)

The future evolution of the Sun's luminosity is approximately given the following expression:

$$\frac{L_{Sun,\,future}}{L_{Sun,now}} = \left(\frac{\text{Time}}{4.5\ \text{Gyr}}\right)^{1/2}.$$

Where "$L_{\text{Sun,future}}$" is the luminosity of the Sun in the future, "$L_{\text{Sun,now}}$" is the current luminosity of the Sun, and "Time" is the time since the beginning of the solar system. For example, the current time since the beginning of the solar system is 4.5 Gyr, so by setting Time=4.5 Gyr in the above equation, we find from the formula that $L_{\text{Sun,future}} = L_{\text{Sun,now}}$,

as we should. What will be the luminosity of the Sun 3.5 Gyr from now? (*Hint: Remember the Sun's luminosity is increasing with time!*)

a) 88% of the current luminosity of the Sun
b) 133% of the current luminosity of the Sun
c) 515% of the current luminosity of the Sun
d) 22% of the current luminosity of the Sun
e) 149% of the current luminosity of the Sun

Another easy one: 3.5 Gyr from today, the time since the beginning of the solar system will be 3.5 Gyr + 4.5 Gyr = 8 Gyr. Using the formula above, the Sun's luminosity will be:

$$L_{Sun,future} = L_{Sun,Now}\left(\frac{8\ \text{Gyr}}{4.5\ \text{Gyr}}\right)^{1/2} = 1.33L_{Sun,now}.$$

Answer b) 133% of the current luminosity of the Sun

Question 5 (20 points)

What will be the width of the habitable zone 5.5 Gyr from now?

a) 1.05AU
b) 0.01AU
c) 1.23 AU
d) 3.51 AU
e) 0.55 AU

First, we need to compute the luminosity of the Sun 5.5 Gyr from now, in the same way as we computed the luminosity 3.5 Gyr from now in the previous question. Following through the same steps, the luminosity of the Sun 5.5 Gyr from now is 1.49 times the luminosity today. Then, we can compute the inner and outer edge of the habitable zone just as we did in Question #2. For the outer edge:

$$d = 1.4\,\text{AU}\left(\frac{1.49L_{Now}}{L_{Now}}\right)^{1/2} = 1.4\,\text{AU}\left(1.49\right)^{1/2} = 1.71\text{AU}.$$

For the inner edge:

$$d = 0.95\,\text{AU}\left(\frac{1.49L_{Now}}{L_{Now}}\right)^{1/2} = 0.95\,\text{AU}\left(1.49\right)^{1/2} = 1.16\text{AU}.$$

For a width of 1.71 AU – 1.16 AU = 0.55 AU.

Answer e) 0.55 AU

Question 6 (10 points)

Will the Earth be in the habitable zone 5.5 Gyr from now?

a) Yes
b) No

Since, as we just calculated, the inner edge of the habitable zone will be outside the current semimajor axis of the Earth, the Earth will not be in the habitable zone 5.5 Gyr from now.

Answer b) No

Extra Credit Questions:

Question 7 (20 points)

An A-type main sequence star has a luminosity that is 20 times larger than the Sun's luminosity. What is the width of the habitable zone around such a star?

a) 0.45 AU
b) 0.01AU
c) 2.01 AU
d) 3.84 AU
e) 450 AU

This is similar to Questions #2 and #5, except that instead of evaluating the change in the width of the habitable zone due to a change in Sun's luminosity, we are evaluating the width around a star of a different luminosity. However, the same equations apply. We can actually make our lives a bit simpler by using a scaling relation for the width of the habitable zone directly. Since both the inner and outer edge scale in the same way with luminosity, the width does as well:

$$\frac{width}{width_{Sun}} = \left(\frac{L_{Star}}{L_{Sun}}\right)^{1/2}.$$

So, for an A-type star, we have that,

$$width = width_{Sun}\left(\frac{L_{Star}}{L_{Sun}}\right)^{1/2} = 0.45\,\mathrm{AU}\left(\frac{20L_{Sun}}{L_{Sun}}\right)^{1/2} = 2.01\mathrm{AU}\ .$$

Answer c) 2.01 AU

Question 8 (20 points)

An M-type main sequence star has a luminosity that is only 0.3% of the Sun's luminosity. What is the width of the habitable zone around such a star?

a) 0.0014 AU
b) 0.45 AU
c) 0.25 AU
d) 0.025 AU
e) 821 AU

Using the same expression from above:

$$width = width_{Sun}\left(\frac{L_{Star}}{L_{Sun}}\right)^{1/2} = 0.45\,\mathrm{AU}\left(\frac{0.003L_{Sun}}{L_{Sun}}\right)^{1/2} = 0.025\,\mathrm{AU}\ ,$$

Answer d) 0.025 AU