Name _____

Astronomy H161 – An Introduction to Solar System Astronomy
Winter Quarter 2009 – Prof. Gaudi
Homework #3

Due Monday, February 2 in class

No late homework will be accepted.

---

1) You are transported back in time to the 4th century BC and enroll yourself in Plato's Academy where Aristotle is now the leading professor. Aristotle's student Eudoxus has just come up with his "nested spheres" theory of planetary motion. You incite Aristotle's outrage by asserting that the Sun is the center of the solar system. Aristotle threatens to expel you from the Academy unless you can prove your point. In response you make two measurements. See Figure 1.
   a) First you wait for a certain Planet X to be in opposition, exactly 180° from the Sun. Mark the position of the Earth at this time in the Figure 1.
   b) Next you wait several years until the planet has returned to exactly the same point. (This time interval, the sidereal period of Planet X, is well known from the work of the Babylonians). If the length of time were an integral number of years, the Earth would also be in the same position. However, the actual length of time is 11% short of an integral number of years, so the Earth is 11% × 360° = 39.6° short of its previous position. Mark the angle which is 39.6° in the Figure.
   c) You measure the angle between the Sun and the planet and find it to be 132.2°. Mark the angle that is 132.2° in the Figure.
   d) You "complete the triangle" in the Figure. Mark the position of Planet X.
   e) Measure the distance (in cm) from the Sun to Planet X. Measure the distance (in cm) from the Sun to the Earth. Record your results.
   f) Find the ratio of the two distances. This is the distance to Planet X in astronomical units (AU). Record your result.
   g) Use the Table 7.1 (p64) of the text to identify the name of the planet. Record your

result.

2) Aristotle is not impressed. He suspends you from the Academy. He demands that if you want to be readmitted, you must use your heliocentric theory to make a valid prediction. You tell him to wait one more sidereal period. You say that you will predict the angle between the Sun and Planet X at that time.
   a) Since the Earth was 39.6° short of a complete orbit after one sidereal period, it will be $2 \times 39.6° = 79.2°$ short after two sidereal periods. Draw a line from the Sun to the Earth at this angle.
   b) Mark the position of the Earth at this time on the diagram.
   c) Draw a line from the Earth to Planet X at this time.
   d) Measure the angle between the positions of the Sun and Planet X as seen from the Earth at this time. Record your answer. Also tell your prediction to Aristotle.

3) Your prediction is verified. Reluctantly, Aristotle agrees to readmit you to the Academy. However, he is still not convinced of your theory. He hands you a table of observations of the planet Mars and says that if you can derive the orbit of Mars from these data he will abandon his geocentric views. See Table. Column 1 of the table shows the date of each observation in days after the initial observation (when the Earth was at the Vernal Equinox – March 21) and column 3 shows the position of Mars relative to the center of the nearest Zodiac constellation. Fortunately for you, Aristotle has already computed for you the angle of Mars' position relative to a fixed point in Virgo that is overhead at midnight on the Vernal Equinox (column 4), although you could have easily done this yourself.
   a) Determine the angle of the Earth's position for each of the observations relative to its position at the first observation. You may assume that the Earth is in uniform circular motion with a period of 365.26 days. So just divide the date by 365.26 and record your answers in column 2 and then multiply by 360°. Since these are angles, all results should be in the interval (0°, 360°). Note that two entries have been filled in for you.
   b) Determine the angle between Mars and the Sun and enter this in column 5. This can be done using the formula:

(Mars–Earth–Sun Angle) = 180° + (Earth Angle) – (Mars–Earth–Virgo Angle)
(The 180° is added because the position of the Sun relative to the Earth is 180° away from the position of the Earth relative to the Sun.)
Note that to make your result useful for part (c) below, you must adjust the angle by multiples of 360° so that it falls in the interval [-180°, 180°]. Also note that two of the entries have been filled in for you.

c) Now find pairs of observations that are separated by a multiple of the orbital period of Mars (686.95 days). Consider the first such pair, the first and the 23rd observation. Since these are separated by exactly two Martian periods, Mars must be in exactly the same physical position. However, the Earth has moved from the Vernal Equinox (0.0° in column 2) to a new position (274.1° in column 2). These two positions (as well as all other positions you will need), are shown as solid dots in Figure 2. Draw a line from the Sun to the first dot (exactly horizontal to the right of the Sun). Note from column 5 that the angle between the Sun and Mars is -167.8°. Use a protractor to draw a line that is 167.8° from the Earth–Sun line in the "negative" direction. (This line has already been drawn for you as a solid line.) Mars must be somewhere along this line. Now repeat the procedure for the other observation (for which the Sun–Earth–Mars angle is +65.5°). This line has been drawn for you as a dashed line. Mars must also be somewhere along this line, so it must lie at the intersection of the two lines. Repeat this procedure to map out the entire orbit of Mars.

d) Mars is in opposition twice during these observations. What are the two constellations it is in during these oppositions? How can you tell from the table? How can you tell from the figure?

e) If one waits long enough, there will be oppositions in every constellation. Which constellation should the opposition be in so that Mars is closest to the Earth? This is called "favorable opposition". What is "favorable" about it?

f) Measure the orbital radius of the Earth $a_{\oplus}$ Measure the distance from the Sun to Mars at perihelion (its closest approach), a_{-}, and at aphelion (its farthest point), a_{+}. [These should be exactly on opposite sides of the Sun.] Record all your results to the nearest millimeter.

g) Evaluate the semi-major axis of Mars in astronomical units,

$$a_{mars} = \frac{a_+ + a_-}{2a_\oplus}$$

and the eccentricity of the Martian orbit

$$e_{mars} = \frac{a_+ - a_-}{a_+ + a_-}$$

Compare your results with those obtained by more modern methods.

h) Measure the distance that Mars travels between two observations separated by 60.88 days near perihelion, d_-, and also between two observations separated by 60.88 days near aphelion, d_+. Record your answers to the nearest millimeter. What relationship do you notice between the pairs (d_-, a_-) and (d_+, a_+)?

Aristotle congratulates you on your excellent analysis and agrees to rewrite most of his work on astronomy.